The Geometric Mean

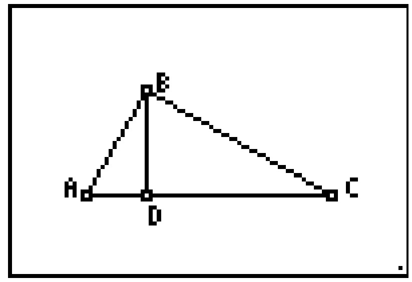
Student Activity

Name ___________________

Class ___________________

Introduction

Consider right triangle ABC with right angle $\angle ABC$ and an altitude from the right angle to the hypotenuse. Therefore, $m\angle ADB = 90°$ and $m\angle CDB = 90°$. Using our previous work with similar triangles, we could conclude that $\triangle ABC$ is similar to $\triangle ADB$ and is similar to $\triangle BDC$. When we create proportions from the two smaller triangles we get:



$\frac{AD}{BD} = \frac{BD}{DC}$. Simplifying this we have $BD^2 = AD \times DC$.

The line segment $\overline{BD}$ is called the **Geometric Mean** between $\overline{AD}$ and $\overline{DC}$.

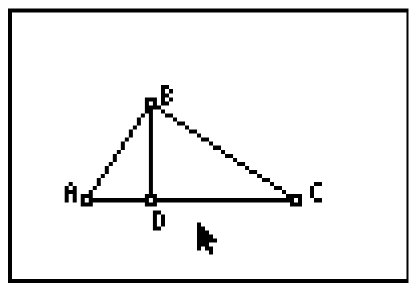
The Geometric Mean

Open the *Cabri™ Jr.* application by pressing [APPS] and selecting **Cabri™ Jr**. Open a new file by pressing [Y=], selecting **New**.

Construct $\overline{AB}$ and a perpendicular to $\overline{AB}$ through *B*. Then, use the **Point On** feature to add a point *C* on the perpendicular line.

Hide the perpendicular line and construct line segments connecting *A* to *C* and *B* to *C*. Now construct altitude $\overline{BD}$, such that point *D* lies on $\overline{AC}$.

You should now have right triangle $\triangle ABC$, with altitude $\overline{BD}$, similar to the triangle shown to the right.

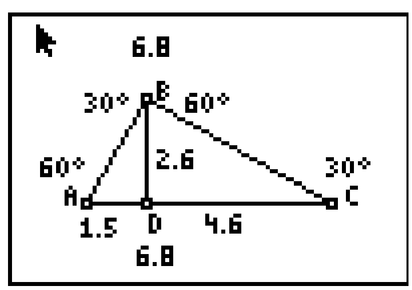


Construct line segments $\overline{AD}$ and $\overline{DC}$. Measure the lengths of $\overline{AD}$, $\overline{DC}$, and $\overline{BD}$.

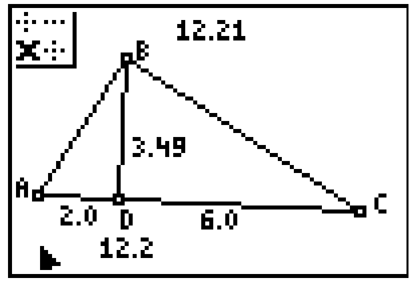
Use the **Calculate** feature to find the product of the lengths of $\overline{AD}$ and $\overline{DC}$.

Then, use the **Calculate** feature again to find the square of *BD*. To do this, click on the measure of $\overline{BD}$, press [×] and click on the measure of $\overline{BD}$ again. The square of *BD* should be equal to the product of *AD* and *DC*.

To verify that the triangles are similar, measure the angles in the figure. All of the angles in this sketch have been measured.

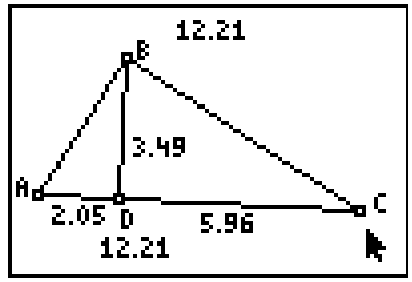


Drag point *A* and point *C* so that *AD* = 2 and *DC* = 6. It may be very difficult to get these values exactly due to the screen resolution. In this sketch, the accuracy of the lengths of $\overline{AD}$ and $\overline{DC}$ are shown to one decimal place.

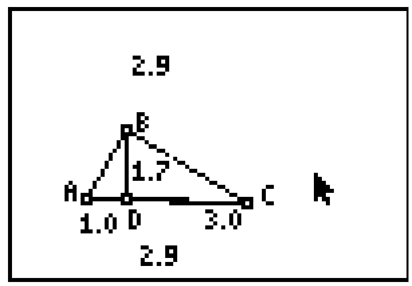


Press CLEAR and move your cursor over the measurements for $\overline{AD}$ and $\overline{DC}$. Press + to increase the number of digits of accuracy. Now you can see that the lengths may or may not be exactly *AD* = 2 and *DC* = 6. Can you make the measurements more exact? If you can, then what you have accomplished is to construct a line segment, $\overline{BD}$, that has length exactly $\sqrt{12}$.

(**Note:** Use the − key to decrease the number of digits of accuracy).



Manipulate your figure again so that *AD* = 1 and *DC* = 3. Note that the product of *AD* and *DC*, in the sketch at the right, indicates that the lengths aren't exactly 1 and 3 units, respectively.



1. If the points had been moved so that *AD* was exactly 1 unit and *DC* was exactly 3 units, what exact length would have been found for the length of $\overline{BD}$?

2. What would the lengths of $\overline{AD}$ and $\overline{DC}$ be in order for the length of segment $\overline{BD}$ to be exactly:

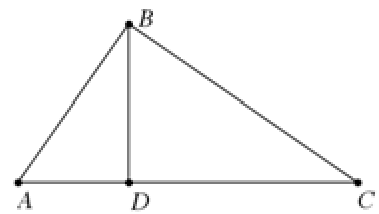
   a. $\sqrt{10}$

   b. $\sqrt{6}$

Exercises

Use the figure on the right to write a proportion using a geometric mean and solve each problem.

$\angle ABC$ is a right angle and $\overline{BD} \perp \overline{AC}$.



1. If AD = 3 inches, and DC = 27 inches, find the length of $\overline{BD}$.

2. If AD = 4 inches, and BD = 8 inches, find the length of $\overline{DC}$.

3. If AD = 6 inches, and DC = 9 inches, find the length of $\overline{BD}$.

4. If AD = 5 cm and DC = 10 cm, find the lengths of $\overline{BD}$, $\overline{AB}$, and $\overline{BC}$.

5. If BD = 2 cm and DC = 8 cm, find the lengths of $\overline{AD}$, $\overline{AB}$, and $\overline{BC}$.