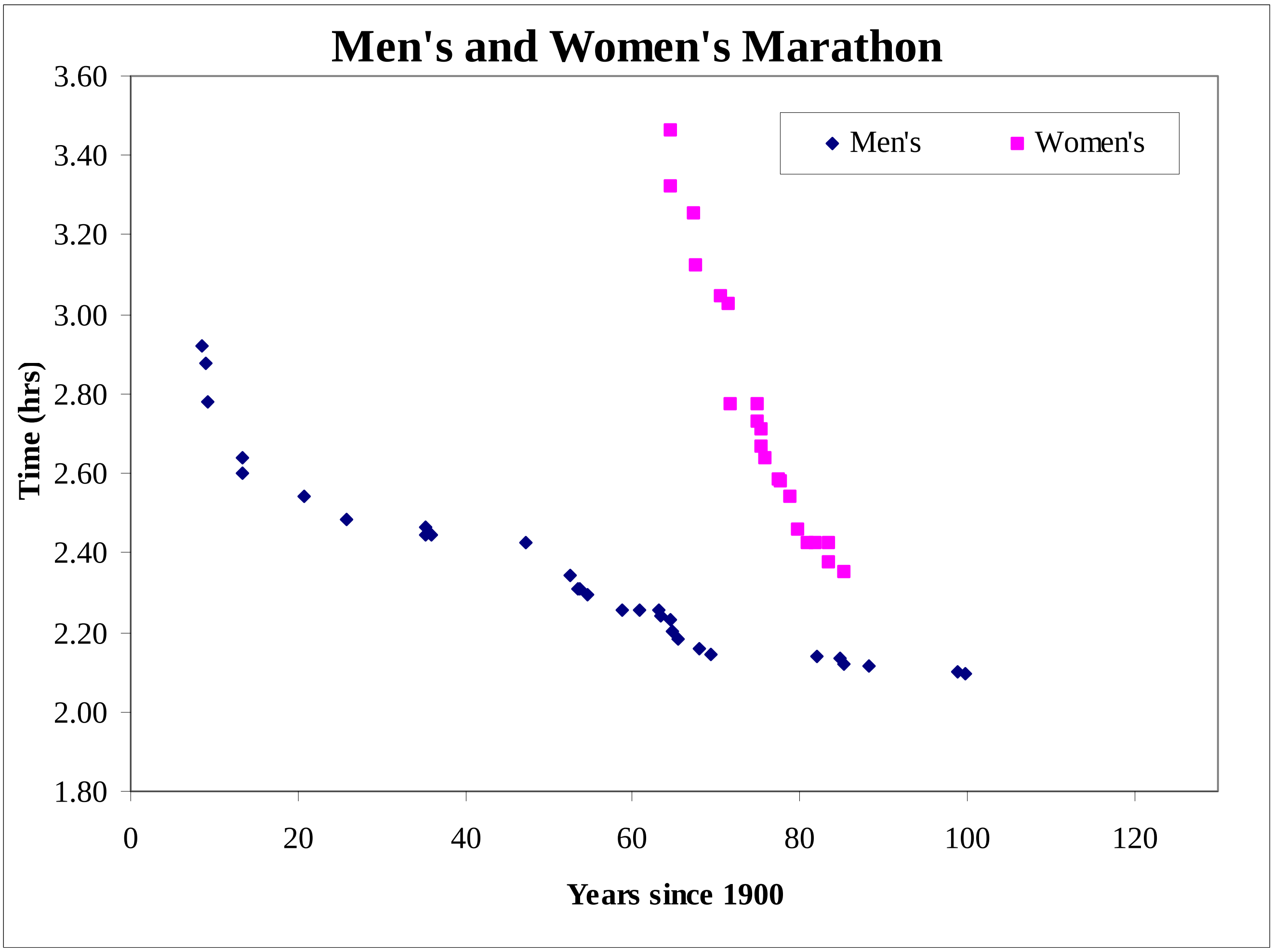
MARATHON RECORDS – USING TI-INTERACTIVE!

The Marathon has long been regarded as one of the "major" events of the Olympic Games and a significant test of an athlete's endurance. In 1964, the world record time for the Men's marathon was 1 hour, 15 minutes and 34 seconds faster than the world record time for the Women's marathon. In 1985, the difference was only 13 minutes and 54 seconds.

This raises an interesting question: Will women beat men in the marathon?

The graph below shows the world record marathon times for men and for women.



The data can be downloaded from the website: www.geocities.com/Colosseum/Arena/3170/index_s.html should you wish to do so.

Data from this site does need to be transformed from dd/mm/yy and hr:mins:secs format into years past 1900 and time in hours.

A copy of the transformed data is available on the school intranet if you need it.

Also available is a graphic calculator program containing the data, stored in lists, for both the men's and women's marathon.

Using TI-InterActive!

Assume that the relationship between T and t is of the form $T = a\,e^{bt} + c$, where a, b and c are constants. One of the limitations of using this approach is that a "reasonable" value for c needs to be chosen. If $c = 2.05$ is selected, then the rule can be re-written as $(T - 2.05) = a\,e^{bt}$.

1. Open the TI-InterActive! data file.

2. Calculate the years since 1900 (Column C), assume that there are 365.25 days in a year. Give your answers correct to four decimal places.

3. Calculate the decimal hours taken for the marathon (Column G). Give your answers correct to four decimal places.

4. Create a new column (Column H) with a heading "modified" which subtracts the 2.05 value from the decimal hours.

5. Use the Statistical regression feature to calculate an exponential model for the data.

6. Graph this function on the same set of axes as the modified data.

7. TI-InterActive! will select an arbitrary base for the calculations. In order to obtain a model in the form $T = a\,e^{bt} + c$, the base must be converted to base *e* by taking the natural logarithm of the *b_* value given in the regression equation. Insert a math box and calculate the value of $\ln(b\_)$.

8. Hence, state the rule for the Men's marathon time in the form $T_m = a \times e^{bt} + 2.05$

9. Check the validity of your rule.

10. Repeat the calculations above to determine the rule for the Women's marathon time.

11. Use the rules generated to determine point of intersection of the two functions. According to the model, will women ever beat men in the marathon? If so, on what date is it anticipated that the women's record will be faster than the men's record?

12. Repeat the process above adding in the last three records for the Women's marathon since 1985. What difference does this make to the results?

Questions

1. In a physical sense, what does the value 2.05 represent? How appropriate is this value?

2. Does it make a significant difference if this value is changed?

3. How accurately do you believe your rules model the given data? Why, why not?

Simultaneous Equations

Assume that the relationship between T and t is of the form $T = a e^{bt} + 2.05$, where a and b are constants.

Since the general equation has two constants, two data points are required in order to establish two simultaneous equations.

Use the following data points to establish two equations for each of the men's and women's marathons:

Mens' Marathon: $A\,(9.0048,\ 2.8792)$ and $B(65.4483,\ 2.1833)$

Women's Marathon: $A\,(64.5558,\ 3.3258)$ and $B(80.8214,\ 2.4281)$

Questions

1. Compare the results obtained with those obtained using the calculator. What differences are there?
2. When do these results predict the women's times will be less than the men's times?
3. What are some of the limitations of using this method to obtain a rule?
4. What differences would you expect if different data points were used?