Math Objectives

- Students will model a real-world application with a linear system of equations with three variables.
- Students will use the technology to solve a system of three equations in three variables and interpret the solutions.
- Students will utilize the information from their solution to solve other problems.
- Students will model with mathematics (CCSS Mathematical Practice).

Vocabulary

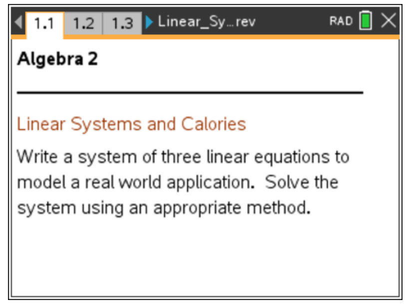
- coefficients
- linear system
- system of equations

About the Lesson

- This lesson involves setting up and solving systems of equations.
- As a result, students will:
  - Write a system of linear equations with three variables to model a real-world application.
  - Solve their system of equations.
  - Utilize their solutions to determine the number of calories contained in a gram of carbohydrate, a gram of protein, and a gram of fat, and also obtain the formula for finding basal metabolic rate (BMR).
  - Write and solve additional equations to further explore information about calories and diet.

TI-Nspire™ Navigator™ System

- Use Live Presenter to have students share their method for solving a system of linear equations with three variables.
- Use Screen Capture to examine students' progress.
- Use Quick Poll to assess students' understanding.
- Use Teacher Edition computer software to review student documents.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Enter a system into a matrix

Tech Tip:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:

Student Activity

Linear_Systems_and_Calories_Student.pdf

Linear_Systems_and_Calories_Student.doc

Linear_Systems_and_Calories_Create.doc

TI-Nspire document

Linear_Systems_and_Calories.tns

Visit www.mathnspired.com for lesson updates.

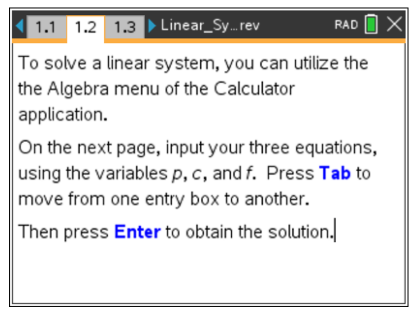
Discussion Points and Possible Answers

Teacher Tip: This activity is designed to provide students with a real-world application of linear systems. It can be used as a review of solving a linear system of equation with three unknowns. A complete TI-Nspire document file is included with this activity. You may give students this file to use or you may want them solve the systems using a method of your choice.

An instructional file on how to create the pages of the TI-Nspire document file has also been provided for your convenience. If you would rather have students create the document as they go, use those directions.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Move to page 1.2.



1. According to the USDA Dietary Guidelines for Americans, females between the ages of 19 and 30 years of age should consume approximately 91 grams of protein, 271 grams of carbohydrates, and 65 grams of fat per day.[1] The total is approximately 2,033 calories.

   The Dietary Guidelines also state that females in the same age bracket but who are following the DASH (Dietary Approaches to Stop Hypertension) eating plan should consume approximately 108 grams of protein, 288 grams of carbohydrates, and 48 grams of fat, totaling approximately 2,016 calories.

   Kristie is training to run a marathon and has to increase her caloric intake. Her trainer suggested that she eat 94 grams of protein, 345 grams of carbohydrates, and 83 grams of fat daily, totaling approximately 2,503 calories.

[1] http://fnic.nal.usda.gov/nal_display/index.php?tax_level=1&info_center=4&tax_subject=256

Write three equations that could be used to determine the number of calories in each gram of protein, carbohydrate, and fat.

