Resolving Vectors – ID: 9899

By Charles W. Eaker

Time required
30 minutes

Topic: Kinematics

- *Graphically resolve a vector into perpendicular components.*
- *Resolve a vector into perpendicular components using the sine and cosine functions.*

Activity Overview

In this activity, students observe the relationship between a vector and its x- *and* y-*components. Students determine the magnitudes and directions of the* x- *and* y-*components through direct measurement and through the use of the sine and cosine functions. Students also learn how to resolve a vector into perpendicular (but not necessarily horizontal and vertical) components.*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Notes, Graphs & Geometry

Teacher Preparation

Before carrying out this activity, review vectors, sine and cosine functions, and the Pythagorean theorem with students.

- *The screenshots on pages 2–4 demonstrate expected student results. Refer to the screenshots on pages 5 and 6 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9899" in the search box.***

Classroom Management

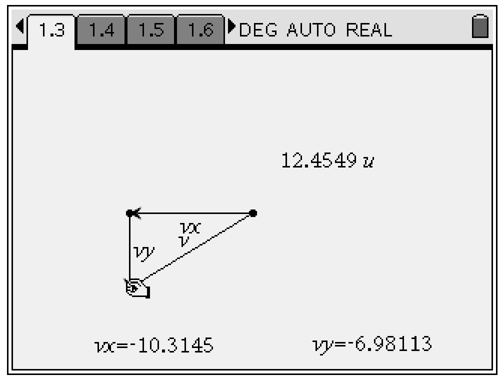
- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:

- How can a vector be resolved into *x*- and *y*-components geometrically?
- What trigonometric functions can be used to determine the components of a vector?
- Can a vector be resolved into components other than the *x*- and *y*-components?

In problem 1, students vary the magnitude and direction of a vector and observe the changes in the *x*- and *y*-components of the vector. They use the Pythagorean theorem to show the relationship between the length of the vector and the components. They also use trigonometric functions to determine the *x*- and *y*-components and use TI-Nspire graphing tools to construct the *x*- and *y*-components of a given vector. In problem 2, students are presented a mass on an inclined plane. They use TI-Nspire tools to construct the components of the gravitational force vector that are parallel and perpendicular to the inclined plane.

Problem 1 – The *x*- and *y*-components of a vector



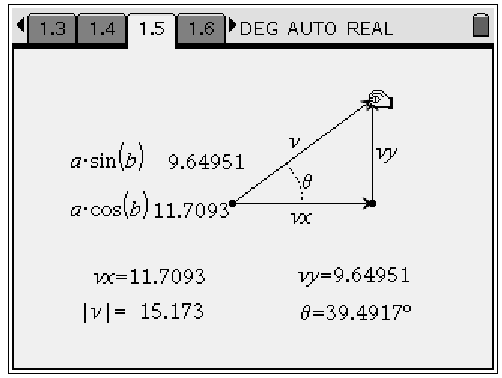
Step 1: Students should open the file **PhyAct_9899_ResolvingVectors.tns**. After reading the first two pages, students should move to page 1.3, which shows a vector (*v*) and its *x*- and *y*-components (*vx* and *vy,* respectively). Students should use the **Length Measurement** tool (**Menu** > **Measurement** > **Length**) to measure the length of vector *v*. They should grab the open point at the end of vector *v* and vary the magnitude and direction of the vector. Then, they should answer questions 1 and 2.

Q1. What is the relationship between the length of vector *v* and the lengths of components *vx* and *vy*?

A. *The square of the length of* v *is equal to the sum of the squares of the lengths of* vx *and* vy *(*$|v^2| = |vx^2| + |vy^2|$*). If students struggle with this, point out that* v, vx, *and* vy *form a right triangle, with* v *as the hypotenuse. Remind them of the equation relating the length of the hypotenuse to the lengths of the two sides of a right triangle (the Pythagorean theorem). Encourage students to test this relationship by varying the direction and magnitude of* v *and observing the results.*

Q2. What is the relationship between the direction of vector *v* and the signs of *vx* and *vy*?

