



Most students instinctively know already that for each consecutive bounce, a ball will reach slightly less height than in the bounce before. Some of you may even have explored the mathematical model that describes how the height is decreasing for a series of bounces. You might have explored the relationship between conservation of mechanical energy in the constant exchange between potential and kinetic energies and the diminishing rebound height.

In this activity, you will explore that relationship from a slightly different perspective. Can we develop a mathematical model that is based on the time interval of the sound the ball makes on impact between consecutive bounces? And can we then use our model to predict how long our ball would (theoretically) bounce?

Equipment & Materials

- TI-Nspire™ CX, CX II or computer with TI-Nspire™ software
- TI-Nspire LabCradle™ & Vernier Microphone
- TI-CBR 2™ (for part 2 of the exploration)
- Cable to connect TI-CBR 2™ to the TI-Nspire LabCradle™
- Ringstand and clamps
- Various small balls (Ping pong, golf, Lacrosse)

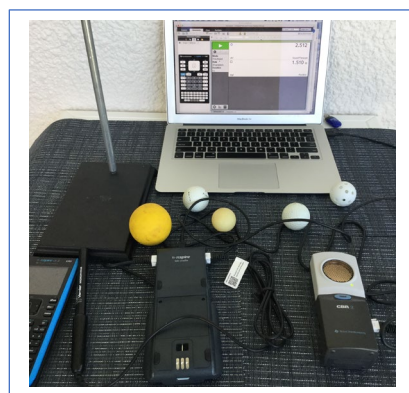


Figure 1

Objectives

Analyze the time intervals between consecutive bounces of a ball;
Determine the decrease in the length of the time interval between bounces;
Compare the time intervals to rebound height measured with CBR 2™;
Determine the relationship between time intervals and rebound height;
Develop a mathematical model for the decreasing time intervals.

Exploration- Part 1

In this first part of the exploration, your group will record the sound of a ball bouncing on a hard surface. Balls that compress very little work best for both parts of the exploration.

Procedure:

1. Connect the microphone to one of the three analog ports on the TI-Nspire™ Lab Cradle;
2. Start a new document on the TI-Nspire™ and open a DataQuest page before attaching the TI-Nspire™ to the Lab Cradle;

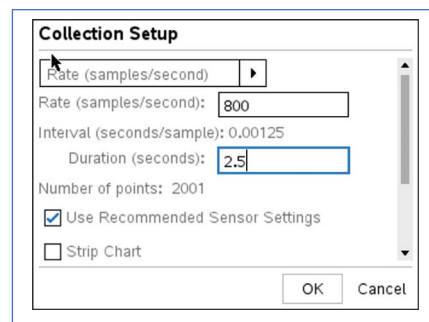


Figure 2



3. Set up the TI-Nspire™ as follows:

- Mode > Time Based, Rate> 800/sec and Duration > about 2.5 sec.

Depending on the settings you choose, a warning window may pop up (see Fig. 3), alerting you to the fact that your sampling rate is not supported for the connected sensor or that it may exceed the available memory.

- Click on OK. Should the total number of samples exceed 2,500, the TI-Nspire™ will adjust the settings or you can adjust the sampling rates if necessary.

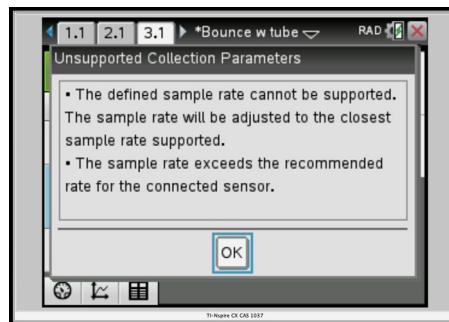


Figure 3

4. “Zero out” the microphone by pressing [menu] > Experiment > Set up Sensors > Microphone, and then tab down to ‘zero.’
5. With one student pressing the green “record” button, another should drop the ball from a height of about 40 – 50 cm above the surface area and have it bounce near the tip of the microphone;
6. If you are satisfied with the way the data collection looks (did you get a graph similar to the one in Fig. 4?), press the check mark icon to save the run;
7. Repeat steps 4 – 6 until you have collected data for three or four successful runs;

