Work and Power – ID: 9898

By Irina Lyublinskaya

Time required
45 minutes

Topic: Work and Energy

- *Calculate work given constant force and displacement in one dimension.*
- *Use the equation for power to solve problems involving power, work, and time.*
- *Compare force, work, energy, and power.*

Activity Overview

In this activity, students explore the relationship between power, work, and time. They also explore the relationship between power, force, and speed. Based on these explorations, students derive the equation P = Fv and solve problems dealing with power in various mechanical problems.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Lists & Spreadsheet, Notes, Calculator, Data & Statistics

Teacher Preparation

Before carrying out this activity, you should review with students the concept of work. Make sure they understand the difference between work and force applied and the relationship between force, displacement due to the force, and work done by the force.

- *The screenshots on pages 2–9 demonstrate expected student results. Refer to the screenshots on pages 10 and 11 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9898" in the search box.***

Classroom Management

- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

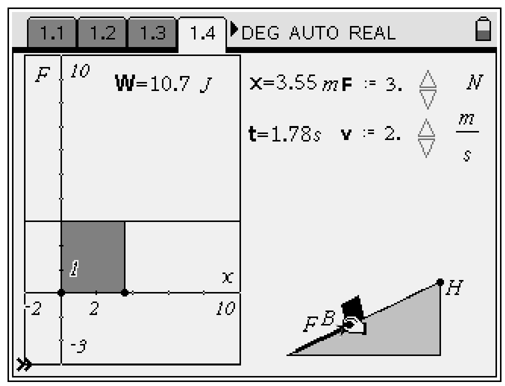
The following questions will guide student exploration during this activity:

- What is power?
- What is the relationship between work and power?
- How does power depend on force and speed?

The purpose of this activity is to provide students with an opportunity to explore the general concept of power as the rate at which work is performed by a force, to analyze several specific cases of motion, and to derive a formula for power in terms of force and speed. After students identify these relationships and derive equations describing them, they will use the equations to solve problems involving force, work, and power.

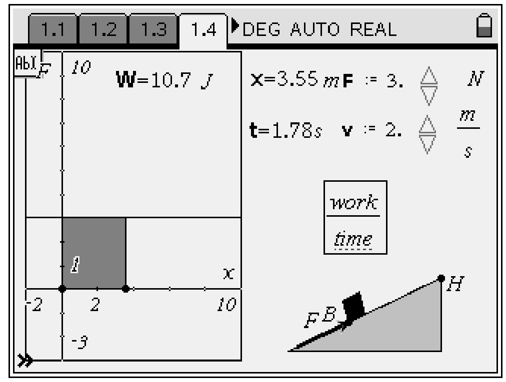
In the first problem, students explore the relationships between work, power, and time. This helps students understand the definition of power as the ratio of work to time. They use this definition to answer conceptual questions and solve problems. In the second problem, students explore the relationships between power, force, and speed and find the equation $P = Fv$ using a simulated experiment. They then apply the definition of power to confirm the equation analytically. Finally, they apply the derived equation to solve problems involving power, force, and speed.

