EXPLORATIONS

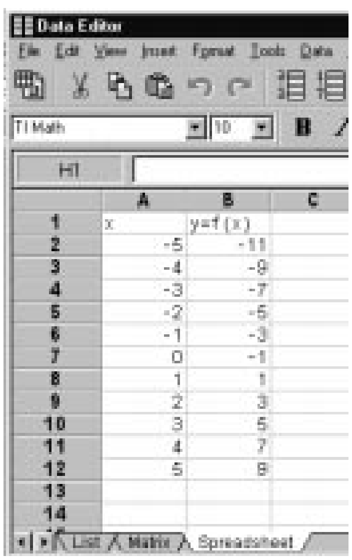
Activity 2

Tight Rope

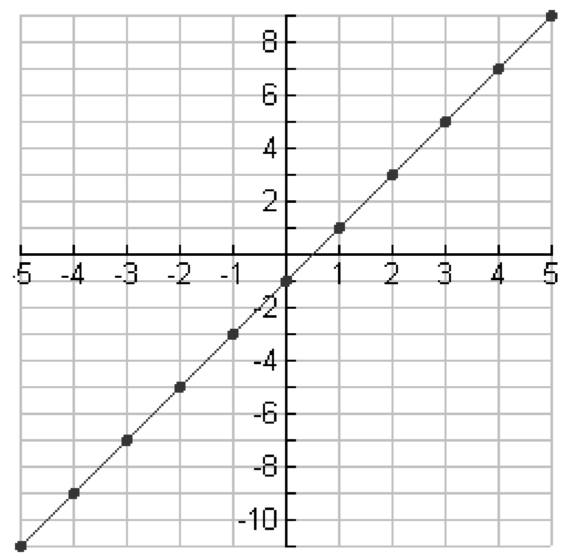
When one quantity changes at a constant rate with respect to another, we say they are related linearly. Mathematically, we describe this relationship by defining a linear function, f(x), which can be represented with a straight-line graph. In many real-world applications, quantities are linearly related and can be visually represented using a straight-line graph.

Introduction

Compare this TI InterActive!™ spreadsheet and the graph of the data points:



	A	B	C
1	x	y=f(x)	
2	-5	-11	
3	-4	-9	
4	-3	-7	
5	-2	-5	
6	-1	-3	
7	0	-1	
8	1	1	
9	2	3	
10	3	5	
11	4	7	
12	5	9	
13			
14			



As the independent variable *x* values increase by one, the dependent variable *y = f(x)*, or function of *x*, values increase by the same constant value of 2 at each step.

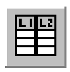
In this activity, you will create constant-speed distance versus time plots and develop linear equations to describe these plots mathematically.

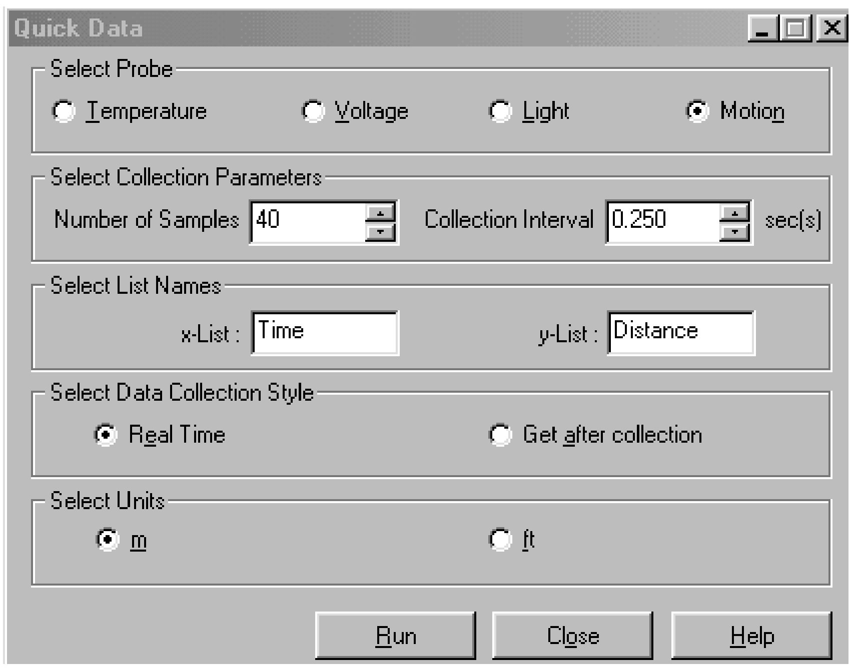
Equipment Required

- Computer
- TI InterActive! software
- CBL™ or CBL 2™ unit
- Motion detector
- TI-GRAPH LINK™ cable

