Closure Tables – ID: 8641

Time required
30 minutes

Topic: From Arithmetic to Algebra

- *Prove that the integers are closed under the operations of addition, subtraction, and multiplication.*
- *Determine the operations under which the even and odd integers are closed.*

Activity Overview

In this activity, students create and complete closure tables to determine if the sets of whole numbers, integers, even numbers, and odd numbers are closed under the operations of addition, subtraction, multiplication, and division.

This activity provides a review of the definitions of whole number and integer as well as practice with integer operations. Students experience abstractions like the commutative properties of addition and multiplication as concrete patterns by performing many operations and recording their answers in an organized way. Because of the large number of simple calculations involved, this activity is also a good way for students new to the graphing calculator to gain confidence with its use and familiarity with the keypad layout.

Teacher Preparation

This activity may be used to introduce the closure axioms and/or review integer operations. You may choose to use the activity in its entirety or break it up into separate activities by sets.

- *Students should be familiar with the definitions of the set of whole numbers and the set of integers before beginning this activity.*
- *Although it will reinforce related concepts, students should have been introduced to integer operations prior to this activity.*
- *It is very important that you thoroughly describe what it means for a set to be* ***closed*** *under an operation prior to the class completing the activity on their own. One way to do so is by completing Problem 1 together as a class.*
- ***To download the CLOSURE program, go to http://education.ti.com/exchange and enter "8641" in the search box.***

Classroom Management

- *This activity is intended to be introduced in a* ***teacher-led****, whole class setting. Once it has been introduced, students can work* ***individually or in pairs*** *to complete the closure tables, following directions on the student worksheet.*
- *The student worksheet is intended to guide students through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*

TI-84 Plus Applications

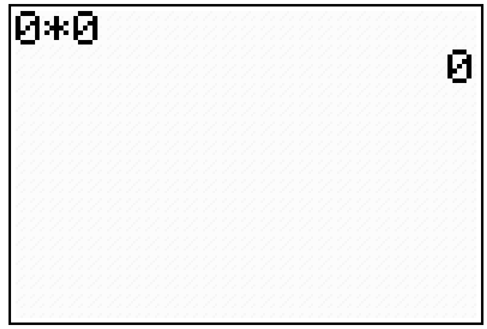
none

Closure Tables

ID: 8641

In this activity, you will explore:

- *the properties of some sets under the operations of addition, subtraction, multiplication, and division*
- *the concept of closure and closure tables*



Listen as your teacher explains what it means for a set to be closed under an operation.

Things to Remember…

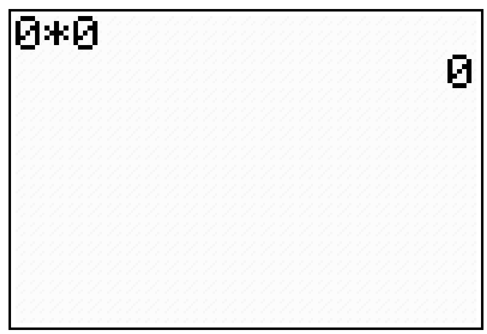
A set is ***closed*** under an operation if for any two numbers in the set, the result of the operation is also in the set. This definition will take on more meaning as you explore examples.

You can think of a set as a bag of numbered tiles. The set is closed under an operation if you can draw any two tiles from the bag, perform the operation with them, and put the answer back in the bag.

Problem 1 – The set {0, 1}

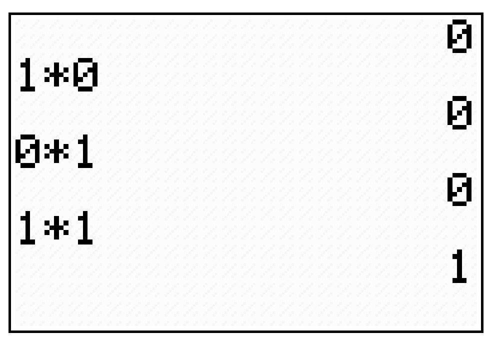
Let's begin with the set {0, 1}. This set has only two elements. Is it closed under multiplication?

Make all the multiplication problems you can using two numbers from this set. You can use a number twice. Try the problems in your calculator.



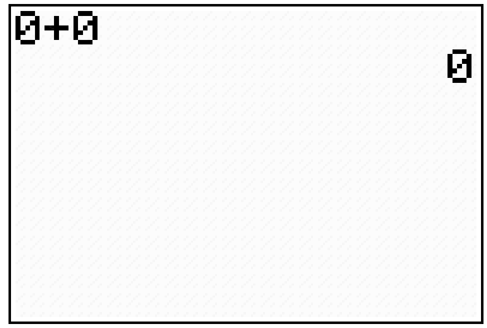
There are 4 possible multiplication problems. Look at their answers. Do the answers belong to the set {0, 1}?