Data Analysis:

1. Add a new problem to your document ([doc] > Insert > Problem). Add an L&S and a D&S page to the new problem;
2. Label the columns on the L&S page *bounce #*, *time_ms*, and *interval_ms_#* (see Fig. 6, replace # with the run number, in this case 2);
3. Enter the bounce numbers (1, 2, 3, ...) in the first column recorded (Figure 4 shows the sound for about 11 clearly distinguishable bounces);
4. Tracing along the graph and determine the time of maximum loudness for each bounce (in *ms*);
5. Zooming in on each individual bounce by adjusting the viewing window will help you identify the time of the bounce more easily. Figure 5 shows the graph window expanded to show the first bounces more clearly. The dotted line and the small circle show a time of .293 s, which corresponds to the first entry in cell B2 in the L&S page (see Fig. 6);
6. Record all those times in the 2nd column;
7. Using these entries, compute the difference in time between bounces and enter those in column 3.

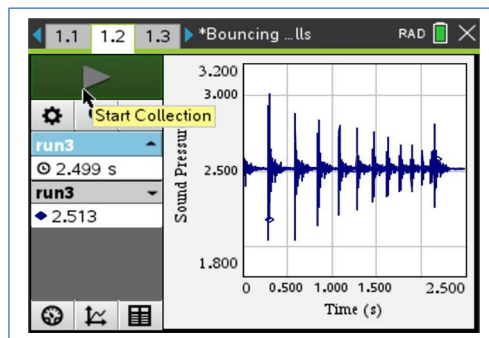


Figure 4

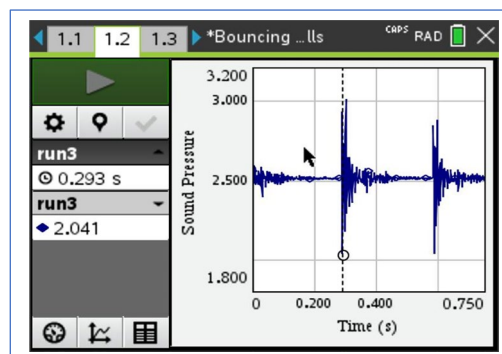


Figure 5



8. You can “automate” that calculation by entering the appropriate formula in column C. Example: C3=B3-B2, etc. By selecting cell C3 and dragging the bottom right corner of the cell, you can “autofill down” for the remaining values;
9. Repeat steps 3. – 9. after first adding three columns per additional run, labeling each *bounce #*, *time_ms*, and *interval_ms_#* (see Fig. 6, replace # with the run number as you did in step 3).
10. Next, plot the data *time_diff_ms* vs. *bounce_#* on the D&S page.
11. Use [menu] > Analyze > Plot Function to find an exponential function that best fits your data from the ball bounces in the form $f(x)=a*b^x$.
12. Alternatively, use the regression analysis function of your handhelds, use [menu] > Analyze > Regression > Show Exponential, to analyze your data and determine how well the mathematical model fits the data you collected.

Answer the following questions based on your data analysis:

- How well does the exponential function fit their data?
- What do the coefficients ‘a’ and ‘b’ in the function $f(x) = a*b^x$ stand for?
- How well does the mathematical model describe the reality of the ball bounce?
- Will the time difference between bounces ever reach zero?

Extension to Part 1:

What defines a good mathematical model? Let’s think about the question how long the ball would bounce? We saw that, eventually, our ball came to rest. Theoretically, the ball would bounce an infinite number of times and the time between bounces would get shorter with each bounce.

1. Is this where the mathematical model breaks down and doesn’t represent physical reality anymore?
2. What would be the total time for an infinite number of bounces?
3. Let’s use an infinite geometric series, in other words, a series with a constant ratio between successive terms.
4. Add a column to your L&S page like the one in Figure 8, verify that the ratio of successive time differences remains approximately constant (Column D, ratio).