Problem 1 – Exploring the relationship between work, power, and time

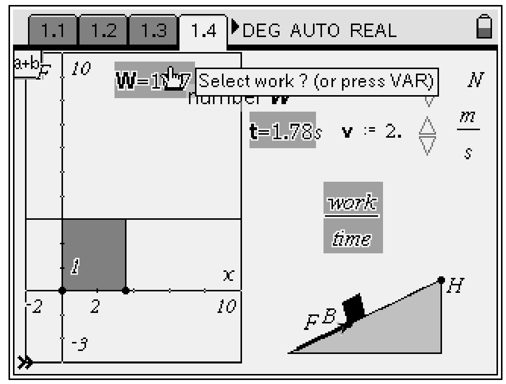


Step 1: Students should open the file **PhyAct_9898_Work_Power.tns** and read the first three pages. Page 1.4 shows a simulation of a block on an inclined plane, together with a graph of force vs. displacement. In the simulation, a constant force *F* is pushing the block up the incline at constant speed *v*. The displacement, *x,* of the block is defined as the distance from the bottom of the incline to point *B*. The time it takes to push the block up the hill through the distance *x* is calculated as $t = \frac{x}{v}$. On the left side of the page, students can observe the graph of force vs. displacement ($f1(x) = F$). Because force is constant, the graph of *F* vs. *x* is a horizontal line. As students drag point *B* to simulate the motion of the block up the incline, the work done by the force is calculated as an area under the graph of $f1(x)$. If desired, the height of the incline can be adjusted by moving point *H*. Students should observe the changes in distance, time, and work that occur as the block is pushed all the way to point *H*.

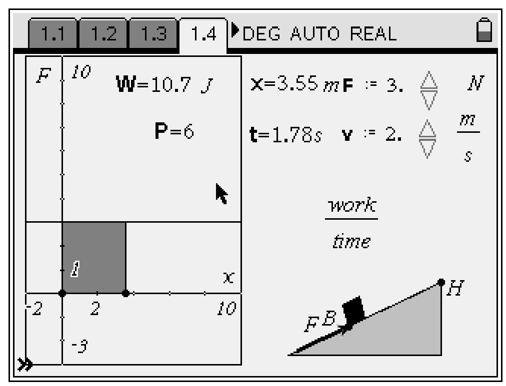
Step 2: Next, students should calculate power for the simulation on page 1.4. First, students should use the **Text** tool (**Menu > Actions > Text**) to place a text box somewhere on page 1.4. They should enter the expression $\frac{\textit{work}}{\textit{time}}$ in the text box. (You may wish to encourage students to discuss why they must enter this expression in order to calculate power.)



Step 3: After entering the expression in the text box, students should use the **Calculate** tool (**Menu > Actions > Calculate**) to determine the value of the expression. After they select the **Calculate** tool, they should click on the expression they entered in step 2. They will be prompted to select values for *time* and *work*. They should click on the value of **t** for *time* and the value of **W** for *work*. After students have selected values of *time* and *work,* they should click on a blank spot on the page to place the calculated value of power in that location.



Step 4: Next, students should store the value of work they just calculated in the variable **P**. To do this, students should click once on the calculated value and press (sto→ var). They should select **Store Var** and then type **P** and press (enter). Then, students should answer questions 1–7.



Q1. Predict what will happen to work, time, and power if you increase the force *F* but keep the speed constant.

A. *Students should answer this question without using the simulation on page 1.4. Encourage student discussion of the effects of changing force while keeping speed constant. Also encourage students to think about what must be happening to the mass of the box if force is increasing but speed is remaining constant. (In this simulation, friction is ignored.) Specifically, students should be able to reason that the mass of the box must be increasing if speed remains constant even as force increases.*

Q2. Vary the magnitude of *F* on page 1.4 to verify your prediction.

A. *Students should use the simulation on page 1.4 to test their predictions. To vary the value of* F*, students should click on the up or down arrows next to the value. They should change the value of* F *several times and observe the results. If you wish, you may have the students record their observations in a table like the one shown below. (The data in the table below were collected for a constant speed of 2 m/s.) Make sure students record force, work, time, and power values for the situation in which the box has been pushed all the way up the incline.*

F, N	*W*, J	*t*, sec	*P*, W
1	6.4	3.2	2
3	19.19	3.2	6
5	31.99	3.2	10
7	44.78	3.2	14

Students should confirm that the time it takes for the box to get to the top of the incline remains constant, because speed and displacement are the same each time. However, the work done increases as force increases. Because work increases but time remains the same, the power increases in direct proportion to force.

Q3. Predict what will happen to work, time, and power if you increase the speed *v* but keep the force constant.

