

Limaçon Curve

ID: 13676

Time required

45 minutes

Activity Overview

Students will understand the role of parameters for equations of polar limaçon curves in the form of $r = a \pm b \sin(\theta)$ or $r = a \pm b \cos(\theta)$. Students will discover the four different types of limaçon curves and their relationship to the ratio of $\frac{a}{b}$.

Topic: Polar Functions

- Limaçon Curves
- Cardioid

Teacher Preparation and Notes

- Students will use sliders to observe different graphs of polar limaçon curves and their equations.
- Students will explore how parameters such as a and b , and the addition or subtraction of the sine or cosine of theta in equations of the form of $r = a \pm b \sin(\theta)$ and $r = a \pm b \cos(\theta)$ affect the graphs.
- Students will summarize their findings in a table provided on the student worksheet.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter “13676” in the keyword search box.**

Associated Materials

- LimaconCurve_Student.doc
- LimaconCurve.tns

Suggested Related Activities

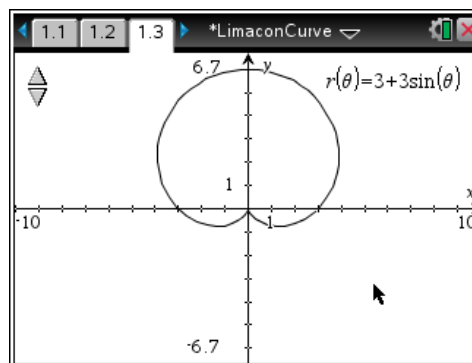
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Exploring Polar Equations — 4692
- Polar Point Plotting (TI-Nspire technology) — 12852

Problem 1 – Limaçon Curve

Polar curves called limaçons have equations of the form $r = a \pm b \sin(\theta)$ or $r = a \pm b \cos(\theta)$. On page 1.3 are special kinds of limaçon graphs called cardioids. Use the up/down arrows to see all of the graphs.

The purpose of these sets of graphs is for students to see: 1) how the addition and subtraction signs affect the graph, 2) when a and b are equal the shape of the graph stays the same, and 3) when the values of a and b increase, the size of the graph also increases.



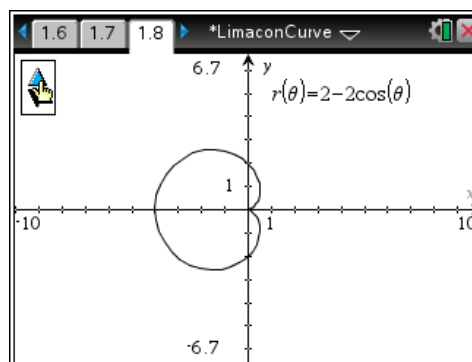
TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

Students are asked to answer questions on page 1.4 – 1.7 and/or on their worksheet.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Move to page 1.8. The purpose of these graphs is for students to see how replacing the sine function with cosine affects the graph of the limaçon.

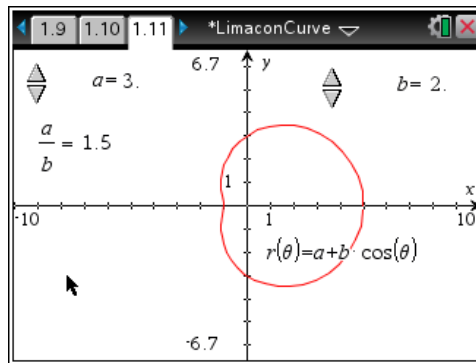
Limaçons have different shapes depending on the ratio $\frac{a}{b}$. We have already seen the cardioid graph that is the result when $a = b$. Students should answer the question on page 1.10. On page 1.11, the values of a and b can be changed to explore different limaçon curves.



Upon completion of the discussion, the teacher should ensure that students understand:

The general form of the equation for polar limaçon curves. How each of the parameters of the equation affects the graph of the limaçon curve.

That the type of limaçon curve can be determined by the ratio of a and b . There are four different shapes of limaçon curves.



Student Solutions

- Answers may vary. Students may notice that the graphs are heart shaped. Also, the word limaçon comes from the Latin word *limax*, which is a snail-like creature.
- Answers may vary. Students should notice that in each equation the values of a and b are the same. Also, each equation contains the trigonometric function sine.
- In the graphs with equations containing addition signs, the heart shape is pointing down. In the graphs with equations containing subtraction signs, the heart shape is pointing up.
- The difference between these equations and the ones on page 1.3 is the trigonometric function sine has been replaced with cosine. Now the heart shape is pointing either left or right depending on the sign, instead of up or down. If the equation contains an addition sign, the heart shape points left. If the equation contains a subtraction sign, the heart shape points right.
- Answers may vary. There are four different-shaped limaçons: one contains an inner loop, one is the cardioid, one is dimpled, and one is convex and looks almost circular. Student descriptions may vary.
- Equations and descriptions may vary. A limaçon in which the ratio of a and b is less than 1 results in an inner loop.
- Equations and descriptions may vary. A limaçon in which the ratio of a and b is between 1 and 2 results in a dimpled shape.
- Equations and descriptions may vary. A limaçon in which the ratio of a and b is greater than or equal to 2 results in a convex shape that is almost circular.

9.

Limaçon Curves $r = a \pm b \sin(\theta)$ or $r = a \pm b \cos(\theta)$	
	Description of the effect on the graph
$\frac{a}{b} < 1$	Limaçon with an inner loop.
$\frac{a}{b} = 1$	Cardioid (heart-shaped) limaçon.
$1 < \frac{a}{b} < 2$	Dimpled limaçon.
$\frac{a}{b} \geq 2$	Convex limaçon (almost circular in appearance).
Sine	Limaçon faces up or down depending on sign.
Cosine	Limaçon faces left or right depending on sign.
addition sign	With the sine function, limaçon faces down. With the cosine function, limaçon faces left.
subtraction sign	With the sine function, limaçon faces up. With the cosine function, limaçon faces right.
value of a	The larger the a value, the larger the limaçon. The smaller the a value, the smaller the limaçon.
value of b	The larger the b value, the larger the limaçon. The smaller the b value, the smaller the limaçon.

TI-Nspire Navigator Opportunities

Note 1

Problem 1, *Class Capture*

This would be a good place to use Class Capture to verify that students are able to click on the slider and answer the questions correctly.

Note 2

Problem 1, *Quick Poll*

Use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.