Setup

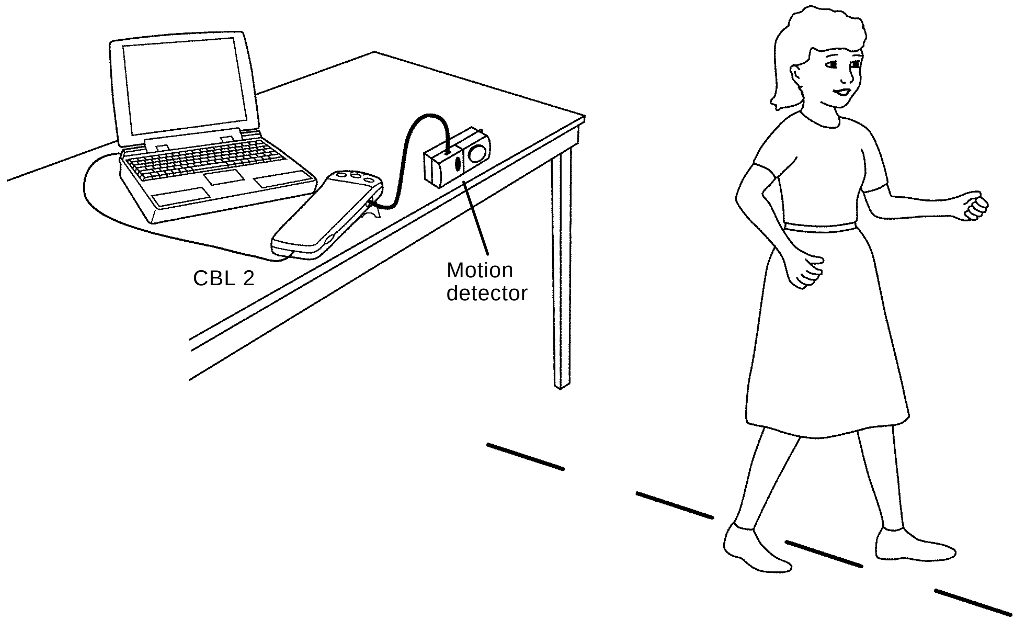
1. Plug the TI-GRAPH LINK™ cable into your computer.
2. Plug the other end of the TI-GRAPH LINK cable into the CBL™/CBL 2™.

   Note: *Alternatively, you can use a CBR™ for this activity.*
3. If you are using a CBL, plug the motion detector into the **Sonic** port. If you are using a CBL 2, plug the motion detector into the **DIG/SONIC** port.
4. Place the motion detector on a table with the front of the unit facing forward. Be sure that you have a clear path in front of the detector.
5. Start TI InterActive!™ The software opens to a new, blank document.
6. Title your new document *Tightrope* and add your name and the date. Press the Save button  to save and name your document.
7. Click the List button  to open the Data Editor.
8. From the **Data** menu, choose **Quick Data**.
9. Adjust the Quick Data dialog box so that the settings match the ones shown below.

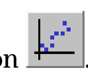


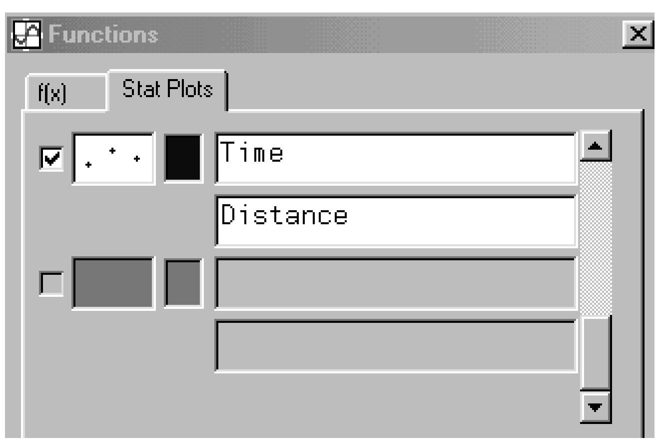
Collecting the Data

1. Place the motion detector on a table or desk and stand at least 50 cm from it. Aim the motion detector at the walker as shown in the illustration below.



