F Distributions

ID: 9780

Time required
30 minutes

Activity Overview

In this activity, students study the characteristics of the F distribution and discuss why the distribution is not symmetric (skewed right) and only has positive values. Students then use the Fcdf command to find probabilities and to confirm percentiles. They move on to find critical values and then compute a confidence interval.

Topic: Continuous Distributions and their Properties

- *Graph the probability density function*
- *Calculate probabilities*
- *Calculate the xth percentile*
- *Calculate a confidence interval*

Teacher Preparation

- *Students should already be familiar with the normal distribution and it characteristics, as well as finding and interpreting confidence intervals for normal distributions. It is recommended, though not required, that students already be familiar with the chi-square distribution for sample variances.*
- *Students will need to have access to charts of F distribution critical values. These appear in most Statistics textbooks.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9780" in the keyword search box.***

Associated Materials

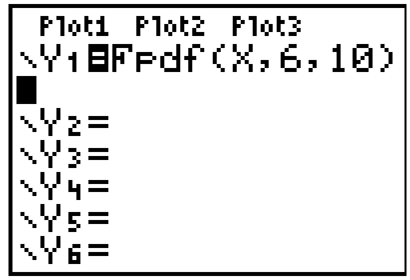
- *FDistributions_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

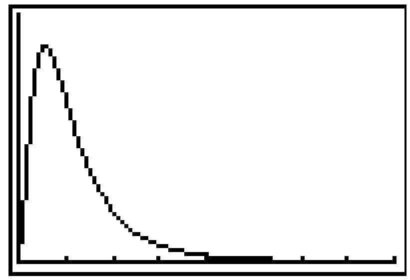
- *Chi-Square Distributions (TI-84 Plus family) — 9737*
- *Is It Rare? (TI-84 Plus family) — 9094*
- *Central Limit Theorem (TI-84 Plus family with TI-Navigator) — 1957*
- *Percentiles (TI-84 Plus family) — 9538*

Problem 1 – Characteristics of the *F* Distribution

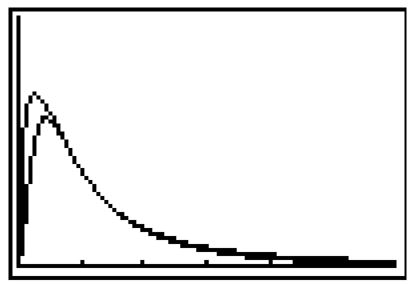


Step 1: Introduce the *F* distribution. Once the students have graphed the F distribution with 6 and 10 d.f., have them compare this distribution to others they have seen.

Allow students to change the degrees of freedom for one or both samples.



Students should note that as with the chi-square distribution, the values of the *F* distribution cannot be negative, and the graph is skewed to the right. Unlike the chi-square distribution, the graph does not become more symmetric as the number of degrees of freedom increases.



Step 2: Students are to determine whether the graph changes when the degrees of freedom are interchanged. They can choose to graph the distribution from the previous step and then enter **Y2** as **FPdf(*x*, 10, 6)**, or type two new functions.

Students will see that the distributions are different when the degrees of freedom are interchanged (unless, of course, the degrees of freedom are the same for each sample).

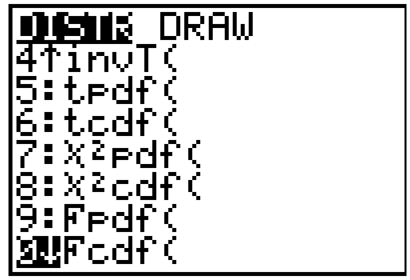
Problem 2 – Probabilities and Percentiles

Step 3: Ask students what would be true about the variances of two samples if *F* was near 1 and why.

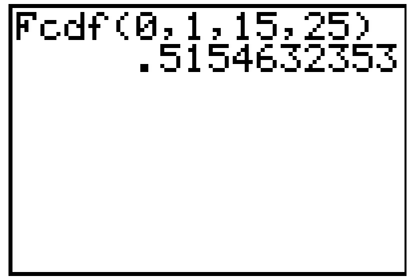
The variances would be almost the same. When the numerator and denominator of a fraction are the same, the fraction has a value of 1. The value gets further from 1 as the numerator and denominator have greater differences.

Discuss why the graph is skewed right. Students can look at graphs in Problem 1 during the discussion. Zero is the leftmost limit, as a fraction of two nonnegative numbers must be nonnegative. There is, however, no limit to how large the ratio can be.

Step 4: Students are to find the probability that *F* is less than 1 for two samples, where $n_1 = 16$ and $n_2 = 26$.

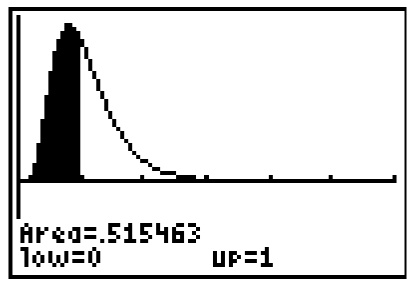


FCdf finds the area under the curve, to the left of the given *F* value. The command can be selected by pressing **MENU > Statistics > Distributions > F Cdf**. When students are typing the values in the parentheses, be sure they enter them in this order: lower bound, upper bound, d.f. for numerator, d.f. for denominator.

