Box It Up (A Graphical Approach)

ID: 4647

Time required
45 minutes

Activity Overview

In this activity, students will graph the relationship between the length of the sides of cut-out squares and the volume of the resulting box.

Topic: Number Sense and Operations

- *Use estimation for multiplication and division problems*
- *Develop estimation strategies*

Teacher Preparation and Notes

- *The activity Box It Up (Activities Exchange ID: 4646) is a tabular approach to this real-world problem.*
- *The games in this activity are designed to help students develop number sense with the operations of multiplication and division.*
- *When forming teams, students should always work with someone else to discuss strategies for number selections. It is better to have teams of three than to have individuals competing.*
- *If competition is not something you wish to foster in your classroom, you can choose to have students work in teams on problems where you have selected the range and start numbers. The goal then would be to have teams demonstrate their reasoning for reaching the goal and how they could reach it in the fewest number of moves.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "4637" in the quick search box.***

Associated Materials

- *BoxItUp_Student.doc*

Suggested Related Activities

To download the activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Box It Up (TI-84 Plus family) — 4646*
- *That's a Stretch (TI-84 Plus family) — 4255*
- *Estimation and Precise Measurement (TI-84 Plus family) — 1906*

Introduction

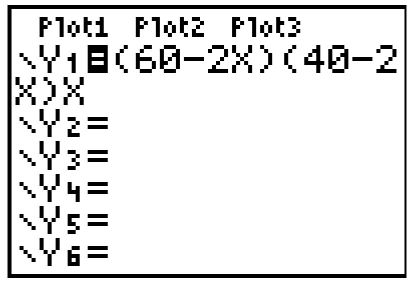
Introduce the box problem on the first page of the student worksheet. They will be finding the size of the square being cut from the metal sheet to produce the maximum volume for the open box.

Explain to students to graph a relationship on a graphing calculator, it is necessary to carry out two tasks:

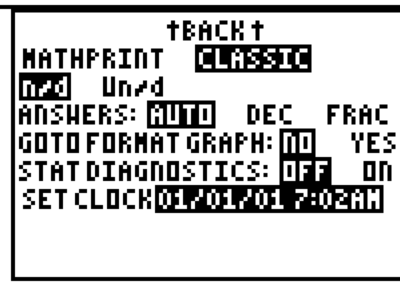
- **a.** Define the relationship in terms of two variables, *x* and *y*.
- **b.** Define the portion of the coordinate plane over which you wish to view the graph.

Question 1

Students will enter the relationship $y = (60–2x)(40–2x)x$ next to Y1 in the [Y=] screen. The keystroke to do this is [(] 60 [–] 2 [X,T,Θ,n] [)] [(] 40 [–] 2 [X,T,Θ,n].



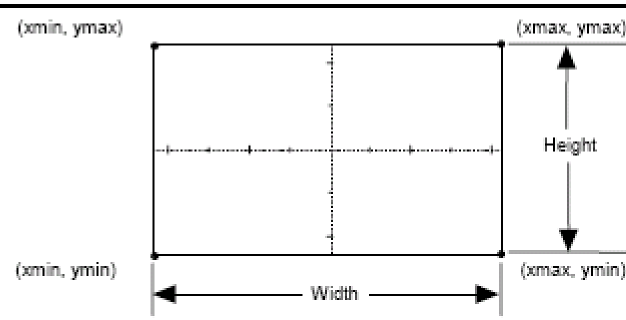
The screenshot shown above is in classic view. If using the TI-84 Plus version 2.53MP operating system, the default view is MATHPRINT. The [Y=] screen can be viewed in classic view by pressing [MODE] and selecting CLASSIC.



Question 2

When students press [WINDOW], they will see a screen that allows them to define six values: **Xmin**, **Xmax**, **Xscl**, **Ymin**, **Ymax**, and **Yscl**.

Explain to students that as illustrated at the right, the values of **Xmin** and **Xmax** define the left and right endpoints of the viewing window and the values of **Ymin** and **Ymax** define the upper and lower limits of the viewing window.



Viewing Window

You may also see a line for defining **Xres** on some calculators. This value can remain at 1 for purposes of this activity.