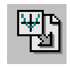
2. When you are ready to start collecting data, click **Run** in the Quick Data dialog box and start walking away from the motion detector at a slow, steady pace. You will have 10 seconds to collect the data.

3. When data collection is done, click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data. You should also see the Functions dialog box as shown below.



4. The plot of straight-line distance versus time should be linear. If you are not satisfied with your data, click **Run** in the Quick Data dialog box to perform another trial. If you are satisfied with your plot, move to the box below the graph and enter a **0**, then press Enter. This will change the minimum value of *y* to 0 and allow you to see the *x*-axis.

5. You can make a sketch of your time versus distance data plot on one of the blank grids in the Appendix. Label the horizontal and vertical axes on your sketch.

6. Click the Save to Document button  to save the graph in your TI InterActive!™ document.

Analysis and Questions

1. The slope-intercept form of a linear equation is:

$$Y = MX + B$$

where M is the slope or steepness of the line and B is the y-intercept or the starting value. In this activity, the control variable, X, represents time, and Y represents distance. Press 

and use the right and left arrow keys in the Trace Value dialog box or use your keyboard to move the cursor along your distance versus time plot. Identify the starting value (the Y-value when $X = 0$) and record this below as the intercept, B.

$B =$

2. Since you have a value for B to find the equation of the line that fits your data, you will use a guess-and-check method to determine the value of M. With your mouse, highlight the B value in the Trace window and press Ctrl+C to copy it.

Close the Trace dialog box, and click the f(x) tab in the upper left corner of the Functions dialog box. Start with an initial guess of $M = 1$. Type **f(x):= 1*x + B** (your value of B) in the uppermost text box of the f(x) tab. Press Ctrl+V to paste your B value. Record your guess for M in the box below. For example, **f(x):= 1*x + .599289.** Press Enter to superimpose the graph on the plotted data.

It is unlikely that your first guess for the value of M produced a model that matched the data closely. Click in the text box of the f(x) tab again and edit the linear function, replacing the old value, $M = 1$, with your new guess for M. Press Enter to update the graph. Repeat the guess-and-check procedure until you find an M-value that models the data well and record your guesses in the spaces below:

Guess #1	Guess #2	Guess #3	Guess #4	Final *M*-value
$M =$	$M =$	$M =$	$M =$	$M =$

Using this value of M and the B-value determined in questions **1** and **2**, complete the slope-intercept form of the equation and record it below.

3. Another way to get a linear function model for the data is to press 

again. Move along the plot with the arrow keys and identify two points (*x*1, *y*1) and (*x*2, *y*2) and record them below. Try to pick the points that are not too close together.

x1	y1	x2	y2

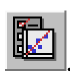
When the coordinates of two points on the same line are known, the slope of the line can be computed by finding the difference in *y*-values divided by the difference in *x*-values:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

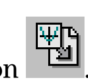
Use this formula to compute the slope of the linear plot and record the result below.

slope =

How does this value compare with the value of *M* you found experimentally in question **2**?

4. TI InterActive!™ lets you check the values of *M* and *B* you just found by calculating the line of best fit. Close the Graph window to return to the Data Editor. Click Statistical Regressions . Click the down arrow  next to Calculation Type, scroll down, and click on **Linear Regression (ax + b).** In the text box labeled **X List**, type **TIME**; in the box labeled **Y List**, type **DISTANCE**. Click **Calculate** to find the regression equation, $y = ax + b$ and its variables. Record the regression equation values of *a* and *b* in the table below.

y* = *ax* + *b
a =
b =

5. Click the Save To Document button . TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document. How does the value of *a* in the linear regression equation compare with the *M*-value you found by guess-and-check?

