"Prodigious!"

Quadratic equations and Vertical Motion

Introduction

In the movie *October Sky*, Homer and Quentin were able to use mathematics to locate a missing rocket. To determine where the rocket landed, they had to use more advanced mathematics than algebra. However, using only algebra, the vertical motion of objects in free-fall (such as a model rocket) can be studied. There is an algebraic function that describes the height in feet of a vertically launched projectile in terms of time:

$$h(t)=-16t^2+vt+s$$

h=height (feet)	t=time in motion (seconds)
s=initial height (feet)	v=initial velocity (feet per sec)

Notice that the function is quadratic, which when graphed will be parabolic.

The Flight of a Rocket

A rocket is fired from the ground with an initial velocity of 160 feet per second. If the rocket does not have a parachute:

A) When is the rocket 400 feet high?
B) When will the rocket hit the ground?
C) How can you show that 400 feet is the maximum height reached by the rocket?

1. To answer question A, first write an equation using the vertical motion function from above:
   height (h) =
   initial velocity (v)=
   initial height (s)=
   equation:

2. Next, write your equation in standard form, and solve by factoring.

3. The solution that you just found is the time that it takes the rocket to reach a height of 400 feet. Write an answer to question A in a complete sentence.

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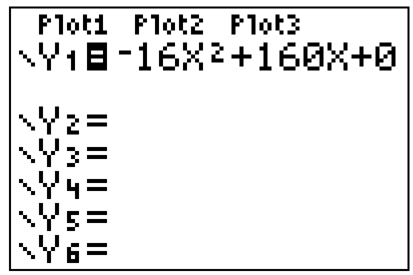
4. Since the height of the rocket when it hits the ground is zero, let h=0 and write an equation that could be used to answer question B.

5. Solve your new equation by factoring. You should get 2 answers.

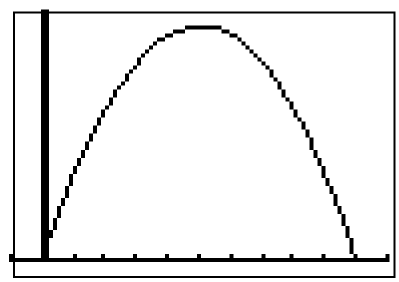
6. Choose the solution from number 5 that is the correct answer to question B. Write your answer in a complete sentence.

Calculator exploration:

Use the given information about the rocket's initial velocity and height to enter the equation in your <Y=> window that gives the rocket's height as a function of time.
(Use Y_1 for h and x for t.)

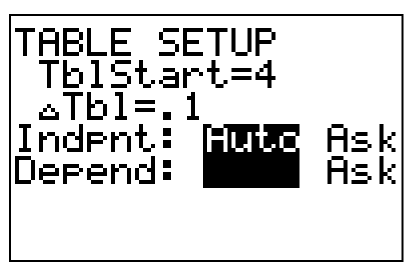


Go to the window and adjust the settings so that you can see the complete graph of the rocket's height vs. time. Notice that x-intercepts match your answers to question 5.



7. Next, use the trace function (and zoom as necessary) to find your best approximation of the maximum point (vertex) on the graph. Label it and the x-intercepts on the graph above.

Now confirm that your answer is correct. Go to <TBLSET>. Set Tblstart at 4 and ΔTbl at .1 Now go to <TABLE>. Scroll through the table until you find the maximum y value (height). Look at the x value (time).



8. Use what you learned from the calculator to answer question C in a complete sentence.

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Baseball Fly Ball

A batter hits a baseball straight up when it is 3 feet off the ground with an initial vertical velocity of 80 feet per second:

A) If the ball is not caught, how long will it take for the ball to hit the ground?

B) What is the maximum height of the ball?

1. As you did for the rocket problem, write an equation that can be solved to find when the ball will hit the ground.

Unlike the rocket equations, the above equation cannot be factored. Therefore, you are going to solve it by using the quadratic formula.

Reminder: For a quadratic equation in standard form $ax^2+bx+c=0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. For your equation:
   a=
   b=
   c=

3. Solve the equation and use a calculator to find decimal values for the solutions. Round your answers to 3 decimal places.

4. Consider your solutions. Remember that x shows the time that the ball is in the air. What does the negative solution mean?

5. Write an answer to question A of the baseball problem in a complete sentence.

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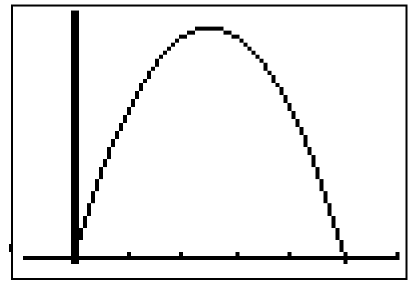
Calculator exploration:
Repeat the process on the calculator that you did with the rocket problem for this equation. Since the equation is different, the window and table settings will be set differently as well.