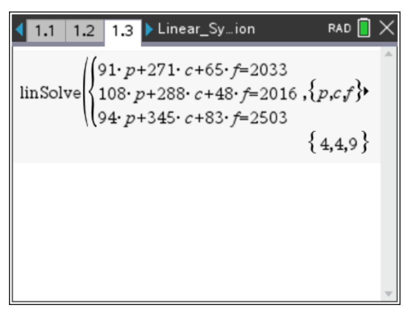
Answer: Using p to represent the number of calories in a gram of protein, c to represent the number of calories in a gram of carbohydrate, and f to represent the number of calories in a gram of fat, the three equations are:

$$\begin{aligned} 91p + 271c + 65f &= 2033 \\ 108p + 288c + 48f &= 2016 \\ 94p + 345c + 83f &= 2503 \end{aligned}$$

Tech Tip: If you prefer that students use the Calculator application, and need assistance on how to use the Algebra menu to solve a linear system, download the file, *Linear_Systems_and_Calories_Create.* This file contains step-by-step directions. Also included in the file are directions for how to utilize a matrix to solve the system.
If you prefer to utilize the TI-Nspire document file that is given with the activity, these directions and screen captures are included below.

Three different solution methods are given below. It is not expected that you use all three methods to solve the first system. Once you decide on a method of solution, scroll down to question 2 to continue with the activity.

Move to page 1.3. (linSolve Method)

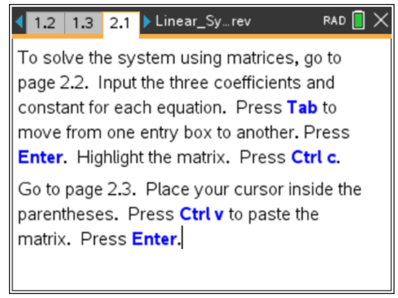


Tech Tip: This page of the .tns document sets up the **linSolve(** command from the Algebra menu. After inputting the equations and pressing enter, your screen should look like the one at the right.

Teacher Tip: Ask students to explain what is being shown and how these values relate to the problem they are solving.

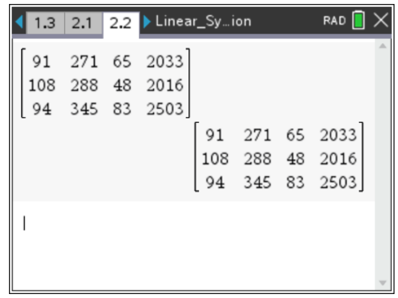
Extension: Ask students if the order in which they enter the equations makes a difference and then ask them to justify their answer.

Move to page 2.1.

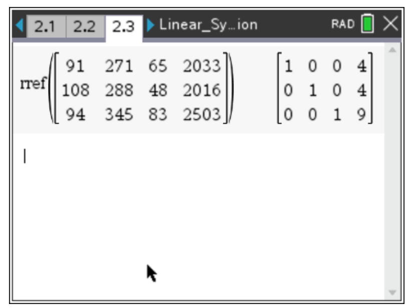


> **Teacher Tip:** You may prefer to have students solve the problem by using the reduced row-echelon form of the matrix.

Move to page 2.2. (Reduced Row-Echelon Method)



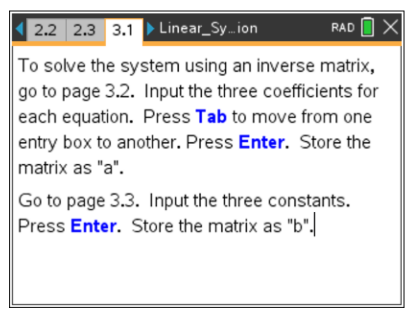
> **Tech Tip:** When you enter the information for each of the rows of the matrix, be sure to enter the three coefficients and the constant, and press [enter].



> **Tech Tip:** Highlight the matrix on page 2.2 as shown in the screen capture above.
>
> Press [ctrl] [C] to copy the highlighted matrix. Move to page 2.3. Move your cursor inside the parentheses and press [ctrl] [V] to paste the highlighted matrix inside the parentheses. Press [enter] to obtain the solution.