A. *Students should answer this question without using the simulation on page 1.4. Encourage student discussion of the effects of changing speed while keeping force constant. Also encourage students to think about what must be happening to the mass of the box if speed is increasing but force is remaining constant. (In this simulation, friction is ignored.) Specifically, students should be able to reason that the mass of the box must be decreasing if force remains constant even as speed increases.*

Q4. Vary the magnitude of *v* on page 1.4 to verify your prediction.

A. *Students should use the simulation on page 1.4 to test their predictions. To vary the value of* v, *students should click on the up or down arrow next to the value. They should change the value of* v *several times and observe the results. If you wish, you may have the students record their observations in a table like the one shown below. (The data in the table below were collected for a constant force of 3 N.) Make sure students record speed, work, time, and power values for the situation in which the box has been pushed all the way up the incline.*

v, m/s	*W*, J	*t*, sec	*P*, W
2	19.19	3.2	6
4	19.19	1.6	12
6	19.19	1.07	18
8	19.19	0.8	24

The work remains constant, because both force and displacement are constant. The time decreases from trial to trial, because the block is moving faster in each trial. Because work is constant but time decreases, power increases from trial to trial. The increase is directly proportional to the increase in speed. Advanced students may benefit from an exploration of the effects of varying both F *and* v*.*

Q5. Two students are lifting weights. Student A lifts a 100 lb barbell over his head 10 times in 1 min. Student B lifts a 100 lb barbell over his head 10 times in 10 sec. Which student does the most work? Which student delivers the most power? Explain your answers.

A. *Both students lift the same weight the same number of times. Therefore, assuming that they are the same height, the amount of work is the same. Because student B does the work in a shorter period of time, student B delivers more power than student A.*

Q6. When doing a chin-up, an athlete lifts her 42.0 kg body a distance of 0.25 m in 2 sec. What is the power delivered by the athlete's body? Explain your answer.

A. *Students should use the* Calculator *application at the bottom of page 1.8 to help them solve this problem. They can switch between the problem and the* Calculator *application by pressing* ctrl tab*. In this problem, the force applied (*F*) is equal to the athlete's weight, which is equal to her mass (*m*) times the acceleration due to gravity (*g*). The work done by her body (*W*) is equal to the product of her weight and the distance she moves (*x*). The calculations are shown below:*

$$P = \frac{W}{t} = \frac{Fx}{t} = \frac{mgx}{t}$$

$$P = \frac{(42.0\ kg)(9.8\ m/s^2)(0.25\ m)}{2\ sec} = 51.45\ W$$

If you wish and time allows, you may discuss with students some of the possible complicating factors in this problem. For example, have students discuss what implications there would be for their calculations if the athlete's displacement were measured by the movement of her chin, but she tilted her head back slightly during the chin-up (as most people do). The displacement of her center of mass in that case would actually be less than 0.25 m.

Q7. A 930 W escalator is used to move shoppers from the first floor of a department store to the second. The escalator can hold 20 people, each with a mass of 55 kg. How long does it take the escalator to move 20 people from the first floor to the second floor, which is 5.20 m above the first floor?

A. *Students should use the* Calculator *application on page 1.10 to help them solve this problem. The power (*P*) of the escalator is given, so students must use the information in the problem to solve for time (*t*). Again, the force applied is equal to the weight of the people moved, which is in turn equal to their total mass (m = 20 · 55 kg = 1,100 kg) times the acceleration due to gravity (*g*). The calculations are shown below:*

$$P = \frac{W}{t}$$

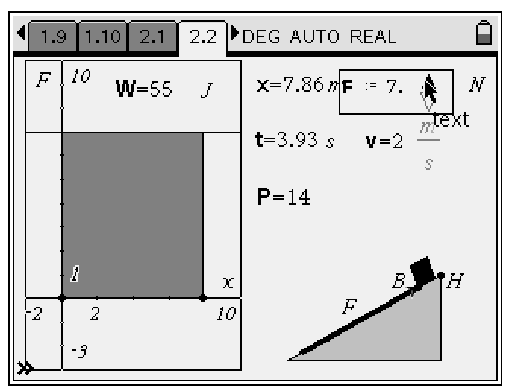
$$t = \frac{W}{P} = \frac{mgx}{P} = \frac{(1{,}100\ kg)(9.8\ m/s^2)(5.20\ m)}{930\ W}$$

