Surface Area of a Cylinder

About the Lesson

Students will explore a net representation for a right cylinder. The surface area will be developed from the parts of the net. As a result, students will:

- Construct 3-dimensional cylinders from nets.
- Calculate the surface area of a right cylinder.

Vocabulary

- net
- surface area

Teacher Preparation and Notes

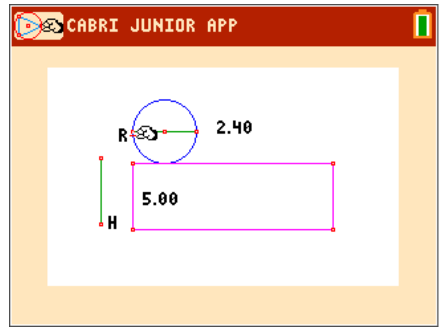
- This activity is designed to be used in a high school or middle school geometry classroom.
- This activity is designed to be student-centered.
- The surface area of a right cylinder with base radius = R and height = H is $SA = 2\pi R^2 + 2\pi RH$; the volume is $V = \pi R^2 H$. The activity asks students to notice that the circumference of the circle is the length of the rectangle in the net.
- The points R and H control the radius and the height of the cylinder. When R is dragged, the length of the rectangle also changes (because the length = circumference of the circle). The height of the rectangle does not change when R is dragged.
- **Note:** Measurements can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To change the number of digits displayed:
  1. Move the cursor over the value so it is highlighted.
  2. Press [+] to display additional decimal digits or [-] to decrease digits.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

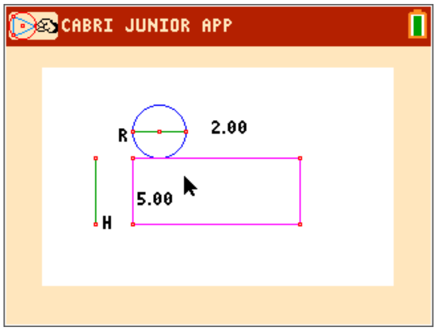
Lesson Files:

- Surface_Area_of_a_Cylinder_Student.doc
- Surface_Area_of_a_Cylinder_Student.pdf
- CYLINDER.8xv

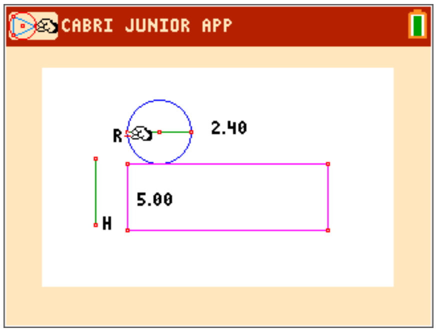
Tech Tip: Before beginning the activity, the file CYLINDER.8xv needs to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE Software.

Problem 1 – Nets

A net is a pattern that can be cut out and folded into a 3-dimensional figure. Students should see a partial net of a right cylinder that models a glass jar 5" tall and 4" in diameter. If the rectangle of the net were rolled up, the circle would be the bottom face of the cylinder (like the bottom of the jar).



The dimensions of the net can be changed by dragging the points *R* and *H*. Students should drag these points and notice what changes with the figure for each point. When *R* is moved, the radius of the circular bottom and the width of the rectangle, which represents the circumference of the jar, are changed.



1. What changes occur to the net and the jar when point *H* is dragged?

<u>Sample Answer:</u> The height of the rectangle, and so the height of the jar, changes.

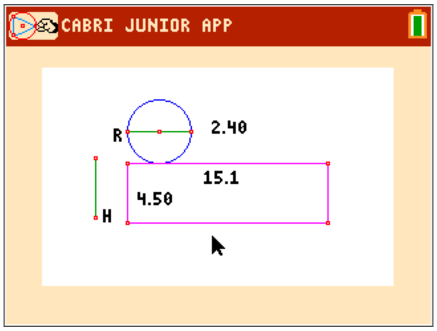
2. What changes occur to the net and the jar when point *R* is dragged?

<u>Sample Answer</u>: The radius of the circle changes and the width of the rectangle changes; both the diameter and circumference of the jar change together.

Students should use the **D. & Length** tool (**F5 > Measure**) to find the length of the rectangle.

Note: Display measurements with 2 decimal digits. To do this, hover the cursor over the measurement and then press the plus key (+).

Next, they should use the **Calculate** tool (**F5**) to divide the length of the rectangle by the radius of the circle to find that the width of the rectangle is the same as the circumference of the circle.



3. Record two sets of measurements of the net in the table. Move both points for a new set of measurements.

Sample Answers:

	Circle Radius	Rectangle Height	Rectangle Length
Set 1	2.00	5.00	12.56
Set 2	2.4	4.5	15.07

4. What is the result when you divide the Rectangle Length by the Circle Radius *R*?

Answer: The result is around 6.28. This is approximately 2 π.

