

Logging In

ID: 12179

Time required

15 minutes




Activity Overview

In this activity, students will use integration by parts to integrate $\ln x$. In the homework/extension problems, they will use a similar process to integrate $\tan^{-1}x$. Self-check questions engage students and help deepen understanding. Multiple-choice exam-like questions are also included to apply what was learned. Additionally, students will review integration using substitution.

Topic: Integration

- *Integration by Parts*
- *Integration using substitution*

Teacher Preparation and Notes

- *Integration by parts is a topic in the AP* Calculus BC curriculum. Students should be familiar with the technique before completing this activity. The suggested activity below may be used prior to this activity to teach integration by parts.*
- *Student responses can be typed directly into the TI-Nspire handheld and/or recorded on the accompanying handout. On self-check questions, after answering the question students can press  and select **Check Answer** (or  + .*
- **To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter “12179” in the quick search box.**

Associated Materials

- *LoggingIn_Student.doc*
- *LoggingIn.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Integration by Parts (TI-Nspire CAS technology) — 9851*

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Part 1 – Explore Integrating $\ln x$

Present students with the problem $\int \ln x dx$. Discuss techniques of integration that could be used, such as direct integration, substitution, or integration by Parts.

Students should eliminate the first two options. Have students recall the formula for integration by parts. *Note: there are a variety of notations. This activity will use the following notation:*

$$\int uv'dx = uv - \int vu'dx$$

Investigate the different choices for u and v' . Together discover $\int \ln x dx = x \ln x - x + C$. Verify the result by taking the derivative. (The CAS software can be used.)

Students will graph $y = \ln(x)$ and their solution. With a partner they will discuss the relationship between the two graphs.

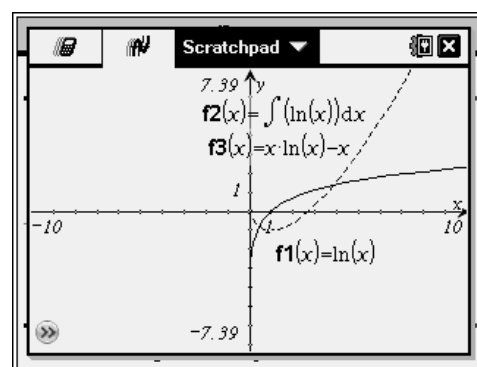
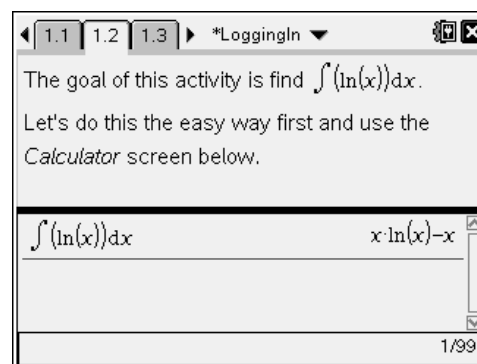
Student Solutions

- $\int \ln x dx = x \ln x - x + C$
- Direct integration, substitution, integration by parts, others that have been taught
- $u = \ln x, v' = 1$
- $\int uv'dx = uv - \int vu'dx$

$$= x \ln x - \int x \cdot \frac{1}{x} dx$$

$$= x \ln x - \int 1 dx$$

$$= x \ln x - x + C$$



Part 2 – Extension/Homework

Note: When doing indefinite integration, the CAS handheld does not include + C. Emphasize to students the importance of including this in their solutions.

This section has problems that allow students to practice techniques of integration. Some questions are multiple-choice and others are free-response. Students can check their answers using the handheld. Answers on the handheld may appear different than the answers students get by hand. This can allow for students to see different forms of the answer and verify (either algebraically by hand or using logic on the CAS system). The methods described below are only one possible way to solve the given problems.

Student Solutions

1. Involves integration by parts first and then substitution.

$$\int \tan^{-1} x dx = x \tan^{-1} x - \frac{\ln(x^2 + 1)}{2} + C$$

2. Involves substitution first and then integration by parts.

$$\int \ln(2x) dx = x \ln(2x) - x + C$$

3. Involves straight integration by parts—typical multiple-choice question.

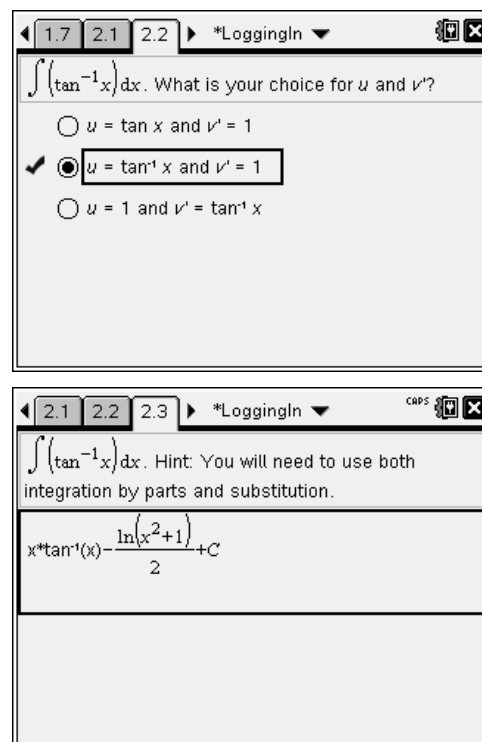
$$\int x e^x dx = x e^x - e^x + C$$

4. Involves integration by parts twice.

$$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x + C$$

5. Involves straight integration by parts.

$$\int x \sin(x) dx = -x \cos(x) + \sin(x) + C$$



1.7 2.1 2.2 *LoggingIn

$\int (\tan^{-1} x) dx$. What is your choice for u and v' ?

☐ $u = \tan x$ and $v' = 1$
☒ $u = \tan^{-1} x$ and $v' = 1$
☐ $u = 1$ and $v' = \tan^{-1} x$

2.1 2.2 2.3 *LoggingIn

$\int (\tan^{-1} x) dx$. Hint: You will need to use both integration by parts and substitution.

$x \cdot \tan^{-1}(x) - \frac{\ln(x^2 + 1)}{2} + C$