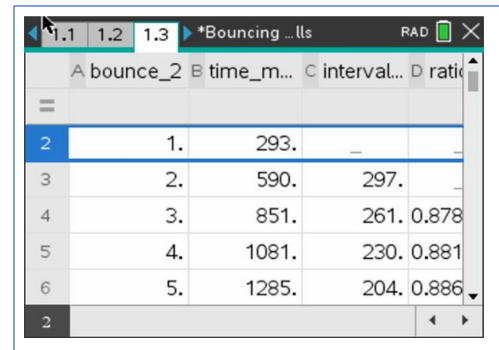


Figure 6

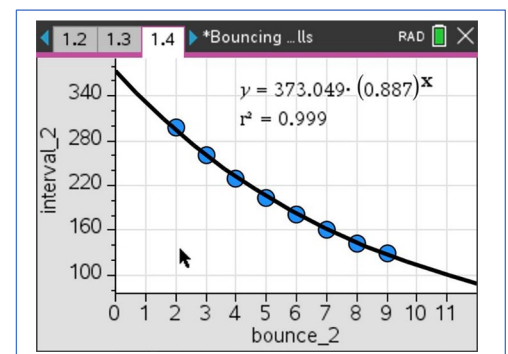


Figure 7

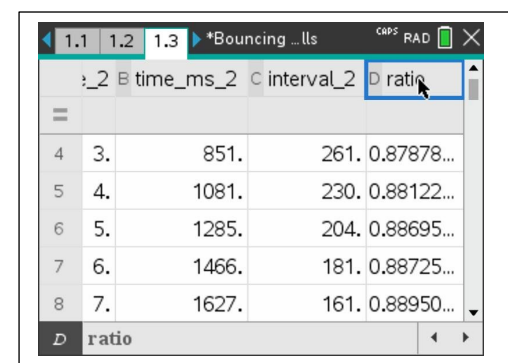


Figure 8



Directions for the TI-Nspire™ CX CAS or CX II CAS models:

$$T_{\infty} = \sum_{x=0}^{\infty} a * b^x$$

Here **a** represents the initial time value and the **b** is the ratio of successive intervals.

On the CAS model, store values for **a** and **b** ($\boxed{\text{ctrl}} \boxed{[\text{sto} \rightarrow]}$) and then insert the summation template $\boxed{\left[\sum \right]}$. Using the values from the function in Fig. 7, for example, we get a total time of T=3,301.3 ms or 3.301 sec, in other words a **finite amount of time** for an **infinite number of bounces**.

Directions for the numeric TI-Nspire™ CX or CX II model:

On a calculator page (or notes page with math template), insert the summation template $\boxed{\left[\sum \right]}$ and then the values for **a** and **b**, using a relatively high number for the upper limit of the sum (we picked 1,000).

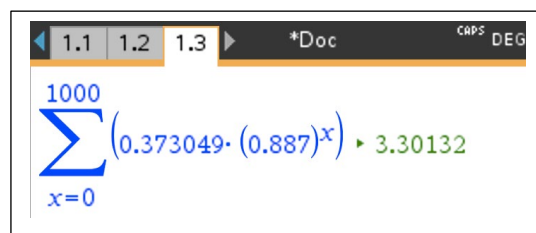


Figure 9

Exploration- Part 2

In the second part of the exploration, you will record the sound of the ball bouncing on a hard surface, while at the same time recording the rebound height for consecutive bounces.

1. Add a new problem to your document: Press ~ > Insert > Problem and open a DataQuest page before attaching the TI-Nspire™ to the Lab Cradle;
2. Connect both sensors to the TI-Nspire LabCradle™ as shown in Fig. 10 and then attach the TI-Nspire™ to the Lab Cradle; The TI-CBR 2™ should be at least 40 – 50 cm above the surface area.