5. Explain this result. Drag point *R* to confirm your conjecture.

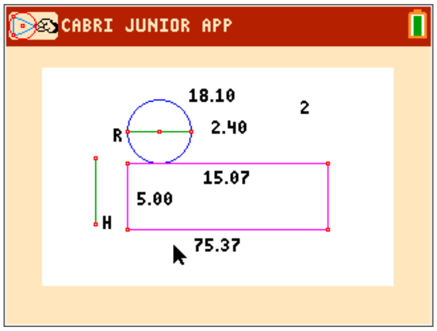
Answer: To form a cylinder, the edge rectangle must wrap around the circle representing the bottom of radius *R*. Therefore the rectangle length should be equal to the circumference of the jar, or $2\pi R$.

Problem 2 – Surface Area

Students should use the **Area** tool from the Measure menu (**F5 > Measure**) to find the areas of the rectangle and the circle. Then, they should use the **Alph-Num** tool (**F5**) to place the number **2** on the screen.

Note: Press the alpha button to access numerical characters. The tool icon in the corner of the screen will display .

Press the enter button to start and end the text.



6. Record these measurements: Circle Radius *R*, Rectangle *L*, Rectangle *H*, Circle Area, and Rectangle Area.

Circle Radius *R*: __________ Rectangle *L*: __________ Rectangle *H*: __________

Circle Area: __________ Rectangle Area: __________

Sample Answers: Circle Radius *R*: 2.4; Rectangle *L*: 15.07; Rectangle *H*: 4.5; Circle Area: 18.10; Rectangle Area: 75.37

Finally, students are to find the surface area of the cylinder. Remind students that this is only a partial net, so one of the faces is missing. To find the surface area, students need to find the area of the circle and the area of the rectangle. They should first use the **Area** tool from the Measurement menu to calculate these areas. Then, they need to use the **Calculate** tool to find the sum of both circle bases by clicking on the area of the circle, pressing [×], and clicking on the **2**. Next, they should click on the number they just calculated, press [+], and click on the area of the rectangle.

7. What is the surface area *SA* of the entire jar?

Sample Answer: 111.57 square units

8. Record the steps you performed to find the surface area of the jar:

(1)____

(2)____

Sample Answer: (1) Calculate the area of the circle and the rectangle. (2) Add up the area of 2 circles and the rectangle.

9. Use your method from Question 8 to develop a formula for the surface area of a cylinder.

Sample Answer: $SA = 2\pi R^2 + 2\pi RH$

Extension/Homework

10. *Windy Colors Painting Company* is painting a smoke stack of a building. In order to know how much paint they need, they must know the surface area of the stack. The smoke stack has a radius of 1.5 feet and a height of 24 feet. What is the surface area of the smoke stack?

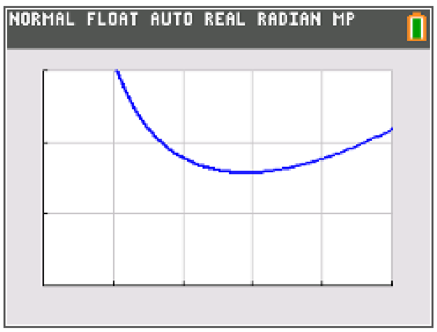
Answer: 240.33 square feet

11. A tin can manufacturer is going to manufacture a new size can. The new can is 4 inches tall and the radius is 3.5 inches. In order to order the correct amount of tin to make the new cans, they need to know how much tin is needed to make one can. What is the surface area of the can?

Answer: 164.93 square inches

12. The same company wants to make sure that they are using the least amount of tin to hold the same amount of product. To the nearest tenth of an inch, what will the radius and the height of the can be that minimizes the surface area of the can while having the same volume as in the previous problem?

Answer: The radius will be 2.9 inches and the height 5.8 inches. If the volume is held fixed, the height, as a function of radius, is $h = \frac{V}{\pi r^2}$. Substituting that into the formula for area gives: $SA = 2\pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right) = 2\pi r^2 + \frac{2V}{r}$, where $V = \pi r^2 h = \pi(3.5)^2(4) = 153.9$ cubic inches. Graphing $SA = 2\pi r^2 + \frac{2(153.9)}{r}$ (plotted below) shows that the minimum occurs at $r = 2.9$ inches. The height $h = \frac{V}{\pi r^2} = \frac{(153.9)}{\pi(2.9)^2} = 5.8$ inches.



Teacher Tip: Depending on grade level, the idea of plotting area versus radius (with volume fixed) can be suggested more (or less) explicitly to the students. (E.g., students grade 8 and up should be able to find the surface area formula in terms of radius and fixed volume, but grades 6 and 7 may need to be given the formula.) Similarly, determining the minimum visually or by using features on the calculator can be suggested.