The Exponential Derivative

ID: 8979

Time required
45 minutes

Activity Overview

In this activity, students will derive the derivative of the function $y = e^x$, work with the derivative of both $y = e^u$ and $y = a^x$, and look at the tangent to the graph of $y = e^x$. In the process, the students will have to find $\lim\limits_{h\to 0}\frac{e^h - 1}{h}$.

Topic: Formal Differentiation

- *Derive the Exponential Rule and the Generalized Exponential Rule for differentiating exponential functions.*
- *Use the Limit command to show $\lim\limits_{h\to 0}\frac{e^{a+h} - e^a}{h} = e^a$ and verify the Exponential Rule for differentiation.*
- *Graph the function $f(x) = e^x$ and measure the slope to the graph at any point $x = a$ to verify that $f'(x) = f(a)$.*

Teacher Preparation

- *This investigation derives the definition of the exponential derivative. The students should be familiar with keystrokes for the limit command, the derivative command, entering the exponential function, drawing a graph, and drawing the tangent to the graph.*
- *Before starting this activity, students should go to the home screen and select **F6:Clean Up > 2:NewProb**, then press* [ENTER]*. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "8979" in the keyword search box.***

Associated Materials

- *ExponentialDerivative_Student.doc*

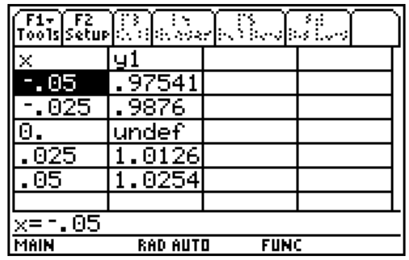
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Functions, Graphs, and Limits (TI-89 Titanium) — 4273*
- *Move Those Chains (TI-89 Titanium) — 11364*

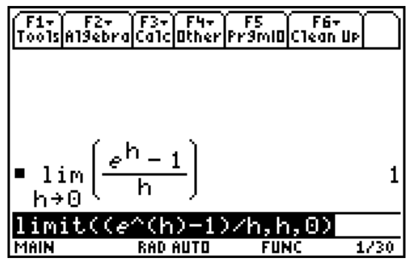
Problem 1 – The Derivative of $y = e^x$

Discuss with the students that we have the definition of e but that doesn't help us find the derivative.



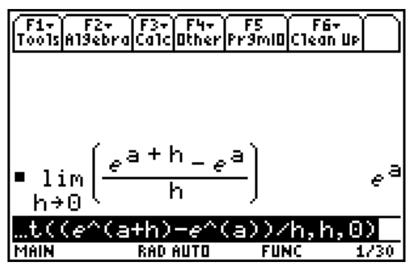
Since $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$, $e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$, and $e^{x+h} = \lim_{n\to\infty}\left(1+\frac{x+h}{n}\right)^n$, we do not have the algebraic tools to use these formulas in the definition of a derivative. In addition, our only formula for the derivative involves a variable base and a constant power. The exponential function, $y = e^x$, is a constant base and a variable power. We do not have the rules for that yet. So we have to look at the definition of the derivative with the function itself instead using the limit definition.

When we look at $f'(x) = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$, we quickly run into a problem. So we have to evaluate $\lim_{h\to 0}\frac{e^h-1}{h}$.

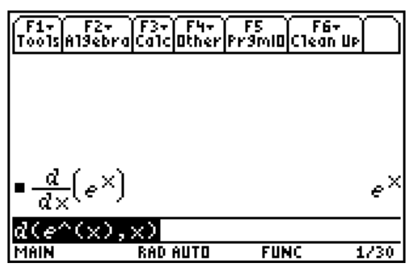


To access the **limit** command while on the home screen, select **F3:Calc > 3:limit(** or type **limit** using the alpha keys.

After going through the derivation, another step is done with the **limit** command. Remind the students to be careful with the parentheses for the exponent and the fraction.

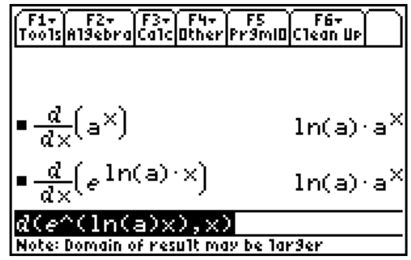


Students will also calculate the derivative directly. To access the **derivative** command while on the home screen, select **F3:Calc > 1:*d*(differentiate** or press [2nd] [d].



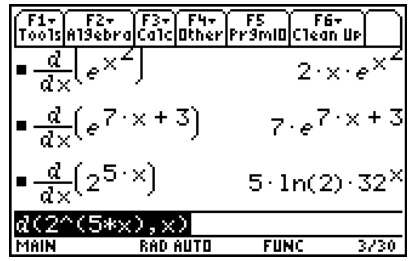
Problem 2 – The Derivative for $y = a^x$

In this portion, the students look at several examples with different number bases. They are asked to conjecture a rule. Then they are asked to attempt the general rule and use the derivative function to get the answer.



There are a few problems involving the chain rule for them to try.

Once again, the use of parentheses is very important.



Problem 3 – Slope of the Exponential Function

Students will draw the tangent to the graph of $y = e^x$ at a point using **F5:Math > A:Tangent**. They will examine the relationship between the y-coordinate of the point and the slope of the tangent line. The student can use just one point or try several points depending on the time left in the class. The instructions ask for a single point, but more points would be much more persuasive. If desired, you can have students draw their tangent lines on the graph on the worksheet.