If all of the answers belong to the set {0, 1}, then the set is closed under that operation. If any of the answers do not belong to the set, then the set is not closed under that operation.



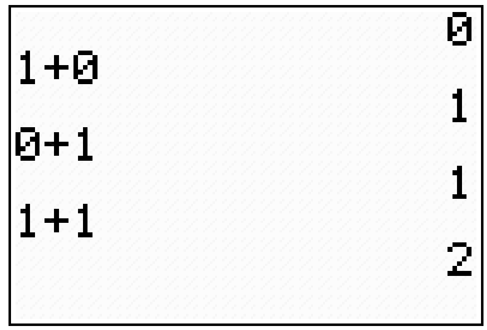
Is the set {0, 1} closed under addition?

- List all the addition problems you can using two numbers from this set. You can use a number twice. Try the problems in your calculator.



There are 4 possible addition problems. Look at their answers.

- Do the answers belong to the set {0, 1}? Is this set closed under addition?



Closure tables are a way to organize your work when checking to see if sets are closed under operations. The closure table for {0, 1} under multiplication is shown.

×	**0**	**1**
0	0	0
1	0	1

Closure tables are similar to multiplication tables, but they can show any operation. To read them, find a number in the first column and a number in the first row. The intersection of these gives the result of the two numbers under that operation.

To use the closure table to determine if this set is closed under multiplication, look at each of the products in the table. All of the products belong to the set {0, 1} so this set is closed under multiplication.

- Complete the closure table for the set {0, 1} under addition.

+	**0**	**1**
0		
1		

- Is the set {0, 1} closed under subtraction? Use the closure table to the right to explain your answer.

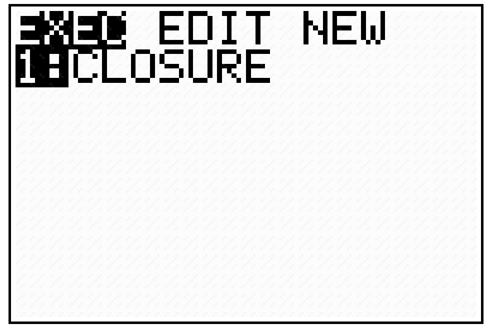
–	**0**	**1**
0	0	–1
1	1	0

Problem 2 – The whole numbers

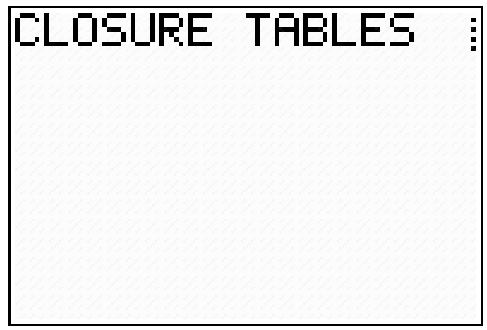
In this problem, you will examine a larger set of numbers, the whole numbers, and determine if they are closed under addition, subtraction, multiplication, and division.

A closure table with all of the whole numbers would be infinitely large! For our purposes, it is enough to use just a few examples of the whole numbers.

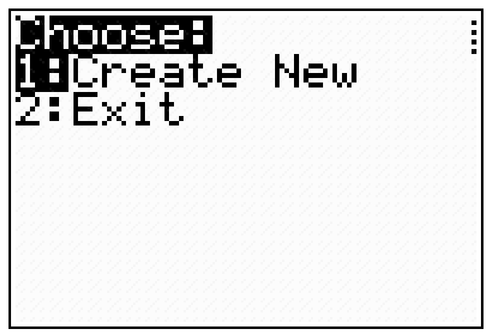
- Use your calculator to complete a closure table for the set of whole numbers under addition. Press PRGM to open the **Program** menu and choose **CLOSURE**.



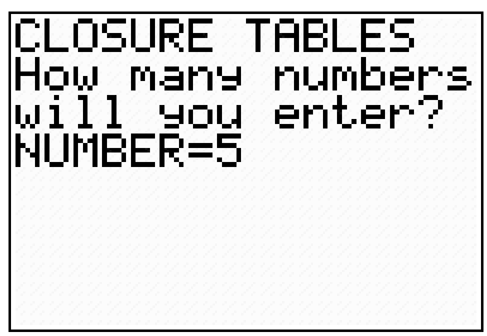
Press ENTER twice to run the program.



