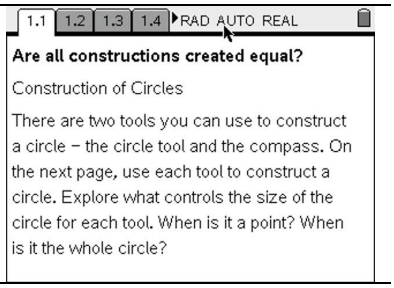
Are All Constructions Created Equal?

STUDENT ACTIVITY

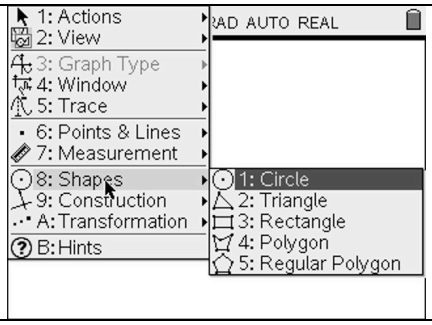
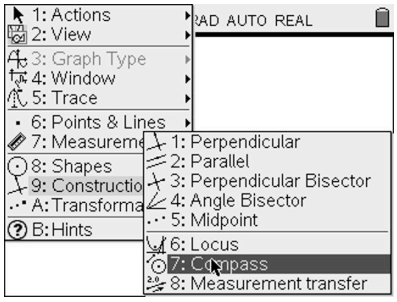
Using a dynamic geometry program opens the door to multiple approaches to problems. This activity is designed to familiarize you with the circle, compass & line tools and how constructions may look the same, but underneath are very different.



- **Open *the* TI-Nspire document *Constructions.tns.***
- **Press ctrl ▸ to move to page 1.2 to begin the lesson.**

1. **Problem 1:Construction of Circles**:
   There are two tools you can use to construct a circle - the circle tool and the compass. The circle tool is used to construct a circle where the first click is the center of the circle and the second forms the radius. The compass tool constructs a circle from a center point with a radius defined by a segment or distance measurement. On the page 1.2, use each tool to construct a circle. Both tools can be access through the menu button (menu). Explore what controls the size of the circle for each tool. When is it a point? When is it the whole circle?

2. Write your findings in the table below

	Circle tool	Compass tool
		
What controls the Size of the circle?		
What moves circle without changing size?		

3. There are many ways to construct two intersecting circles. On the pages 1.4 – 1.8, are two intersecting circles that look the same but are constructed differently. Note how each figure is different and what controls the size of the circles.

4. Write your observations in the table on the following page.

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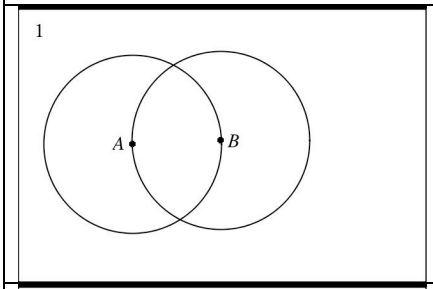
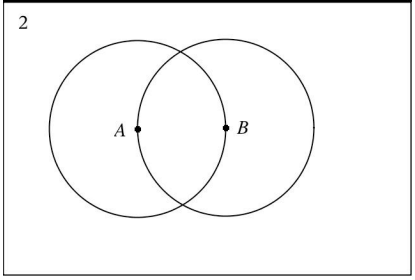
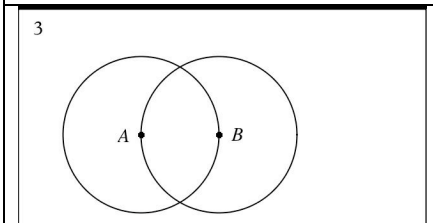
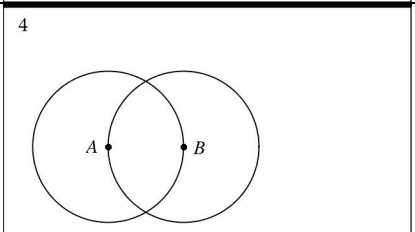
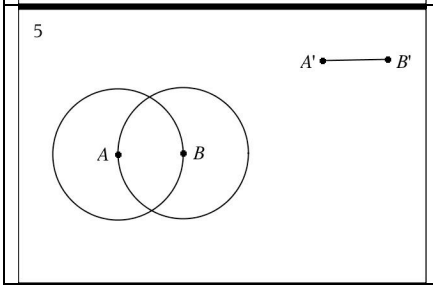
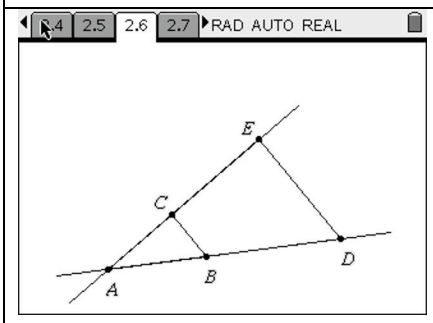
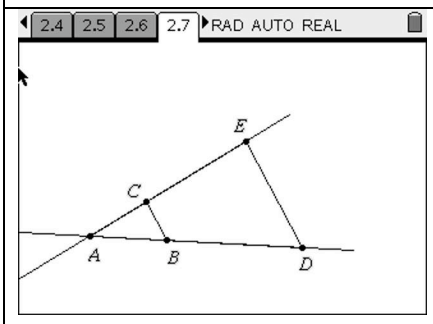
		What controls the Size of the circles?	What moves circle without changing size?
1 	The two circles were constructed using the Circle tool. Both Circles have radius equal to $\overline{AB}$. Circle A was constructed first.		
2 	The two circles were constructed using the Compass tool. Point A was constructed first, point B was added and the distance between point A and B was used for the radius.		
3 	Point A was constructed first using the point tool. With the circle tool, point B was constructed and point A clicked.		
4 	A segment $\overline{AB}$ was constructed first (hidden) and the circle tool was used to construct two circles with centers A and B and radius $\overline{AB}$.		
5 	A segment $\overline{A'B'}$ was constructed first. The compass tool was used to construct two circles with centers A and B and radius $\overline{A'B'}$.		