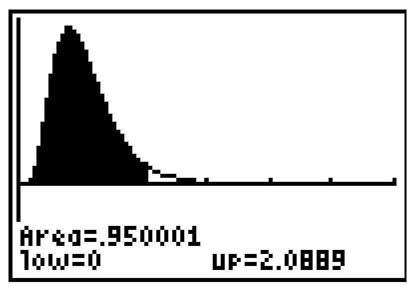


Students may think to use negative infinity for the lower bound, but since the values must be nonnegative, 0 can also be used.

Step 5: To check the area under the graph, students should use the **ShadeF(** tool (press =, move to the **DRAW** menu). Type **0, 1, 15, 25** inside the parentheses.



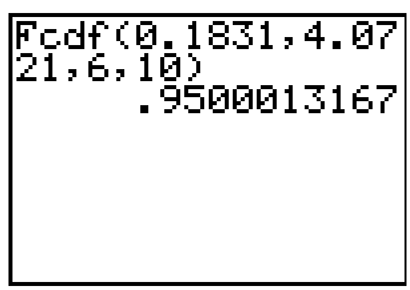
Step 6: If needed, explain or review the notion of a percentile. Students are to find the *F* value at the 95th percentile using an *F* distribution chart.



Students can check their value on the graph using the **ShadeF** tool. The command entered on the Home screen for the graph at the right is **ShadeF(0, 2.0889, 15, 25)**.

Problem 3 – Critical Values for an F Distribution

Step 7: Now that students can find an *F* value for a given area to the left of it, students can construct confidence intervals. As with the chi-square distribution, the critical values are denoted as left and right with subscripts L and R.



Using an *F* distribution chart, have students find the critical values that would be used for *F*(6, 10) at the 95% level.

Step 8: Students are then asked to find the critical values for *F*(20, 15) at the 98% level.

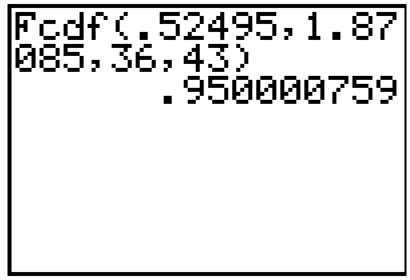
Problem 4 – Constructing a Confidence Interval

Step 9: Now that students can find critical values, they can construct confidence intervals. Introduce the expression for the confidence interval for a ratio of variances.

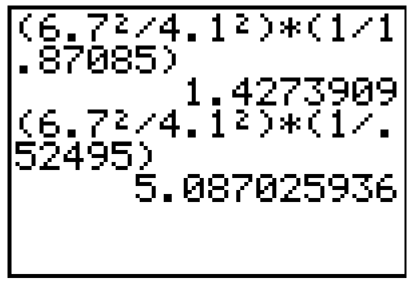
$$\frac{s_1^2}{s_2^2}\left(\frac{1}{F_R}\right) < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2}\left(\frac{1}{F_L}\right)$$

Step 10: A scenario is given on the student worksheet. Students are to construct a 95% confidence interval for the ratio of the population variances.

The screenshot at the right shows a check of finding the critical values using the **Fcdf** command.



We are 95% confident that the ratio of the population variances is between 1.42739 and 5.087

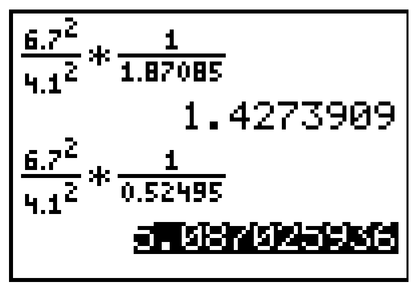


If using Mathprint OS:

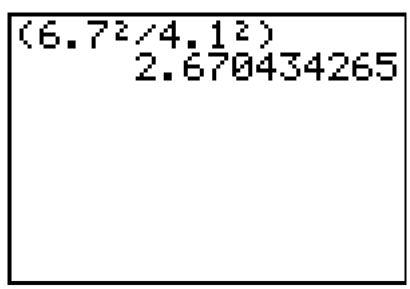
Students can the values in the formula as stacked fractions on the Home screen. To do this, press [ALPHA] [F1] and select **n/d**. Then enter the value of the numerator, press [▼], and enter the value of the denominator. Press [ENTER] to evaluate.

To move out of the fraction, press [▶]. Then press the multiplication key.

Note: Students do not need to place parentheses around the fractions.



Step 11: Have students discuss their answer to Question 10. Students should say that because $F = \frac{(6.7)^2}{(4.1)^2} = 2.67043$ is in the confidence interval, there is no significant difference in the variances.



Solutions – student worksheet

1. Sample: As with the chi-square distribution, the F values cannot be negative, and the graph is skewed to the right. Unlike the chi-square distribution, the graph does not become more symmetric as the number of degrees of freedom increases.
2. No, not unless the degrees of freedom are the same for each sample.
3. The variances would be almost the same. When the numerator and denominator of a fraction are the same, the fraction has a value of 1. The value gets further from 1 as the numerator and denominator have greater differences.
4. About 51.5%
5. 2.0889
6. 4.0721
7. 0.1831
8. 0.3238 and 3.3719
9. between about 1.263 and about 5.646
10. No, because 1, the ratio of equal variances, is outside of the confidence interval, the variances, and therefore the standard deviations, are probably not equal.