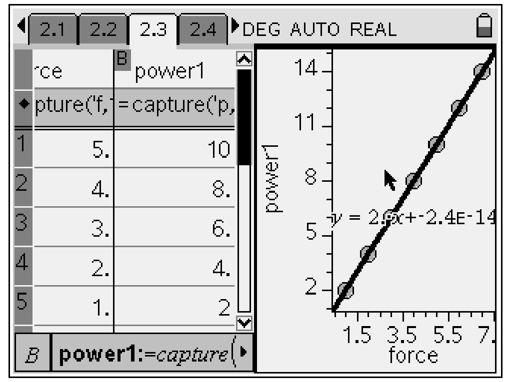
$$t = 60.3\ sec$$

Thus, the escalator moves 20 people from the first floor to the second floor approximately every minute.

Problem 2 – Finding the relationship between power, force, and speed

Step 1: Next, students should move to page 2.1 and read the text there. Then, they should move to page 2.2, which is a copy of page 1.4 but also includes the calculated value for power (**P**). Students should use the up and down arrows to vary the value of **F** and observe the effects on **P**. Students should vary **F** in a regular way over a range of values (e.g., from 1 N to 4 N in increments of 0.5 N). The *Lists & Spreadsheet* application on page 2.3 has been set up to automatically capture the values of **F** and **P** each time the value of **F** is changed. Note: Students should not vary **v** in this simulation. However, if you wish, you may have different students carry out the data collection using different values of *H* and share their results with the class.





Step 2: After students have collected several values of **F** and **P,** they should move to page 2.3, which contains a *Lists & Spreadsheet* application and a *Data & Statistics* application. Students should press ctrl tab to move between the two applications. The *Lists & Spreadsheet* application should contain data on force and power for the values the students entered in step 1. Students should move to the *Data & Statistics* application and plot **power** vs. **force**. Then, students should answer questions 8–11.

Q8. Find the best-fit line for the scatter plot on page 2.3. What is the relationship between power and force?

A. *Students should use the **Linear Regression** tool (**Menu > Analyze > Regression > Show Linear (mx + b)**) to find the best-fit line for the data. The best-fit line should confirm that power is directly proportional to force—that is, that power and force have a linear relationship. The best-fit equation for these data should be **power** = 2 · **force**. (Note: If students have changed the value of **v** in the simulation on page 2.2, their results will be different. The coefficient on **force** in the equation will be equal to the magnitude of **v**.) If you wish, you may have students use several methods to calculate and confirm the best-fit equation. Because the points lie along a line, students can determine the equation by calculating the equation of a line based on any two points. They could also use the **Movable Line** tool to find the best-fit line visually, or they could perform a linear regression on the data using the **Regression** tool in the* Lists & Spreadsheet *application.*

Q9. What is the meaning of the slope in this equation? Write a general equation for power as a function of force.

A. *The slope of this line is equal to speed, **v,** so the general form of the equation is P = Fv.*

Q10. Confirm the equation you wrote in question 9 by deriving the equation for power using the definition of power and the definition of work.

A. *If necessary, remind students that power (P) is equal to work (W) divided by time (t) and that work is equal to force (F) times displacement (x). You may also need to remind them of the definition of speed (displacement divided by time). The derivation is shown below:*

$$P = \frac{W}{t} = \frac{Fx}{t} = F \bullet \frac{x}{t} = Fv$$

Q11. A 500 kg car is moving with a constant speed of 25 m/s along a rough horizontal road. The coefficient of friction between the road and the tires of the car is 0.2. What is the power of the car's engine? Ignore air resistance, and assume the car's engine is 100% efficient.