5. **Problem 2**: **Shapes that maintain their characteristics**:
   Problem 2 is an investigation into the construction of triangles. On page 2.2 is triangle that maintains it shape no matter how you move the points, that is all the triangles that are constructed as you move are similar to each other. This construction is based on the intersection of two circles. The circles are hidden, but you can see them by using the Hide/Show tool. Choose Menu, Actions, Hide/Show (menu 1 3.) The circles should be seen but dimmed in the background. Based on your investigation above, how could these circles have been constructed? What kind of triangle is ΔABC? Justify your choice.

Our constructions above were based on the circles. We can construct other kinds of triangles that remain similar, but we'll need more tools.

6. **Constructions with lines**: Use page 2.4 to experiment with the line tool in Menu, Points & Lines, Line (menu 6 4) and segment tool (menu 6 5). Notice how the line changes when you grab the line or the point. What is the difference between a line and a segment? Write your findings below.

7. On pages 2.5 & 2.6, you will see triangle ADE, which can be manipulated to form similar triangles. Notice in both cases, that point D is the control point for size change. What do point A and E control for each? These constructions are based on lines and circles. Again to see the hidden parts of the construction, use the Hide/Show tool. The circle in this case depends on the ratio of the measurements of the segments to guarantee the similarity. Triangle ABC forms the original triangle. Show the points A & C, the sides of the triangle and the lines AB and AC by clicking them. When the points, sides and lines are no longer dim, press esc. Your picture should look similar to those in the table on the next page. Let us focus on how the lines were constructed and used. In the table on the following page, read how the sketches were constructed and note the differences in how the triangles can be moved or reshaped using the points or the lines/segments.

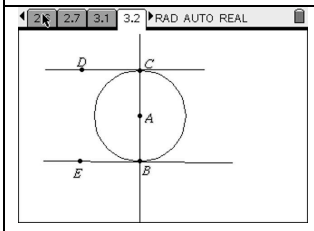
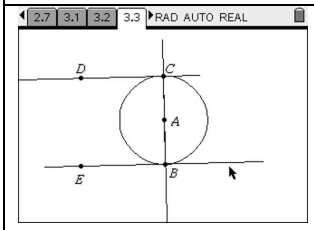
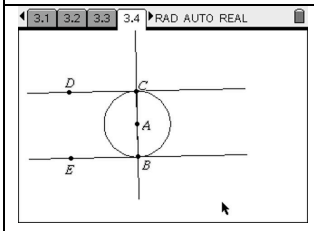
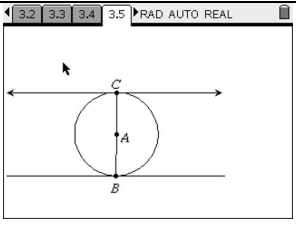
	Triangle ABC was constructed using lines, whose intersection is the control point A for each as one vertex and free points C and B the other vertices. Triangle ADE was constructed with free point D on line AB and the length of segment AE was based on the ratio of the sides AD/AB times the length of AC.	
	Triangle ABC was constructed using lines, whose intersection is point A. The vertices at points C and B are the control points for lines AC and AB. Triangle ADE was constructed with free point D on line AB and the length of segment AE was based on the ratio of the sides AD/AB times the length of AC.	

Describe how the sketches are similar.

Describe how the sketches are different.

8. Problem 3: Construction of Tangent Lines.

Problem 3 provides a look at the construction of tangent lines. There are four different constructions. Determine how the sketches were constructed and what these constructions can tell you about the student's understanding of tangent lines. Write your observations in the table below.

	How was the sketch constructed?	What does the construction tell you about the student's understanding of tangent lines?
		
		
		
		

9. Problem 4: Construction of Squares:

In problem 4, you will find 4 sketches of square ABCD that look the same, but are constructed differently. The sketches were modified for the NSpire and correspond to the following article:

Scher, Daniel, (2005) "Square or Not? Assessing Constructions in an Interactive Geometry Software Environment." in Technology Supported Mathematics Learning Environments. NCTM.Yearbook, pages 113 – 124.

Before reading the article, go through the four sketches and determine how they were constructed and what the construction can tell you about a student's understanding of a square. Write your observations below. Read the article and reflect on your observations.

	How was the sketch constructed?	What does the construction tell you about the student's understanding of the properties of squares?