NUMB3RS Activity: Is It Really Rare?
Episode: "The O.G."

Topic: Poisson Distribution **Grade Level:** 11 - 12
Objective: Understanding a probability distribution of rare events that occur in a time interval
Time: 30 minutes

Introduction

Have you ever wondered how to find the probability that a rare event will occur? For example, what is the probability of having 6 people in front of you in a line at a store, or of getting robbed in a parking lot? Mathematicians solve these problems by using historical data to find the probability of the event in question. For example, if police know that on average one person gets robbed at a particular parking lot every two days, they can use this information to predict the probability that someone will get robbed in one day or in three weeks. **Poisson distributions** are useful for finding the probability that a rare event happens at a constant rate over time – that is, finding the probability of an event that typically has occurred a small number times over a large number of trials. The distribution is a "reasonable probability distribution" for infrequent events that have occurred in the past an average of *m* times in a given time period. A condition for the use of this distribution is that the events have to be independent.

We can calculate the probability of future occurrences of an event based on the historical probability *m* that the event has occurred (for a large number of trials). The probability that the event will occur exactly *k* times (*k* is a non-negative integer: 0, 1, 2, ...) is given by $P(m, k) = \dfrac{e^{-m}m^k}{k!}$, where *e* is the base of the natural logarithm (*e* = 2.71828...) and *m* is a positive real number, equal to the expected number of times the event will occur during the given time interval. The value of *m* is based upon the historical probability of the event occurring in a specific time period, and the expectation of it happening again. For a given value of *m*, the formula might be shortened to $P(k) = \dfrac{e^{-m}m^k}{k!}$.

Numb3rs Example

Suppose that historically, a shooting has occurred in Los Angeles an average of once every 6 hours. Suppose that Charlie Eppes is interested in the probability of 2 shootings occurring in a given 24-hour period. He could use a Poisson distribution to find that probability.

If the average is 1 shooting for every 6 hours, then in a 24-hour period Charlie should expect there to be 24 ÷ 6 = 4 shootings. So, let *m* = 4. Because he is interested in 2 shootings in that time period, then he should use a Poisson distribution with *k* = 2. The probability of 2 shootings in one 24-hour period is found by computing $P(2) = \dfrac{e^{-4}4^2}{2!}$,

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which is approximately 0.147. So, he should expect that about 14.7% of the time, 2 shootings would take place during this 24-hour period.

- Suppose Charlie wants to know the probability of 2 or fewer shootings occurring in that 24-hour period. To find the probability of 2 or fewer shootings, add the probabilities when k = 0, 1, or 2:

$$\begin{aligned} P(\text{2 or fewer shootings}) &= P(\text{0 shootings}) + P(\text{1 shooting}) + P(\text{2 shootings}) \\ &= 0.0183 + 0.0733 + 0.1465 \\ &= 0.2381\text{, or about } 23.8\% \end{aligned}$$

- Suppose Charlie wants to know the probability of more than 2 shootings occurring in that 24-hour period. He could use the fact that 2 or fewer shootings and more than 2 shootings are complementary events:

$$\begin{aligned} P(\text{more than 2 shootings}) &= 1 - P(\text{2 or fewer shootings}) \\ &= 1 - 0.2381 \\ &= 0.7619\text{, or about } 76.2\% \end{aligned}$$

In this activity, students will be asked to find the probability that an event will happen more than k times, less than k times, exactly k times, or any combination thereof for a given expectation m of the event happening. Because m represents the average number of shootings, its value does not change in a specific time period.

You may want to spend time showing students how to simplify the calculations by using the lists on a TI-84 Plus calculator. To compute the individual and cumulative probabilities of 0 – 6 shootings in a 24-hour period, do the following:

Press [STAT] [1] to access the list editor. Input 0, 1, 2, 3, 4, 5, and 6 in L_1.

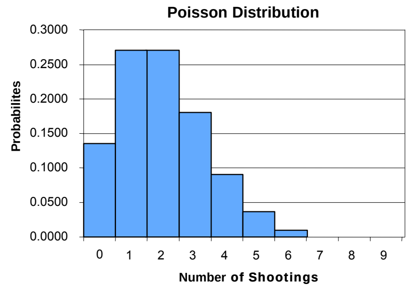
Next, type in the Poisson distribution formula as follows in the very top of L_2: $\dfrac{e^{-4}4^{L_1}}{L_1!}$ (to find L_1, press [2nd] [1] and to find !, press [MATH] [◄] [4].)

Finally, type in cumSum(L_2) in the top of L_3 (to find the cumSum command, press [2nd] [STAT] [►] [6], and to find L_2, press [2nd] [2]). The results should look like the screen shot below.

L1	L2	L3 1
0	.01832	.01832
1	.07326	.09158
2	.14653	.2381
3	.19537	.43347
4	.19537	.62884
5	.15629	.78513
6	.1042	.88933

L1(1) = 0

Student page answers: 1a. *24 ÷ 12 = 2 (that is, m = 2)* **1b.** *P(0) = 0.1353, or about 13.5%; P(1) = 0.2707, or about 27.1%; P(2) = 0.2707, or about 27.1%; P(3) = 0.1804, or about 18.0%; P(4) = 0.0902, or about 9.0%; P(5) = 0.0361, or about 3.6%; P(6) = 0.0120, or about 1.2%* **1c.** *1 or 2 shootings; This is reasonable because there are an average of 2 shootings expected in a 24-hour period.* **1d.**



