1. In class discussion, revise the meaning of 'factors'. 2. Provide pairs of students with copies of Game Board 1 from Factor Games. 3. Read through the game instruction with students and explain how to play the game using the Sample Game on the PowerPoint. 4. Have students play the Factor Game in pairs – maybe change partners after two games. 5. After students have played the game, lead the class in a shared discussion about the game. Ask such questions as: - "How many points would you score (at the start of a game), if I chose the number 24?" Closed question, designed to clarify the rules and establish if students understand factors and the point scoring system. Personalising the question encourages greater student engagement. - "How many points would you score (at the start of a game), if I chose the number 18?" Closed question, rote rehearsal, but also used to as a precursor for the next question. - Suppose I start the game and choose 18. It's now your turn and you choose 24. How many 'factor' points will I get? - Why is this result different than before? What other numbers would be better? This question further develops understanding of the game rules and leads students into thinking about common factors and subsequent strategies. 6. Refer to notes Using Your Calculator – Factor Game at the end of this section and PPT discuss with students how they can use their TI-30XB MultiView[™] to speed up the game and make it more interesting. 7. Have students play more games incorporating use of the TI-30XB MultiView[™]. 8. Ask students to complete the reflection questions on Worksheet 1. W1 9. Repeat steps 4-8 using Game Board 2. 10. For more detailed Teacher Notes please refer to section following Using Your Calculator – Factor Game. 11. Extension: Refer to Interactive Learning Objects sheet for links to web-sites that IWB present the factor game and other activities dealing with factors. You may wish to use these as whole-class activities on the interactive whiteboard. They can be found at the end of the Detailed Teacher Notes – Factor Game.

Using your Calculator

It is possible to expedite the 'factor – finding' process using the calculator. The screen opposite shows that 11 is a factor of 55 since no remainder is present for $55 \div 11$. Furthermore, this calculation helps reinforce the 'factor pair' concept and will lead to subsequent identification that numbers such as 25, 36, 49... are different.

Optional Calculator Use

Additional functionality on the TI-30XB Multiview[™] X can expedite the factor finding process. Suppose a student wishes to search for factors of 36.

Begin by entering an equation:

Press the table key.	Ч=
Type in the expression: $36 \div x$ Press $x_{abc}^{y \equiv t}$ once for x. Press enter to store the equation: $y = 36 \div x$	9=36÷n∎
Select 'Ask – x' followed by OK	Start=O Step=1 Auto <mark>Hsk=2</mark> OK
Type in values for x to see the result. This shows the factor pairs for 36: 2×18 and 3×12 etc	x 2 3 18 12 x=4

DEG

55÷11 55÷22 2.5

Using the table feature has a couple of benefits:

- Speeds up the calculation process
- Students learn more about calculator navigation, which may be useful for future explorations
- Introduces other concepts such as a *table of values*.

Detailed Teacher Notes

The purpose of this game is for students to become familiar with factors. Games increase student motivation and engagement. It is very important that some discussion take place about 'factors' before and after the game. Discussion is important for verbal/linguistic² students and will provide feedback with respect to current student knowledge and understanding of factors. Use this dialog as a formative assessment strategy to determine whether students are ready to proceed to factor trees as a visual way of representing factors.

Playing the Game:

When students are playing the game, much discussion about who goes first generally takes place. Students can take it in turns to go first, this dialog between students should not be dismissed. This is an opportunity to enquire *is there an advantage to going first and what do you think is the best starting number*? The role played game at the start of the lesson provides an implicit stimulus to this question. As students are playing the game, explicitly challenge them to think about why an advantage might exist for the player going first. This is one of the reflection questions for students at the conclusion of the first round of game playing.

The significance of common factors is one of the most obvious facets to the game. Leading from this understanding, students can be encouraged to think about which numbers are the *most frequent* common factors. This leads into another important aspect to this game, prime factors. An understanding of prime factors is useful for identifying the best starting number. Students are unlikely to make this connection initially. The purpose of the second game board, and any subsequent game boards is not just to increase rote – rehearsal, but also to develop generalisation. It is the generalisation that helps establish the connection between the ideal starting number and an understanding of prime factors.

Prime Factors - Perfect Squares - Starting Number

To stimulate student thought about prime factors, ask students to consider *what factors will automatically be included if 6 is a factor? What factors will automatically be included if 10 is a factor?* These questions aim to increase conceptual understanding in students in relation to prime factors.

The best starting number is the one with the least number of prime factors. The expression *least number* includes repeated factors. For example, consider the prime factor expression for 36:

$$36 = 2^2 \times 3^2$$

Compare this with the prime factor expression for 6

$$6 = 2 \times 3$$

These two numbers have the same prime factors, but these factors are repeated for 36 resulting in a large quantity of factors for the number 36.

On the first board the number 25 is a perfect square and an excellent starting value. The factors of 25 are:

25, 5 and 1

This means the opponent scores:

5 + 1 = 6

Not all perfect squares will be a good starting number. For example: 36 is a perfect square, however 6 is not a prime number. It has prime factors 2 and 3. These factors can then be used to form other factors. This reasoning and logic drives later lessons in this unit on prime factorisation.

On the second board, the number 49 appears. Since $7^2 = 49$ and 7 is a prime number, the only factors of 49 are:

1, 7 and 49

This results in the opponents score:

1 + 7 = 8.

A larger board consisting of 70 squares would include the perfect square 64. The number 8 is not a prime number; it can be expressed as:

 $8 = 2 \times 2 \times 2$ or $8 = 2^3$

The number 64 can be written as:

 $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ or $64 = 2^6$

Other factor representations would therefore include:

 $2 \times (2 \times 2 \times 2 \times 2 \times 2) \text{ or } 2 \times 32$ $(2 \times 2) \times (2 \times 2 \times 2 \times 2) \text{ or } 4 \times 16$ $(2 \times 2 \times 2) \times (2 \times 2 \times 2) \text{ or } 8 \times 8$

The factors of 64 are therefore 1, 2, 4, 8, 16, 32 and 64. So even though 64 is a perfect square, it is not a good starting number. This posses the question, on a 70 square board, what would be the best number to choose? A good starting point for this search is to consider the product of prime numbers, examples include: 5×13 , 3×23 or 7×7 . As a problem solving task this is an example of working backwards.

Review Questions

The second game board provides the opportunity to extend student knowledge of factors of numbers and could be extended to a 100 square game board (possible homework game). As mentioned, the second board is not only about rote rehearsal, it is the driving force towards generalisation.

Key questions for students moving towards generalisation:

- What if the board had 100 squares...
 - What would be the best starting number?
 - What would be the worst starting number?
 - What would be the combined score for a 100 square board?
- Can you determine the best starting number for any sized board? (Open)

Learning Objects – Factor Game

This applet is designed to be used by teachers / students for the first worksheet 'Factor Game'. An applet for this game already exists on the Illuminations web site by NCTM. http://illuminations.nctm.org/activitydetail.aspx?ID=12 Identical games can also be found at: http://connectedmath.msu.edu/CD/Grade6/FactorGame/ http://www.phschool.com/atschool/cmp2/active_math/site/Grade6/FactorGame/index.html

References:

1. Rote rehearsal - How the brain learns mathematics (David A. Sousa, Hawker Brownlow)

2. Verbal / Linguistic – Gardiner's Multiple Intelligences