A. *A free-body diagram of this scenario is shown on page 2.7. Students should use the* Calculator *application on page 2.8 to help them solve this problem. Because the speed of the car is constant, its acceleration is zero, and the net force on the car is also zero. This means that the force produced by the engine (F_e) must be equal to the force of friction (f)—that is, $F_e = f$. By definition, $f = \mu N$, where N is the normal force. According to Newton's second law, $N = mg$. Thus, $F_e = \mu mg$. Based on the equation derived in question 10, $P = F_e v = \mu mgv$. Substituting the given values and solving yields the following:*

$P = \mu mgv = (0.2)(500\ kg)(9.8\ m/s^2)(25\ m/s)$

$P = 2.45 \times 10^4\ W$

Encourage student discussion of how to solve this problem. You may need to remind students of the equations relating frictional force and normal force.

Work and Power – ID: 9898

(Student)TI-Nspire File: *PhysWeek12_Work_Power.tns*

1.1

WORK AND POWER

Physics

Work, Energy, and Power

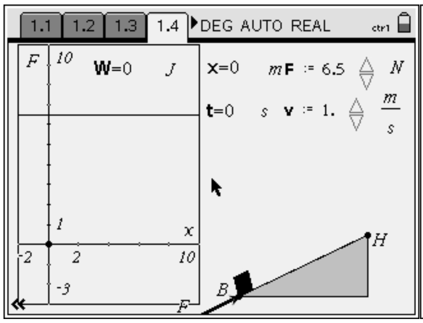
1.2

Recall that work is defined as the product of force and displacement. The same work can be done very quickly or rather slowly. The rate at which the work is done is called power. In cases when work changes steadily with time, power is defined as the ratio of work to time. The unit of power is the watt (W). 1 W = 1 J/s

1.3

Page 1.4 shows a force *F* pushing a block up an incline at constant speed *v*. The work done on the block by the force is displayed on the graph of force vs. distance. Drag point *B* to move the block up the incline, and observe the work done and the time required to move the block all the way up the incline.

1.4



1.5

1. Predict what will happen to work, time, and power if you increase the force *F* but keep the speed constant.

2. Vary the magnitude of *F* on page 1.4 to verify your prediction.

1.6

3. Predict what will happen to work, time, and power if you increase the speed *v* but keep the force constant.

4. Vary the magnitude of *v* on page 1.4 to verify your prediction.

1.7

5. Two students are lifting weights. Student A lifts a 100 lb barbell over his head 10 times in 1 min. Student B lifts a 100 lb barbell over his head 10 times in 10 sec. Which student does the most work? Which student delivers the most power? Explain your answers.

1.8

6. When doing a chin-up, an athlete lifts her 42.0 kg body a distance of 0.25 m in 2 sec. What is the power delivered by the athlete's body? Explain your answer.

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1.9

7. A 930 W escalator is used to move shoppers from the first floor of a department store to the second. The escalator can hold 20 people, each with a mass of 55 kg. How long does it take the escalator to move 20 people from the first floor to the second floor, which is 5.20 m above the first floor?

1.10

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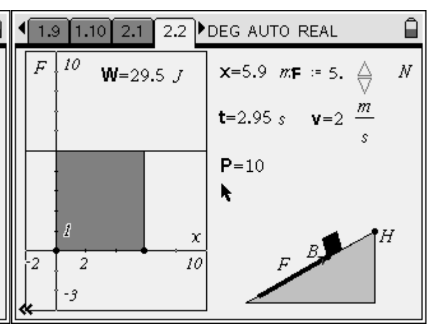
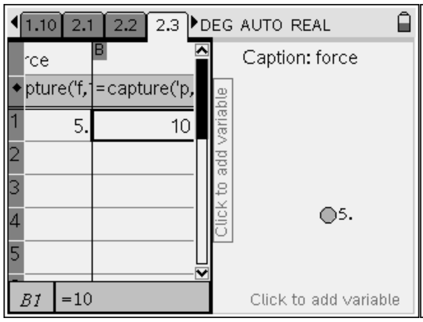
2.1

Now you will collect data to explore the relationships between power, force, and speed. On the next page, vary the value of *F* and observe the effects on *P*. The data will be recorded in the spreadsheet on page 2.3.