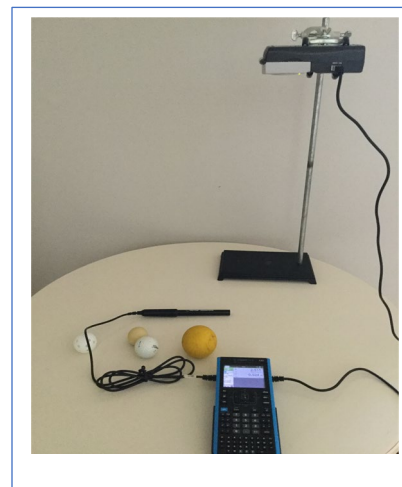


Figure 10

For this part of the investigation it is important to keep in mind that you are using a digital and an analog sensor at the same time. Their recommended collection rates are quite different. We found a sampling rate of < 100 to work best.

3. Set up the TI-Nspire™ as follows: Mode > Time Based, Rate > 95/sec and Duration > about 3 sec.
4. You might still get a warning window like the one in Fig. 3, alerting you to the fact that your sampling rate is not supported for the connected sensor. Allow the TI-Nspire™ to adjust it.
5. Zero out” the microphone as described in part 1.
6. Set up the TI-CBR 2™ by first placing the ball you are going to use directly underneath the TI-CBR 2™. Then press $\boxed{\text{menu}}$ > Experiment

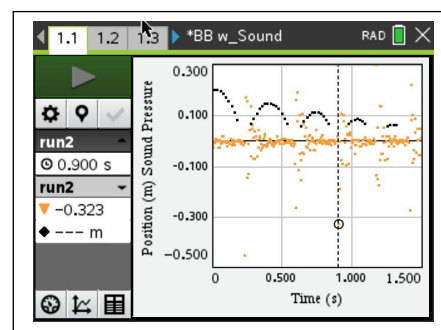


Figure 11



> Set up Sensors > Motion Sensor, tab down to and select *Reverse Readings*, and finally, tab down to 'zero' and hit .

Before starting the data collection, think about why we select "Reverse Reading" and how that setting manifests itself in the graph. In a sentence or two, explain your thinking.

The data collection for this part will probably take quite a few trials, since the balls you are working with are quite small. Unless the bounce is well aligned with the motion sensor, the height vs. time readings can be messy.

Data Analysis:

1. Add a new problem (~ > Insert > Problem) and then add an **L&S** and a **D&S** page to the new problem.
2. Label the columns on the L&S page as shown in Fig. 12. It is a good idea to add the run# to each column label to keep track of the run that each data set represents as well as match up the correct variables on the D&S page.
3. Trace along the data points to locate the loudest sound between any two rebounds. In Fig. 11, the dotted vertical line indicates that point between bounces three and four at 0.9 sec.
4. As in part 1, plot your data, *time_diff_run#* vs. *bounce_#* as well as *height_run#* vs. *bounce #* on the D&S page.
5. Use b > Analyze > Plot Function to come up with an exponential function that best fits the data from your ball bounces in the form $f(x)=a*b^x$.
6. Alternatively, using the regression analysis functionality of your handheld, use b > Analyze > Regression > Show Exponential, to analyze your data.

	bounce_run1	time_run1	height_run1
1	0	—	0.22
2	1	0.23	0.18
3	2	0.6	0.14
4	3	0.9	0.1
5	4	1.17	0.07

Figure 12

Answer the following questions based on your data analysis:

- How are decreasing rebound heights and the time intervals between loudest sounds of the bouncing ball related?
- How well does the exponential function fit their data?
- For what do the coefficients 'a' and 'b' in the function(s) $f(x) = a*b^x$ stands?
- How well does the mathematical model describe the reality of the ball bounce?



Extension:

The time between bounces, in other words the time the ball spends in the air, can be thought of as the ball's *hangtime*.

- What are some other variables you might be able to change to affect the hang time of the ball.
- What is the average hang time a human being might be able to achieve?
- Can you think of ways to measure it?
- What about top athletes? Did Michael Jordan really have such exceptional hang time?