A. *When* v *points from left to right,* vx *is positive;* vx *is negative when* v *points from right to left. When* v *points up,* vy *is positive; when* v *points down,* vy *is negative.*



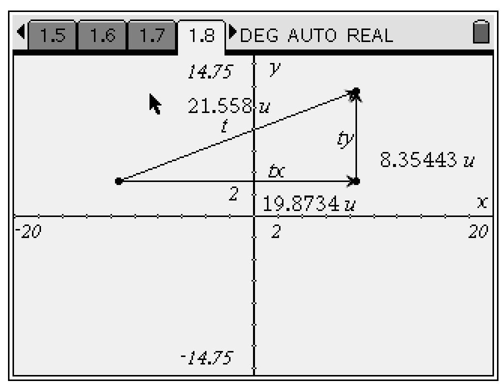
Step 2: Next, students should move to page 1.5, which again shows vectors *v, vx,* and *vy*. This simulation also shows the length of vector *v* and the angle (*θ*) between *v* and *vx*. Students should use this simulation to determine the relationship between the magnitude of vector *v* and the magnitudes of its *x*- and *y*-components using trigonometric functions. Students should use the **Text** tool (**Menu > Actions > Text**) to enter the expressions *a · cos b* and *a · sin b* in text boxes on the page. They should then use the **Calculate** tool (**Menu > Actions > Calculate**) to determine the values of these two expressions for the vectors shown on the screen. (They should use the magnitude of *v* for *a* and the value of *θ* for *b* in each expression.) Then, they should answer question 3.

Q3. What are the relationships between the magnitude of vector *v,* angle *θ,* and the magnitudes of components *vx* and *vy*?

A. *Students should compare the values calculated for the two expressions with the lengths of* vx *and* vy*. The correct relationships are given below:*

$$|vx| = |v| \cos\theta$$

$$|vy| = |v| \sin\theta$$



Step 3: Next, students should read page 1.7 before moving to page 1.8, which contains a *Graphs & Geometry* page with vector *t* on it. Students should construct the *x*- and *y*-components of vector *t* (*tx* and *ty,* respectively) on this page. Students should use the **Parallel** tool (**Menu > Construction >Parallel**) to construct lines parallel to the *x*- and *y*-axes that pass through the endpoints of the vector.

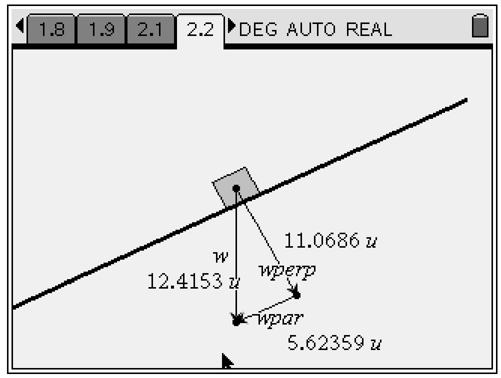
Step 4: Then, they should use the **Intersection Point** tool (**Menu > Points & Lines > Intersection Point(s)**) to find the intersection of the two lines they constructed in step 3. Students can now use the **Vector** tool (**Menu > Points & Lines > Vector**) to construct the vectors *tx* and *ty*. The **Length** tool gives the magnitudes of *t*, *tx*, and *ty*.

Q4. Determine the length of vector *t* and the magnitudes of *tx* and *ty*.

A. *Students should use the* ***Length Measurement*** *tool to measure the lengths of* t, tx, *and* ty*. For this simulation,* $|t| = 21.558$ *,* $|tx| = 19.873$ *, and* $|ty| = 8.354$ *. If you wish and time allows, you may have students confirm that the relationships they derived in steps 1 and 2 hold for this situation.*

Problem 2 – Parallel and perpendicular vector components

Step 1: Next, students should read page 2.1 and then move to page 2.2. Page 2.2 shows a box resting on an inclined plane. The weight of the box is represented by vector *w*. Students should use the **Parallel**, **Perpendicular**, **Intersection Point(s)**, and **Vector** tools to construct the components of *w* that are parallel and perpendicular to the inclined plane (*wpar* and *wperp*, respectively). Then, they should answer questions 5–7.