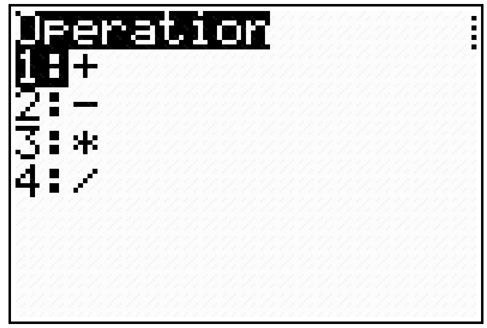
The program asks you what you would like to do. You want to create a new closure table, so choose **Create New**.



The program asks you how many numbers you want in your closure table. In this case, you want 5 numbers.



Enter the numbers 0, 1, 2, 3, and 4 and choose *addition* as the operation. Copy the result into the table.



- Do you see any patterns in the table? Do any of the sums repeat?
- What property of addition causes this pattern?
- Are all of the sums whole numbers?
- Is the set of whole numbers closed under addition?

+	**0**	**1**	**2**	**3**	**4**
0					
1					
2					
3					
4					

Addition is commutative, so it doesn't matter what order you add two numbers. Subtraction is **not** commutative. It does matter in what order you subtract.

- With this in mind, use the **CLOSURE** program to create the closure table. Copy your results into the table.
- Do you see any patterns in the table?
- Are all of the differences whole numbers?
- Is the set of whole numbers closed under subtraction?

–	**0**	**1**	**2**	**3**	**4**
0					
1	1				
2		1			
3			1		
4				1	

- Use the **CLOSURE** program to complete the closure tables. Round your answers to the nearest tenth if necessary. (***Note:*** *When creating a division, if you see an entry of 999, this means that there was a division by zero and this value is undefined.*)

×	0	1	2	3	4
0					
1					
2					
3					
4					

÷	0	1	2	3	4
0					
1					
2					
3					
4					

- Do you see a pattern in the table?
- What property causes this pattern?
- Is the set of whole numbers closed under multiplication?
- Is the set of whole numbers closed under division?

Problem 3 – The integers

In this problem, you will determine if the set of all integers is closed under addition, subtraction, multiplication, and division.

The set of integers is even larger than the set of whole numbers!

- What types of numbers belong to the set of integers that do not belong to the set of whole numbers? List a few examples.
- Choose some integers to use in the closure tables. Be sure to choose some negative and some positive integers.

- Use the **CLOSURE** program to complete each closure table.

+			**0**		
0					

–			**0**		
0					

×			**0**		
0					

÷			**0**		
0					

- What pattern do you see in the multiplication and division tables? Where are the products and quotients positive? Where are they negative?

- Under which operations are the integers closed?

Problem 4 – The even and odd integers

In this problem, you will determine if the set of even integers and the set of odd integers are closed under addition, subtraction, multiplication, and division.

- Choose some even and odd integers to use in the closure tables.

- Use the **CLOSURE** program to complete each closure table.

EVEN

+					

ODD

+					

EVEN

–					

ODD

–					

EVEN

×					

ODD

×					

EVEN

÷					

ODD

÷					

- Under which operations is the set of even integers closed?
- Under which operations is the set of odd integers closed?
- Complete the table to summarize your findings.

set	closed under +	closed under –	closed under ×	closed under ÷
{0,1}	no		yes	
whole numbers				
integers				
even integers				
odd integers				

Solutions:

Problem 1

+	0	1
0	0	1
1	1	2

- No. The sum 2 appears in the closure table, and 2 is not the set {0, 1}.

Problem 2

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

- There is a pattern along the diagonal of the table. Sums above and below the diagonal repeat.
- The commutative property of addition
- Yes
- Yes

–	0	1	2	3	4
0	0	–1	–2	–3	–4
1	1	0	–1	–2	–3
2	2	1	0	–1	–2
3	3	2	1	0	–1
4	4	3	2	1	0

- There is a pattern along the diagonal of the table. The differences above the diagonal are the opposite of the differences below the diagonal.
- No
- No

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	6	8
3	0	3	6	9	12
4	0	4	8	12	16

- There is a pattern along the diagonal of the table. Products above and below the diagonal repeat.
- The commutative property of multiplication
- Yes

÷	0	1	2	3	4
0	und	0	0	0	0
1	und	1	0.5	0.3	0.2
2	und	2	1	0.7	0.5
3	und	3	1.5	1	0.7
4	und	4	2	1.3	2

- No

Problem 3

- The negative numbers, like –3 and –67.
- Tables will vary.
- The products and quotients are positive when both factors are positive or both factors are negative. The products and quotients are negative when one factor is positive and the other is negative.
- Addition, subtraction, and multiplication

Problem 4

- Tables will vary.
- Addition, subtraction, and multiplication
- Multiplication

set	closed under +	closed under –	closed under ×	closed under ÷
{0, 1}	no	no	yes	no
whole numbers	yes	no	yes	no
integers	yes	yes	yes	no
even numbers	yes	yes	yes	no