6. Using the graph on the right, label the points that show the initial height, the maximum height, and the time when the ball is on the ground.



7. Use what you have learned to answer question B in a complete sentence.

Solving Quadratic Equations on a Spreadsheet*
Since using the quadratic formula on paper and pencil can be time consuming (it's also very easy to make a mistake), you are going to create a spreadsheet that will solve quadratic equations for you.

Open up your spreadsheet program. You will be using the first 5 columns. Label cell A1, "a", cell B1, "b", and cell 1C,"c". Label cells D1 and E1 "solution #1" and "solution #2", respectively. In row 2, enter the values for a, b, and c from the baseball problem. Since you already know the correct solutions, you will be able to check if your spreadsheet formulas are correct.

	A	B	C	D	E
1	a	b	c	solution #1	solution #2
2	-16	80	3		
3					
4					
5					

You know what the quadratic formula is. You will need to rewrite it in a form that the spreadsheet can recognize. Some things to remember:

A) All spreadsheet formulas start with "="
B) Variables are identified by cells, not letters. For example, the variable "a" in the quadratic formula will be entered as "A2" in the spreadsheet formula.

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C) Since you can’t enter a ± into the spreadsheet, you will have to enter the formula twice. (cell D2 for “+” and E2 for “-“).

D) It is very important that you remember order of operations! Carefully use parentheses to make sure the computer does things in the correct order.

E) You will know that you have correctly entered the formula when your solutions in cells D2 and E2 match your answers to question 3 of the baseball problem.

1. Write the correct formula that you entered in cell D2 exactly as it appears on the spreadsheet.

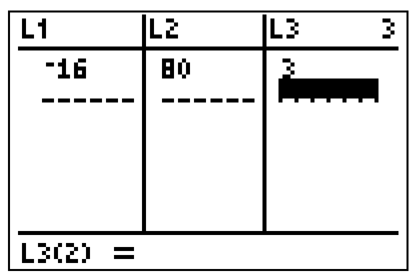
Use the edit menu or drag the cell to fill down your formulas to at least row 10. (Note: you will get error messages in your solution columns while your cells in column A are empty. You will also get an error message if you enter values for a quadratic equation that has no solution. For this lesson, all equations will have a real solution.)

***Graphing calculator option:**

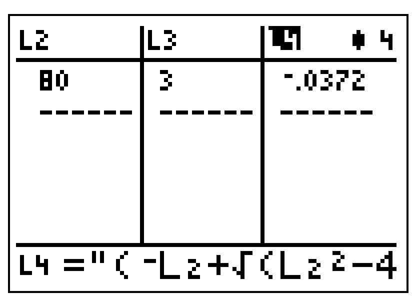
Instead of using a spreadsheet, you can use the lists on your calculator.

First, clear all lists <2ND><MEM><4>
Press <ENTER>

Press <STAT><Edit><Enter> and enter the values below.



L4 and L5 will be your solution columns. Highlight L4 and begin the quadratic formula. Start with “ marks. Use <2nd><L1> for a, <2nd><L2> for b, and<2nd><L3> for c. Read C), D), and E) in the spreadsheet directions, substituting L4 for D2 and L5 for E2.



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You will finish this investigation by exploring several free-fall or vertical motion problems. For each problem,

- write a quadratic equation or equations
- solve the equation or equations by any method you choose
- sketch the graph of the equation, labeling all points that are part of the solution (x-intercepts, maximum heights, final height, point of intersection, etc…) If a problem involves 2 different quadratic equations, sketch them together using the same set of axes.
- clearly state the solution to the problem in a complete sentence or sentences.

Problems.

1. You are standing on a bridge over a river, holding a rock 36 feet above the water.
   A) If you release the rock, how long until it hits the water?
   B) If you toss it up instead, with an initial velocity of 20 ft/s, how long until it hits the water?

2. Bradford threw a tennis ball to Dina, who was on a hotel balcony 24 feet above him, with an initial speed of 44 ft/s. Dina missed the ball on the way up, but caught it on its way down. How long was the ball in the air?

3. You and a friend, armed with water balloons, go to the roof of a 15-story building and look over the edge to the ground 200 feet below. You drop your water balloon at the same time that your friend throws his straight down at 40 ft/s. By about how many seconds does his balloon beat yours to the ground?

4. Miguel says that he can throw a baseball straight up at 30 mi/hr (about 44 ft/s). Kevin says he can throw it up twice as fast, and that it will go twice as high and stay up twice as long. Assuming that Kevin really can double Miguel's velocity, is Kevin right? Explain.

5. Some time in the future…A man is on a scaffold 100 ft above the ground on the earth. A woman is on the moon, also 100 feet up. Both are going to drop a hammer and they want the hammers to land at the same time. If the equation for the hammer's vertical motion on the moon is $h=-2.7t^2+100$, how long should the man wait before he releases the hammer?

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