Discuss with students that he values of **Xscl** and **Yscl** (abbreviations for *x-scale* and *y-scale*) have no effect on the window limits nor on the appearance of the graph. They are used to define the distance between reference (or *tick*) marks that will appear on the two axes or along the left and bottom edges of any generated graph. Later in the activity when students graph the function, they can change the values for **Xscl** and **Yscl** to verify this and to help them understand why changing the scale does not change the appearance of the graph.

It will be worth the time to let some students explore what happens when **Xscl** is changed. Some will think the graph should shrink or enlarge. Let them think about why this does not happen. Avoid the temptation to just tell them. Suggest that they think of a ruler. Does the distance between decimeters change when centimeter or millimeters marks are added?

Defining the limits for the viewing windows for graphs provides students much needed practice in thinking about the function and its connection to the problem situation. They need to ask themselves "What input values make sense for this problem?" (a restricted domain) and "What output values do I get with those inputs?" (the range). When drawing graphs by hand, all too often students will just draw the two perpendicular axes, make tick marks, each 1 unit apart, and then find corresponding *y*-values to *x* = 0, 1, 2, and so on, even if it doesn't make sense to use those values for the problem situation.

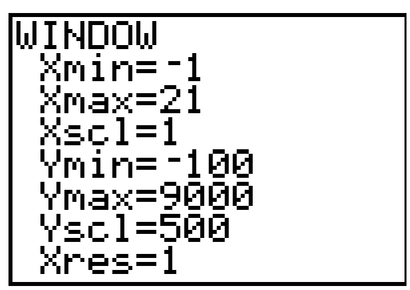
It makes sense in this problem to define **Xmin** to be 0 (or slightly less than 0 to allow for viewing near 0) and **Xmax** at 20 (or slightly more than 20 to allow for viewing near 20). If tick marks are desired, student can set **Xscl** to 1 and have tick marks at every unit along the *x*-axis.

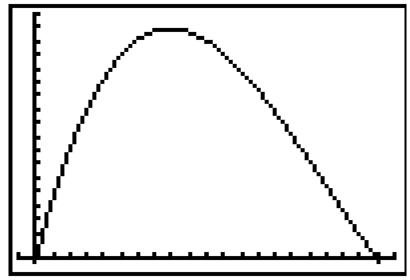
Defining the limits on *y* is a little more challenging. A given value of **Y1** represents the volume of a box for some height *x*. Volume cannot be negative, so you can define **Ymin** to be slightly less than 0. You are looking for the *x* that would produce the largest value of *y* possible. This doesn't give much of a hint about how large *y* can get. However, students previous work with tables showed that all volumes were less than 8500 cubic centimeters, so a reasonable value for **Ymax** is 9000. Since the height of this view is very large, the **Yscl** should be large. Enter 500 for **Yscl** so that tick marks on the *y*-axis are placed at 500, 1000, 1500, … , 9000.

Allow the students to reason and explain why there are many reasonable values. Number sense can be think about the choices for reasonable values for scaling the axes.

Once students understand why the ranges for *x* and *y* have been chosen, they can enter the values given on the worksheet in the WINDOW screen.

Xmin = –1	Xmax = 21	Xscl = 1
Ymin = –100	Ymax = 9000	Yscl = 500



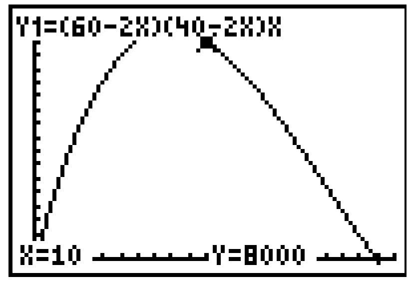


Since the tasks of defining the expression and setting the window limits is completed, it is time to view the graph.

When students press [GRAPH], they should see a graph similar to the one at the right.

Starting with x = 0, the volumes increased (fairly quickly) up to a maximum volume between x = 7 and x = 8. The volumes then decreased beyond x = 8. The graph shows this pattern of increasing and decreasing also with the maximum occurring between x = 7 and x = 8.

Question 3



When students press [TRACE], the cursor should automatically be on the point (10, 8000). The x-value of 10 cm indicates the side lengths in centimeters of the cutout square and the y-value of 8000 cm^3 is the volume of the corresponding box. The coordinates of the blinking point are displayed at the bottom of the screen.