Q5. Determine the lengths of vectors *w, wpar,* and *wperp*.

A. *Students should use the* ***Length Measurement*** *tool to measure the lengths of* w, wpar, *and* wperp*. For this simulation,* $|w| = 12.145$ *,* $|wpar| = 5.624$ *, and* $|wperp| = 11.069$ *.*

Q6. Do these vectors obey the relationships you derived in problem 1? Explain your answer.

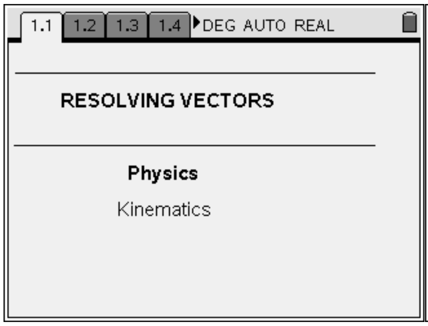
A. *The vectors should obey the same relationships (i.e.,* $|w^2| = |wpar^2| + |wperp^2|$ *;* $|wpar| = |w| \cos \theta;$ *and* $|wperp| = |w| \sin \theta$ *). If you wish and time allows, you may have students carry out further constructions and calculations to verify these relationships. It may also be useful for students to determine the relationship between* wpar *and the angle of the incline (relative to the horizontal).*

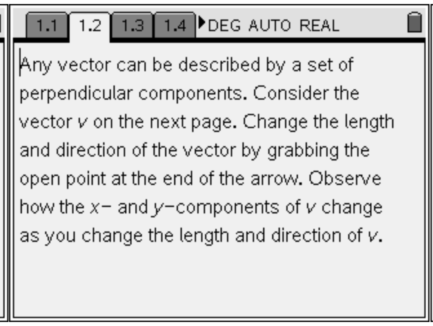
Q7. Under what general conditions do the relationships you derived in problem 1 apply?

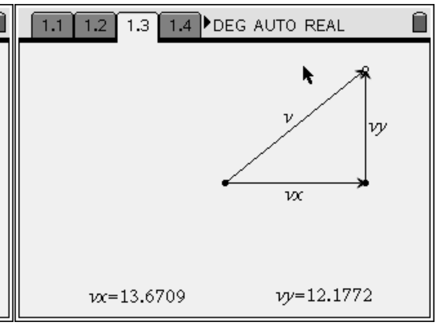
A. *The relationships derived in problem 1 apply to any vector that is resolved into two perpendicular vectors. Encourage student discussion of this concept.*

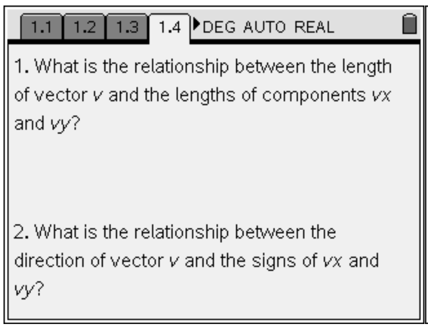
Resolving Vectors – ID: 9899

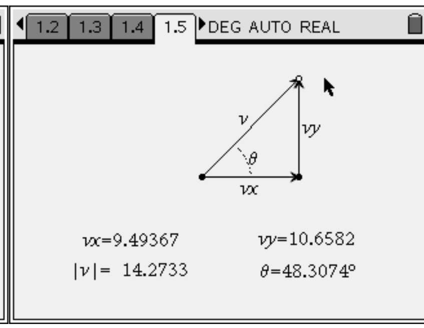
(Student)TI-Nspire File: *PhyAct_9899_ResolvingVectors.tns*