2.2

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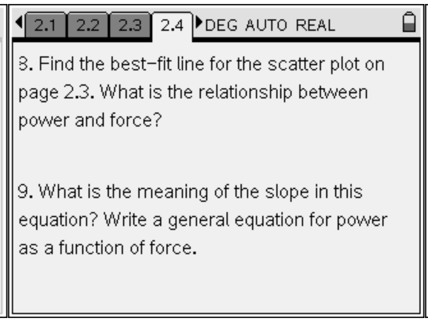
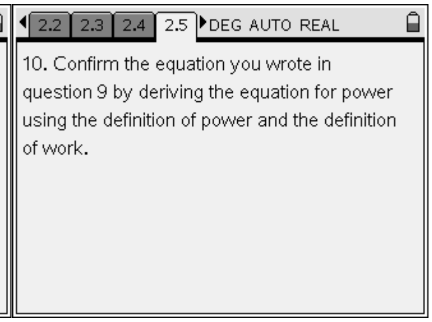
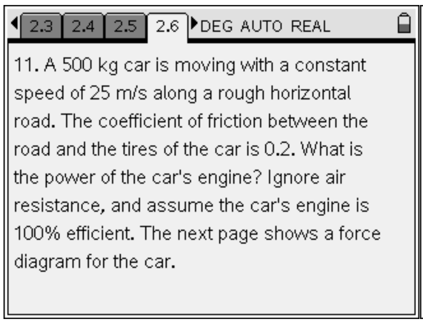
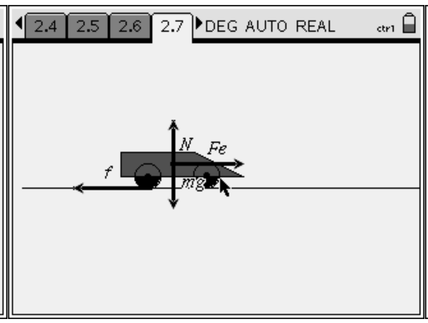

2.1 2.2 2.3 2.4 DEG AUTO REAL
8. Find the best-fit line for the scatter plot on page 2.3. What is the relationship between power and force?
9. What is the meaning of the slope in this equation? Write a general equation for power as a function of force.


2.2 2.3 2.4 2.5 DEG AUTO REAL
10. Confirm the equation you wrote in question 9 by deriving the equation for power using the definition of power and the definition of work.


2.3 2.4 2.5 2.6 DEG AUTO REAL
11. A 500 kg car is moving with a constant speed of 25 m/s along a rough horizontal road. The coefficient of friction between the road and the tires of the car is 0.2. What is the power of the car's engine? Ignore air resistance, and assume the car's engine is 100% efficient. The next page shows a force diagram for the car.


2.4 2.5 2.6 2.7 DEG AUTO REAL
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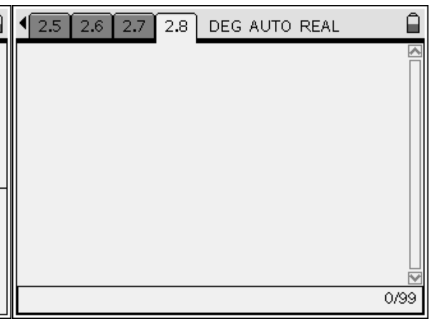
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