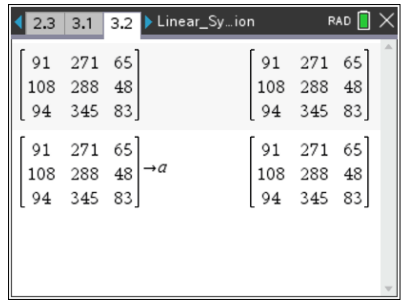
> **Teacher Tip:** Be sure to ask students to explain what they are seeing and how this applies to the problem they are solving.

Move to page 3.1.



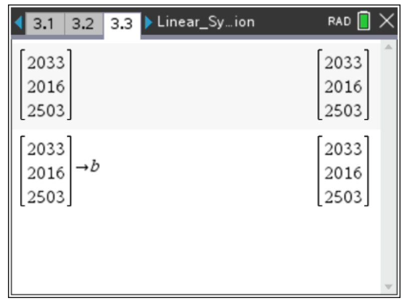
> **Teacher Tip:** Alternatively, you may prefer to have students solve the problem by using the inverse of the coefficient matrix.

Move to page 3.2. (Matrix Inverse Method)



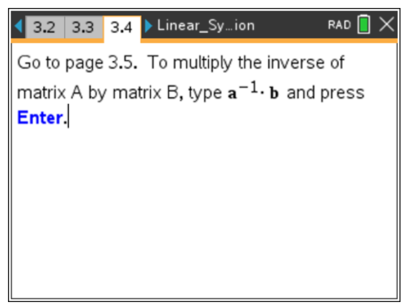
> **Tech Tip:** When you enter the information for each of the rows of the matrix, be sure to enter the three coefficients.
>
> After you enter the matrix, to store the matrix as *a*, press ctrl var A and press enter.

Move to page 3.3.



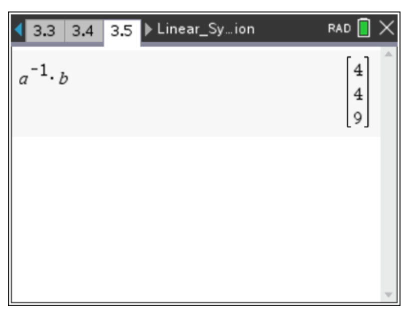
> **Tech Tip:** When you enter information in the second matrix, be sure to enter the three constants.
>
> After you enter the matrix, to store the matrix as *b*, press ctrl var B and press enter.

Move to page 3.4.



> **Teacher Tip:** Make sure that students understand the mathematics of the operations. For example: if [A] represents the coefficients, [V] represents the variables, and [B] represents the constants, then
> [A] * [V] = [B]. You need to solve for [V].
> $[A]^{-1}*[A]*[V] = [A]^{-1}*[B]$
> $[I]*[V] = [A]^{-1}*[B]$ where [I] is the identity matrix
> Therefore: $[V] = [A]^{-1}*[B]$
>
> It may be a good idea to ask students what a^{-1} represents and why you are multiplying a^{-1} and *b*.

Move to page 3.5.



> **Tech Tip:** As directed, type $a^{-1}\cdot\ b$ and press enter to see the solution of the system.

2. Solve the system of equations from question 1, as directed by your teacher.
   a. Explain what your answer tells you about calories.

      Sample answer: Solving this system, $p = 4$, $c = 4$, $f = 9$. This indicates that there are 4 calories in every gram of protein, 4 calories in every gram of carbohydrates, and 9 calories in every gram of fat.

   b. Use this information to write a formula that can be used to determine the total number of calories in a food item that contains protein, carbohydrates, and fat.

      Sample answer: To determine the total number of calories in a food item that contains protein, carbohydrates, and fat, multiply the number of grams of protein, carbohydrates, and fat by 4, 4, and 9, respectively, and add the three products.

      Thus, if x represents the number of grams of protein, y represents the number of grams of carbohydrates, z represents the number of grams of fat, and N represents the total number of calories in the food item, then $N = 4x + 4y + 9z$.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 2 at the end of this lesson.