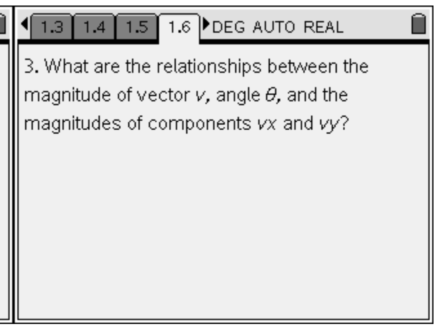


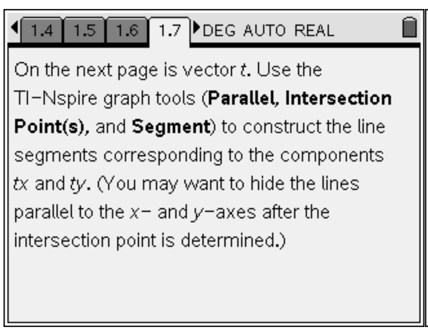


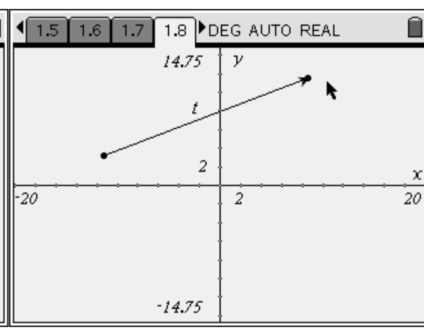


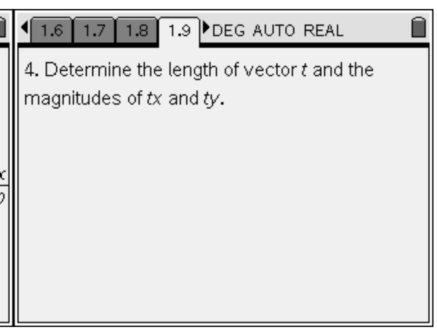


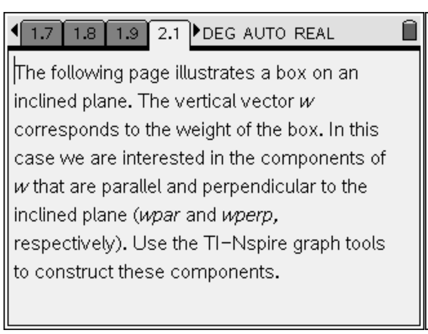


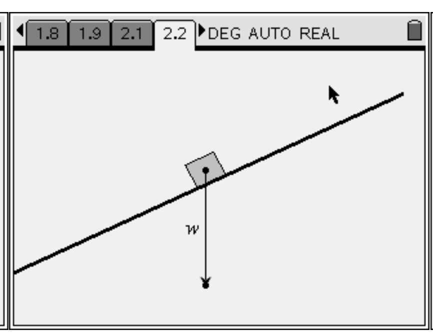


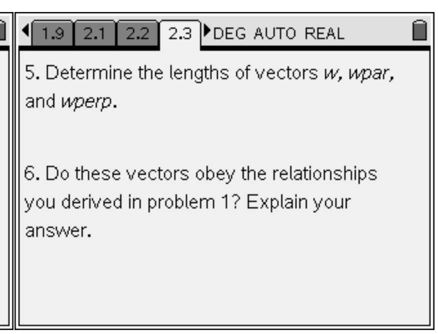


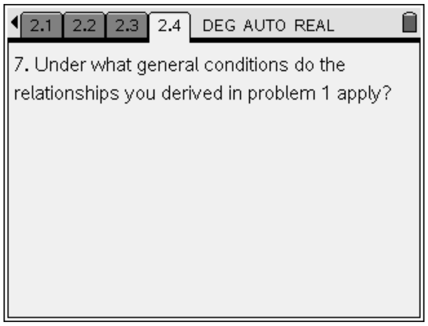











2.1 2.2 2.3 2.4 DEG AUTO REAL
7. Under what general conditions do the relationships you derived in problem 1 apply?