## Handy Reflections

## Introduction:

The purpose of this investigation is to:

- Revise plotting points on the Cartesian plane
- Use and understand aspect ratios
- Improve estimation skills
- Use a spreadsheet or a graphing calculator to draw a graph
- Learn how to reflect lines and points in the x \& y axis.


## Instructions

## Drawing your hand:

- Carefully trace around your right hand on the grid provided.
- Mark points along your trace that will become a 'dot - to - dot' representation of your hand.
The more dots the better the final graph.
- Use the scale on the graph to determine the co-ordinates of each of your points.
Write the co-ordinates next to each corresponding point.
- Fill in all your points in either $L_{1}$ (Abscissa) and $L_{2}$ (Ordinate) on the graphical calculator or use columns $A$ and $B$ on a spreadsheet. The calculator instructions are on page 4.
- Draw a connected scatter plot for your points.


## Aspect Ratio:

Figure 1- Image of right hand



Television screens come in a variety of shapes and sizes. Traditionally television screens had an aspect ratio of $4: 3$. This means if the screen was 40 cms wide, it would measure 30 cms high. With wide screen television a range of aspect ratios are possible including 2.35:1. If a wide screen movie is viewed on a standard television there are three possibilities:

- Black bars may appear at the top and bottom of the screen representing the unused portion of the screen.
- The movie can take up the whole screen, characters and objects in the movie take on a 'tall - skinny' appearance.
- The movie can take up the whole screen but a large portion of the action on the left and right will be missing.
When you graph the image of your hand, it may take on a different view because the aspect ratio has been changed. The original aspect ratio was 20 cm (wide): 25 cm (high) or simply 4:5. The aspect ratio of your graph should be the same.
- Determine the aspect ratio of your initial graph.

Measure the width and height of your graph.

- Change either the width or the height of your graph to obtain the correct ratio. Write down your final measurements and explain how you determined them.


## Reflecting your hand - The $\mathbf{y}$ - axis

- Suppose you wanted to draw a graph of your left hand. You could trace the hand and repeat all the measurements. A quicker solution is to imagine the $y$-axis is a mirror. The left hand could be drawn by making a reflection of the right hand in the $y$-axis. What would happen to your co-ordinates?
Write down some of your right-hand co-ordinates and explain how the co-ordinates for your left hand were obtained.
- Enter the new co-ordinates for your left hand into the calculator or spreadsheet and draw a graph of your left hand.
- Determine the correct aspect ratio to display both hands on the same


Figure 2 - Image of both hands graph.
Write down the dimensions for the graph and explain how you arrived at these dimensions.

## Reflecting your hand - The $\mathbf{x}$ - axis

- Suppose you wanted to draw an identical hand but upside down. Once again you could trace your hand and repeat all the measurements. A quicker solution is to imagine the $x$-axis is a mirror. What would happen to your co-ordinates if you wanted to reflect your hand in the $x$-axis?
Write down some of your original right-hand co-ordinates and explain how the new co-ordinates were obtained.
- Enter the new co-ordinates for your right hand into the calculator or spreadsheet and draw a graph of your right hand reflected in the x-axis.
- Determine the correct aspect ratio to display both images of your right hand on the same graph. Write down the dimensions for the graph and explain how you arrived at these dimensions.


## Double reflections - The x \& y-axis

- By now you should already know how to create reflections by changing your co-ordinates. How could you change your co-ordinates if you wanted to reflect your hand in the $x$ - axis followed by the $y$-axis? Write down a down a selection of your co-ordinates and explain how the new co-ordinates were obtained.
- Draw a graph of your right hand after it has been reflected in the $x$ and $y$-axis. Include the original image in your graph.
- Determine the correct aspect ratio to display both hands on the same graph.

Write down the dimensions for the graph and explain how you arrived at these dimensions.

## Translating the image of your hand(s)

- Figure 3 shows a picture of the right hand but in a new position. It has been shifted to the left. This shift is called a translation. Each point on your graph can be translated to the left.
Write down a selection of your co-ordinates and explain how the new co-ordinates were obtained.
- Draw a graph of your right hand after it has been translated to the left or parallel to the x -axis.
- Determine the correct aspect ratio to display both hands on the same graph.
Write down the dimensions for the graph and explain how you arrived at these dimensions.
- Figure 4 shows a picture of a 'pair of hands' that have been reversed. This can be achieved by translating your left hand to the right and your right hand to the left. Complete both of these translations and explain how this was achieved.


## Challenging Problems:

## Diagonal reflections - The line $\mathbf{y}=\mathbf{x}$



Figure 3 - Translated right hand


Figure 4 - Reversed hands

- Imagine a mirror is placed on the line $y=x$.
- By now you should already know how to create reflections by changing your co-ordinates. How could you change your co-ordinates if you wanted to reflect your hand in the line $y=x$ ?
Hint: Where would the point $(1,0)$ be reflected? What about the point $(0,2)$ ?
Write down a selection of your co-ordinates and explain how the new co-ordinates were obtained.
- Draw a graph, using an appropriate aspect ratio of your right hand reflected in the line: $y=x$.
- Draw a graph, using an appropriate aspect ratio of your right hand reflected in the line: $y=-x$. Explain how you your new co-ordinates were obtained.



## Graphical Calculator Instructions:

| Key Stroke \& Instructions | Screen |
| :---: | :---: |
| Press the MODE button and use the $\triangle \square \square \square$ arrow keys and press ENTER on your selections to match those shown in the screen opposite. |  |
| Press $Y=$, use the $\Delta \square \square$ arrow keys to move up and down through the equations and press CLEAR to remove any equations that may be entered on the calculator. In the screen shot displayed two statistical graphs have also been selected, "Plot1" and "Plot2". If your calculator has any plots switched on, use the $\triangle \square \square \square$ arrow keys to move the cursor on top of each of the plots and press ENTER to switch them off. |  |
| Press 2nd ZOOM and use the $\Delta \square \square \square$ arrow keys and press ENTER on your selections to match those shown in the screen opposite. |  |
| Data may already be contained in the calculator. To remove any existing data press 2nd $⿴$ and selection option 4 to clear all the lists. <br> Press ENTER to execute the command. |  |
| Finally, occasionally lists may be accidentally removed from the main editing screen, to chose a custom set up press STAT and select option 5 Set up editor. | ```EDID CHLC TESTS 1BEdt. 2: Sort.Aく S: Sortos 4: ClnL . 5: SetupEditor``` |
| The calculator is now set up ready to accept data entry. |  |
| Press STAT and select option 1 to edit/enter data. The headings $L_{1}, L_{2} \ldots$ represent List one, List two... These lists are similar to a spreadsheet's columns. Use $L_{1}$ to enter your x co-ordinates (abscissa) and $\mathrm{L}_{2}$ to enter your y co-ordinates (ordinates). |  |
| When you have finished entering your data press 2nd $Y=$ to enter the Stat - Plot menu. Press ENTER to select Plot 1. Use the $\square \square \square \square$ arrow keys to match the settings shown opposite. <br> Press ZOOM and select option 9 to view your graph. |  |
| The window settings for the graphing screen can be viewed or changed by pressing the WINDOW button. <br> $X \min =$ Minimum $X$ value displayed $\quad X \max =$ Maximum $X$ value displayed <br> Ymin $=$ Minimum Y value displayed $\quad \quad Y$ max $=$ Maximum $Y$ value displayed. <br> $\mathrm{Xscl}=$ Tick mark frequency on X - axis. $\quad$ Yscl $=$ Tick mark frequency on Y - axis. |  |

Note: The aspect ratio of the graphical calculator screen is: 95:63