3. Kristie's friends plan to watch the marathon and want to stop at Subway™ to buy turkey breast mini sub sandwiches. Each of these sandwiches contains 2.5 grams of fat, 31 gram of carbohydrates, and 12 grams of protein.[2] Based upon your answer to question 2, write and solve an equation to find the total number of calories in each sandwich.

   Answer: Using the formula $N = 4x + 4y + 9z$, substitute 2.5, 31, and 12 for *z, y,* and *x*.
   $N = 4 \cdot 12 + 4 \cdot 31 + 9 \cdot 2.5 = 194.5$. Subway's turkey breast mini sub sandwich contains 194.5 calories.

[2] http://www.subway.com/applications/NutritionInfo/index.aspx?CountryCode=USA&LanguageCode=ENG

4. Is it possible for a food item to contain 194 calories if all of the calories come from carbohydrates and fat? Explain your answer.

   Sample answer: Yes, it is possible for a food item to contain 194 calories if all of the calories come from carbohydrates and fat. The solution is the set of ordered pairs, (y, z), that satisfy the equation $4y + 9z = 194$, where *y* represents the number of grams of carbohydrates and *z* represents the number of grams of fat.

   For example, a food item could with 44 grams of carbohydrates and 2 grams of fat has a total of 194 calories.

5. In Phase One of the Atkins Diet™ [3], carbohydrate consumption is restricted to 20 grams per day.[4] For the rest of her caloric intake, Gail would like to obtain twice as many calories from protein as from fat. If she is restricted to a total of 2,100 calories per day and expects to consume 20 grams of carbohydrates, how many grams of protein and fat can she consume?

   Sample answer: Using the formula $N = 4x + 4y + 9z$, substitute 2100 for *N* and 20 for *y*. Since she is consuming twice as many grams of protein as fat, you have $x = 2z$.

   Substituting the above information and solving the resulting equation for *z*, you have:

$$2100 = 4(2z) + 4(20) + 9z$$
$$2100 = 17z + 80$$
$$2020 = 17z$$
$$z = 118.8235$$

   Thus, Gail can consume approximately 119 grams of fat and 238 grams of protein per day.

> **Teacher Tip:** Use whatever method you prefer to have students solve the following problem. You may want them to use a method different from the one they used to solve the system in question 1.

[3] Subway and Atkins Diet are trademarks of their respective owners.
[4] http://www.the-atkins-diet.info/

6. Most nutrition labels are based on a 2,000-calorie diet. However, height, weight, and age as well as activity level and gender affect the number of calories needed. The basal metabolic rate (BMR) is the amount of energy a person needs to function at rest. When combined with information about a person's level of activity, it can be utilized to get a more accurate calculation of the number of calories a person needs.

Juan would like to determine his BMR. His three friends each know their BMRs but do not know the formula.[5] However, they do remember that they added 66 to some number times their weight in pounds + another number times their height in inches – another number times their age in years.

Jeff weighs 180 pounds, is 5'8" tall and is 19 years old. His BMR is 1,948.
Larry also weighs 180 pounds. He is 5'10" tall, is 25 years old, and his BMR is 1,933.
Mario weighs 168 pounds, is 5'6" tall and is 21 years old. His BMR is 1,833.

a. Write and solve a system of equations to obtain the coefficients to be used in determining BMR for men.

Answer: Let *w* represent weight in pounds, *h* represent height in inches, and *a* represent age in years. Input the information above to get:

$$66 + 180w + 68h - 19a = 1948$$
$$66 + 180w + 70h - 25a = 1933$$
$$66 + 168w + 66h - 21a = 1833$$

$w = 6.3$
$h = 12.9$
$a = 6.8$

TI-Nspire Navigator Opportunity: *Screen Capture* and/or *Live Presenter* and *Quick Polls (Multiple Choice* or *Open Response)*
See Note 3 at the end of this lesson.

b. Use your information from part 6a to write a formula to find BMR for a male.