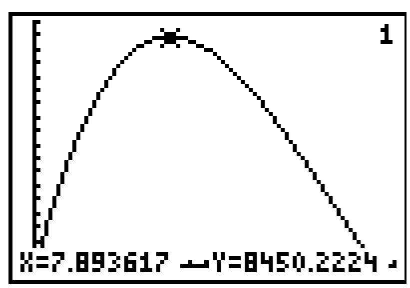
The equation graphed is displayed at the top of the screen.

You may want to ask students why the graph on the calculator is "flat" at the top. Are there several x-values that produce a maximum volume?

Question 4

Students can use the left and right arrow keys ([◄] and [►]) to see the coordinate of other points on the graph. Explain to students that the crosshair will move only along the graph and not just anywhere in the coordinate plane.

Discuss with students what they notice as they press the arrow keys. They should notice that the coordinates at the bottom of the screen change. Explain that the x-coordinates are determined by the calculator and are based upon the current values of **Xmin** and **Xmax**. The y-coordinates are computed using the expression *(60–2x)(40–2x)x*. Students will probably see that the x-values are not always "nice" numbers like those when tables of functions are generated. This sometimes makes it more difficult to decide on solutions to real world problems being modeled by the graphs.



Students are to try to locate the highest point on the graph. Since the equation of the graph crashes with the top of the graph, students can press [2nd] [FORMAT] and select **ExprOff**. Then press [GRAPH] to return to the graph and press [TRACE] to view the coordinates.

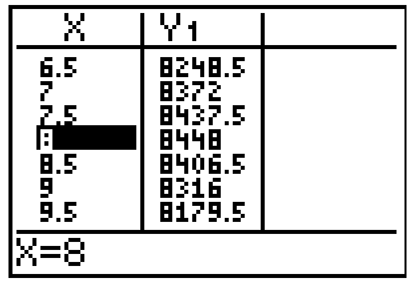
Discuss with students that not all of the decimal places shown in the coordinates of this point are meaningful. In fact, this point may not be the one you are looking for. It is just the closest point on this view of the graph. By examining the *x*-coordinates of the points on *each* side of this point, you can find an interval that contains the best value of *x*.

Question 5

Students are to use the arrow keys to determine the *x*-coordinates of the points immediately to the left and to the right of the point found in Question 4 and record them on their worksheet.

Explain to students that the solution to the problem lies somewhere between these two *x*-values. Students should check to see that the tabular solutions fall somewhere between the two values they have written for Question 5.

Press [2nd] [TABLE] to access the table of numbers.

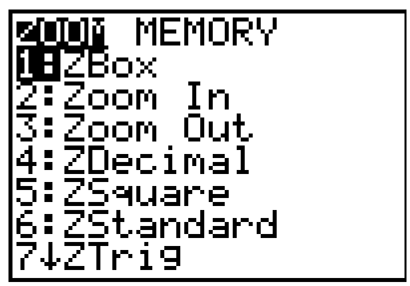


Question 6 and 7

Discuss with students that it is easy to examine the volume function over a smaller interval in order to obtain a more precise approximation to the desired solution. This can be accomplished a number of ways, but the fastest is to use the [ZOOM] capabilities of the calculator.

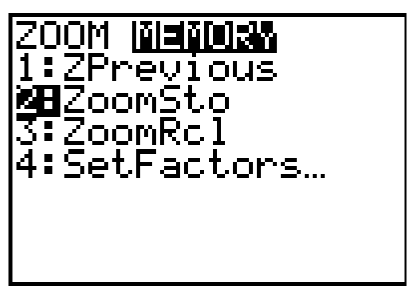
Examine the [ZOOM] menu with students. Two of the options shown are **Zoom In** and **Zoom Out**.

Zooming in is similar to bringing a portion of your current view closer to you for a finer examination—much like looking through a pair of binoculars. *Zooming out* is somewhat equivalent to enlarging your field of view so that you see more but in less detail.



Since students need a closer, more detailed look at the points near the apparent maximum of the graph, they must zoom in around the region containing that maximum. A quick way to zoom in is to use the option named **ZBox**. **ZBox** lets you use the cursor to select opposite corners of a box to define the portion of the current view that you wish to enlarge.