1e. *On a day when only 2 shootings are normally expected, the probability of 10 shootings occurring is about 0.004% or 0% . So, it is not likely that it is a random occurrence.* **2a.** *about 67.7%* **2b.** *about 32.3%* **2c.** *Part a and part b describe complementary events. So, the probabilities have a sum of 1.* **2d.** *about 0.0045 or 0.5%; The probability that 6 or fewer crimes will be committed in the time period is about 0.9955 or about 99.6%, so the probability of more than 6 crimes occurring is about 1 – 0.9955 = 0.0045 or about 0.5%.* **2e.** *about 48.7%*

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Name ____ Date _______________

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Suppose *m* is the expected number of times that an event will occur based on what has happened in the past. Also suppose that the occurrences of such an event are independent. A **Poisson distribution** gives the probability that such a random event will occur in a time interval when the probability of the event occurring has a known historical distribution. The probability that an event occurs exactly *k* times is given by

$$P(m, k) = \frac{e^{-m}m^k}{k!}, \qquad k = 0, 1, 2, \ldots$$

where *e* is the base of the natural logarithm ($e \approx 2.71828\ldots$). For a given value of *m*, the formula might be shortened to $P(k) = \dfrac{e^{-m}m^k}{k!}$.

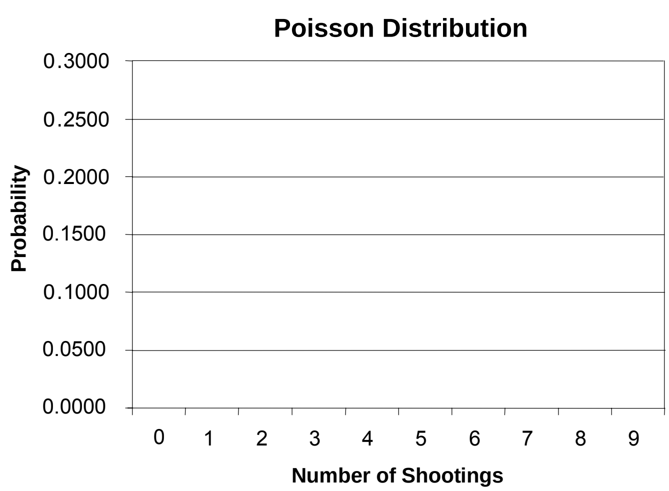
1. Suppose that on average, 1 gang-related shooting occurs every 12 hours in Los Angeles.

a. In a 24-hour day, how many shootings are expected?

b. Find the probabilities of 0, 1, 2, 3, 4, 5, and 6 gang related shootings occurring in a 24-hour day.

c. Which numbers of shootings have the highest probability of occurring? Why is this answer reasonable?

d. Using the information found in part b, draw a probability distribution below.



e. Suppose 10 shootings occur in a 24-hour day. Do you think that this could be a random occurrence? Use the probabilities you found to explain your reasoning.

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For any sample space, the sum of the probabilities of the outcomes is 1. So, for a given value of m,

$$P(0) + P(1) + P(2) + ... = 1$$

From this, you can use the **complement** of an event to help you find a probability. For example, given m, to find the probability that an event will happen more than 2 times ($k > 2$), subtract the probability that the event will happen 2 or fewer times ($k \leq 2$) from 1.

$$P(\text{2 or fewer times}) + P(\text{more than 2 times}) = 1$$
$$P(\text{more than 2 times}) = 1 - P(\text{2 or fewer times})$$
$$P(\text{more than 2 times}) = 1 - P(\text{0 times}) - P(\text{1 time}) - P(\text{2 times})$$
$$P(\text{more than 2 times}) = 1 - (P(\text{0 times}) + P(\text{1 time}) + P(\text{2 times}))$$

2. The number of crimes that are typically committed in one local precinct of Los Angeles between 11 P.M. and 2 A.M. can be thought of as a Poisson variable. In this precinct, an average of 2 crimes are committed during this 3-hour period.

a. What is the probability that 2 or fewer crimes will be committed between 11 P.M. and 2 A.M.?

b. What is the probability that more than 2 crimes will be committed between 11 P.M. and 2 A.M.?

c. How are the answers to part a and part b related?

d. What is the probability that more than 6 crimes will be committed between 11 P.M. and 2 A.M.? [Hint: Refer back to what you know from 2b.]

e. **Challenge** What is the probability that at least 1 crime will be committed between midnight and 1 A.M.? [Hint: Find the value of m for this time interval.]

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Given a set of historical data, a Poisson distribution can be used to predict the probability of certain events occurring in the future. Here you will see how Poisson distributions can be used to make predictions about natural phenomena.

- The following website has an applet that allows you to predict how many square miles of the earth's surface will **not** be hit by meteors. First predict how many you think, and then go to the site and see what the computer prediction is: **http://www.anesi.com/poisson.htm**

- Research the following web site: **http://www.nhc.noaa.gov/pastdec.shtml?** What do you think is the expected number of major hurricanes, categories 3, 4, or 5, in a given decade? Suppose the number of hurricanes hitting the United States each year is a random variable whose probability distribution can be approximated by a Poisson distribution. Use a Poisson distribution to help you decide whether or not you think the hurricane season for 2005 was typical.

Reference

Byrkit, D. *Statistics Today: A Comprehensive Introduction*. Menlo Park, CA: The Benjamin/Cummings Publishing Company, 1987.

Other Resources

- These websites provide information on how to use the TI-83 plus calculator to calculate Poisson distributions.
  http://www.cbu.edu/~wschrein/media/Stat/TI83Pman.pdf

  http://www.pballew.net/TIPoiss.html

  http://www.bbn-school.org/us/math/ap_stats/papers_folder/articles_folder/ti-83_enhanced_statistic_folder/ti-83_enhanced_statistics.html

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