How does the value of *b* compare with the y-intercept value, *B*, you identified earlier? Explain.

6. Double-click on the graph in the TI InterActive!™ document to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regEQ(x)** and press Enter. TI InterActive! graphs the equation that was created as the Stat Regression result. Which equation seems to fit the data better? Which equation is a better linear function model? Why?

7. Remember, slope is defined as change in *y*-values divided by change in *x*-values. Complete the following statement about slope for the linear data set you collected.

In this activity, slope represents a change in ______________________________

divided by a change in ______________________________.

Based on this statement, what are the units of measurement for the slope in this activity?

8. As mentioned earlier, the intercept value, *B*, can be interpreted as the starting position or the starting distance from the motion detector. What does the value of *M* represent physically? (Hint: Think about the units of measurement for the slope you described in question **6**).

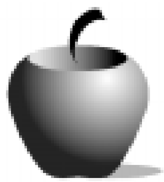
9. Save and print your TI InterActive! document.

Extensions

- Change your rate of walking speed and redo the activity with this new data. Describe any differences in the linear models.
- Start about 3.5 meters away from the motion detector and walk towards it. Describe any differences in the linear models.
- Stand in front of the motion detector and do not move. Describe any differences in the linear models.

Teacher Notes

Activity 2: Tight Rope



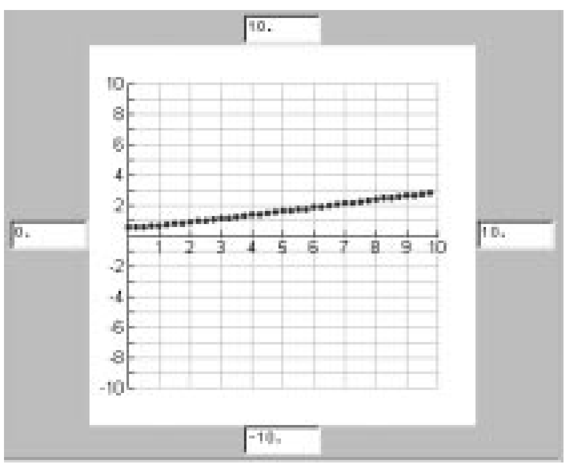
Math Concepts

- CBL™/CBL 2™
- Linear Function

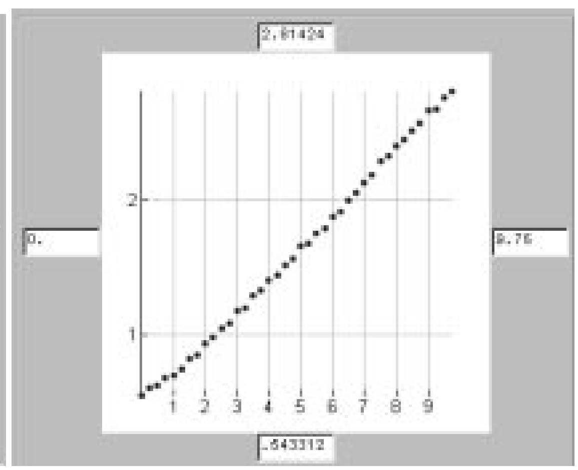
Activity Notes

- This activity investigates linear functions as models with a constant rate of change. Students will create constant-speed distance versus time plots.

Sample Data



Original StatPlot



After ZoomStat

Analysis and Questions - Answer Key

1. Answers will vary but $B \approx .5$ m (50cm). B is the distance from the motion detector at time 0.
2. Answers will vary but M represents the rate the distance from the detector is increasing with respect to time.
3. The calculated slope should be approximately equal to the slope determined by guess-and-check.
4. The regression equation for the sample data is: $regEQ(x) = .240356x + .454713$.
5. Although all the guess-and-check linear functions should ultimately be similar, one could argue that the linear regression equation is a better fit because we would all get the same linear function for the data set.
6. Change in distance divided by the change in time.
7. Slope = $\dfrac{\Delta \text{distance}}{\Delta \text{time}}$; meters per second (m/s).
8. M is the rate the person is walking away from the motion detector. In the sample data this rate is $\approx$ .24 m/s.