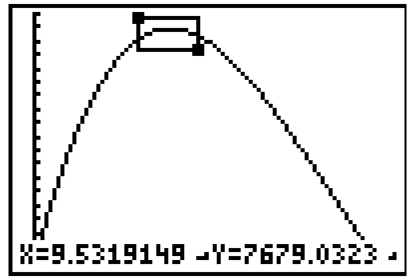
On some calculators, it is possible to save the current graph prior to zooming. That way if the students are not satisfied with their "zoom", they can easily start over again. Once they press [ZOOM], have them arrow over to **MEMORY** and select the **ZoomSto** option. Then press [ZOOM] again and continue with the instructions given. Whenever they want to return to the stored zoom, they need to press [ZOOM], **MEMORY**, and select **ZoomRcl**.



When students hold down the arrow key as they create the box, the box disappears until they stop pressing the key.

Directions are given on the student worksheet for how students should select the box. Essentially they will need to press [ENTER] at the location for the upper left corner and press [ENTER] at the location for the lower right corner.

When student press [ENTER] the second time, the calculator should redraw the graph in the window they have just defined.

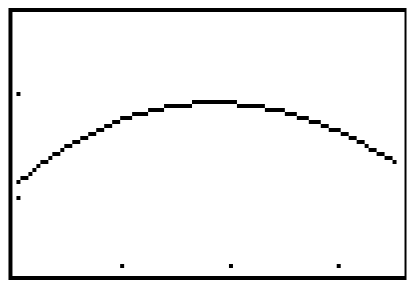


Question 8 – 10

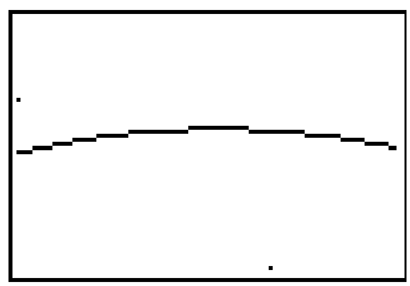
If the tenths position in the *x*-interval is not the same for both values, or if both *x*-values when rounded to the nearest tenth are not the same, then the best choice for *x* we can make has to be a whole number. If students zoom one more time on the place where they believe the maximum volume will be, they will be able to provide a value for *x* that is accurate to the nearest tenth.

Students are now learning a bit about false precision and accuracy. To ensure that the *x*-value chosen is accurate to the nearest tenth, we need to find an interval of values where the tenths digits are the same when any of the *x*-values are rounded to the nearest tenth.

Students are to press [ZOOM] and select **Zbox** again. They should make another box around the area on the graph where they think the maximum volume lies. They should also press [TRACE] to find new *x*-values for an interval containing the maximum volume as they have done previously.



Students may have to do several zooms in order to get values in an *x* interval whose tenths digits round to the same number. If they zoom in with a tiny box around the maximum value, they may arrive at such an interval sooner, for example (7.846, 7.847).



Ask students if they notice how the graph appears to be getting flatter and flatter as one zooms in on the maximum value. Several interesting “upper level” mathematical ideas can spring from this investigation: the fact that every graph is “locally linear”, that is, that all graphs will appear to be a straight line as we zoom in on smaller and smaller intervals of *x*, and that the slope of the tangent line at the local maxima and minima of a function is zero.

Remind the students that they cannot simply read off the volume from the graph as the volume is a computed value based on the *x*-value given. They need to compute the volume based on their chosen value for *x*.

Student worksheet – Selected sample answers

1. Because 1 centimeter is the smallest whole centimeter cut and if more than 19 centimeters is cut, the box would have no volume (one of its dimensions becomes less than or equal to zero.)
4. x = 7.893617, y = 8450.2224
5. x left = 7.6595745, x right = 8.1276596
6. x = 7.8388411, y = 8450.4392
7. x interval: (7.78, 7.89)
9. Side length of square: 7.8 cm
   H = 7.8 cm L = 44.4 cm
   W = 24.4 cm V = 8450.208 cm^3
10. Advantages will vary based upon the group discussion. Some students prefer the numerical/tabular approach. The Δ**Tbl** feature makes "zooming in" on a table fairly easy. Others prefer a picture and would rather zoom in on the graph for a maximum value. Sometimes when viewing a finite table of values, it is difficult to get a sense of how the function "behaves" over a larger interval; the graph provides a "bigger picture". One can find a maximum value fairly quickly on the table without having much sense about how the function behaves. When creating a graph, you have the additional challenge of defining the window. This requires some knowledge of the behavior of the function.