Answer: For a male, $BMR_m = 66 + 6.3w + 12.9h - 6.8a$.

[5] http://www.bmi-calculator.net/bmr-calculator/bmr-formula.php

c. Use your formula to calculate Juan's BMR if he is a 22-year-old who weighs 170 pounds and is 5'4" tall.

Answer: For a male, $BMR_m = 66 + 6.3w + 12.9h - 6.8a$.
Thus $BMR_{Juan} = 66 + 6.3 \cdot 170 + 12.9 \cdot 64 - 6.8 \cdot 22 = 1813$.
Juan's BMR is 1813.

Extension: If students need extra practice in solving systems, or you want them to use a different method to solve a system, you can give them the question below. Note: This question is not in the Student Activity. It is only provided to you to give your students extra practice.

7. Nicole also does not know her BMR. Three of her friends know their BMRs and remember that their formula was different from the boys'.[6] They added 655 to some number times their weight in pounds + another number times their height in inches – another number times their age in years.

Jaya weighs 128 pounds, is 5' tall, and is 22 years old. Her BMR is 1,384.
Carra weighs 142 pounds, is 5'2" tall, and is 20 years old. Her BMR is 1,463.
Katie weighs 138 pounds, is 5'4" tall, and is 26 years old. Her BMR is 1,427.

a. Write and solve a system of equations to determine the coefficients to be used in determining BMR for women.

Answer: Let *w* represent weight in pounds, *h* represent height in inches, and *a* represent age in years. Input the information above to get:

$$655 + 128w + 60h - 22a = 1384$$
$$655 + 142w + 62h - 20a = 1463$$
$$655 + 138w + 64h - 26a = 1427$$

$w = 4.3$
$h = 4.7$
$a = 4.7$

b. Use your information from part a to write a formula to find BMR for a female.

Answer: For a female, $BMR_f = 655 + 4.3w + 4.7h - 4.7a$.

[6] http://www.bmi-calculator.net/bmr-calculator/bmr-formula.php

c. Use your formula to calculate Nicole's BMR if she weighs 155 pounds, is 5'5" tall, and is 20 years old.

Answer: For a female, $BMR_f = 655 + 4.3w + 4.7h - 4.7a$.
Thus, $BMR_{Nicole} = 655 + 4.3 \cdot 155 + 4.7 \cdot 65 - 4.7 \cdot 20 = 1533$.
Nicole's BMR is 1533.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to model a real-world application with a linear system of equations with three variables.
- How to solve their system of equations and interpret the solutions.
- How to utilize the information from their solution to solve additional problems.

Assessment

Utilize *Screen Capture* and *Quick Poll* to assess students' understanding of how to solve linear systems.

TI-Nspire Navigator

Note 1

Whole Activity, *Live Presenter:* If you prefer to have students choose their own method for solving this linear system and do not use the .tns file, you may want to use Live Presenter to have several students share their methods with the class.

Note 2

Question 2, *Quick Poll (Open Response):* Tell students that you are going to send several *Open Response Quick Polls.* Ask them to input the number of calories in one gram of protein, one gram of carbohydrates, and one gram of fat. If you see that students had trouble obtaining these answers, ask them how they obtained their answers. This is a good opportunity to see where students are having difficulty.

If most of the students obtained the correct answer, you can then send another *Open Response Quick Poll* to input the equation.

Note 3

Question 6, *Screen Capture* and/or *Live Presenter Quick Poll (Open Response):* You may want to take several screen shots of students' progress to see if they are understanding how to solve the system. Perhaps you might make one of the students a Live Presenter to share his or her method of solving with the class.
Tell students that you are going to send several *Open Response Quick Polls.* Ask them to input the equations they wrote. If you see that students had trouble obtaining these equations, stop and ask them how they obtained their equations.

After students have the correct equations, send additional *Quick Polls* to check if they were able to solve the system correctly.