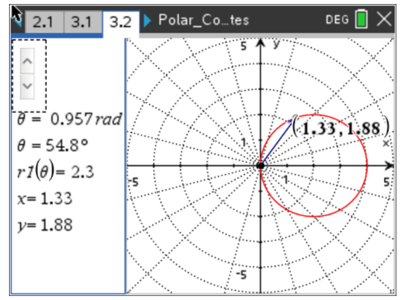
Polar Coordinates



Math Objectives

- Students will develop a basic understanding of the polar coordinate system and locate points given in polar form.
- Students will convert points between polar and rectangular coordinates.
- Students will sketch graphs of polar equations and compare them to their rectangular coordinate function counterpart.
- Students will use appropriate technological tools strategically, and look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- polar coordinates
- rectangular coordinates
- absolute value
- polar axis
- pole
- argument

About the Lesson

- This lesson involves a brief introduction to the polar coordinate system.
- As a result, students will:
  - Determine the location (quadrant) of various points given in polar form.
  - Recognize cases in which a point lies on an axis.
  - Convert points between polar and rectangular form.
  - Discover that polar coordinates for a point are not unique.
  - Use their calculators to check graphs sketched using paper and pencil.

TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

Polar_Coordinates_Nspire_Student.pdf

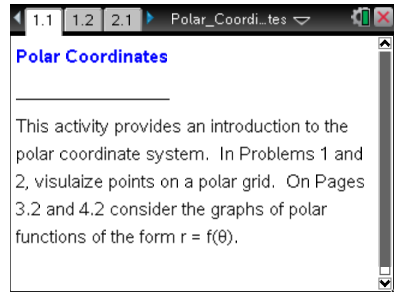
Polar_Coordinates_Nspire_Student.doc

Polar_Coordinates.tns

Special_Types_of_Polar_Functions.pdf

Open the TI-Nspire document *Polar_Coordinates.tns.*

In this activity, you will be introduced to the polar coordinate system. You will plot points given in polar form, convert polar coordinates to the rectangular coordinate system, and sketch the graphs of polar equations.



The polar coordinate system is a two-dimensional coordinate system defined by a point, called the pole, and a ray from the pole, called the polar axis. In a rectangular coordinate system, the **pole** is usually placed at the origin, and the **polar axis** is represented by the positive x-axis. A point in the polar coordinate system is represented by the ordered pair (r,θ) where r is the distance from the pole and θ is the angle (in radians) measured counterclockwise from the polar axis.

Tech Tip: This activity uses sliders. To change slider settings, right-click in the slider box and select **Settings**. You might want to consider different values for the minimum, maximum, or step size.

Problem 1 – Identifying The Quadrants in a Polar System

1. **Move to page 1.2.** On this page, the left work area contains a slider for r and a clicker for θ. The point (r,θ) is plotted in the right panel along with a position vector. Change the values of r and θ as needed to answer the following questions.
   a. Complete the following tables by finding the quadrant in which the point (r,θ) lies.

<u>**Solution:**</u>

r	1.7	1.3	–0.6	–4.2	–3.2	3.1	–1.5	–2.5
θ	$\frac{5\pi}{6}$	$-\frac{3\pi}{4}$	$-\frac{7\pi}{6}$	$\frac{3\pi}{4}$	$-\frac{4\pi}{3}$	$-\frac{13\pi}{4}$	$\frac{13\pi}{12}$	$-\frac{7\pi}{4}$
Quadrant	**II**	**III**	**IV**	**IV**	**IV**	**II**	**I**	**III**

r	0.8	2.1	2	–2.7	4	3.5	–1.4	–3
θ	$\frac{19\pi}{6}$	$\frac{\pi}{4}$	$-\frac{17\pi}{12}$	$\frac{11\pi}{4}$	$\frac{7\pi}{6}$	$-\frac{\pi}{3}$	$\frac{11\pi}{3}$	$\frac{\pi}{3}$
Quadrant	**III**	**I**	**II**	**IV**	**III**	**IV**	**II**	**III**

r	-4	2.7	1	3.9	-5	-2	-1	1.5
θ	$-\frac{\pi}{6}$	$-\frac{11\pi}{3}$	$\frac{23\pi}{12}$	$-\frac{11\pi}{6}$	$-\frac{9\pi}{4}$	$\frac{11\pi}{6}$	$-\frac{7\pi}{6}$	$\frac{9\pi}{4}$
Quadrant	**II**	**I**	**IV**	**I**	**II**	**II**	**IV**	**I**

TI-Nspire Navigator Opportunity: ***Class Capture and Quick Poll.*** **See Note 1 at the end of this lesson.**

b. Describe the location of the point with the following polar coordinates:

(i) $r > 0$ and $\theta = 0$

Sample Solution: The point lies on the polar axis, or the positive x-axis.

(ii) $r < 0$ and $\theta = \frac{3\pi}{2}$

Sample Solution: The point lies on the positive y-axis.

(iii) $r < 0$ and $\theta = \frac{\pi}{2}$

Sample Solution: The point lies on the negative y-axis.

(iv) $r > 0$ and $\theta = -3\pi$

Sample Solution: The point lies on the negative x-axis.

Problem 2 – Matching Polar Coordinates with Rectangular Coordinates

Move to page 2.1. If a point has polar coordinates (r,θ), then the rectangular coordinates are given by $x = r\cos\theta$ and $y = r\sin\theta$. Similarly, if a point has rectangular coordinates (x, y), then the polar coordinates are (r,θ) such that $r^2 = x^2 + y^2$ and $\tan\theta = \frac{y}{x},\ x \neq 0$.

2. Complete each of the following tables. Use Page 2.1 to enter polar coordinates and/or rectangular coordinates, to plot the points, and to check your answers. Enter coordinates in the left work area in the appropriate Math Box. The polar coordinates are represented by the point P and the rectangular coordinates are represented by the point R. Remember that there are an infinite number of polar coordinates that can be plotted in a single location. For example, $\left(1, \frac{\pi}{6}\right)$ is equivalent to $\left(-1, \frac{7\pi}{6}\right)$.

a. For each given polar coordinate, find two different polar coordinates that represent the given point.

Sample Solutions:

(r_1,θ_1)	$\left(2,\frac{\pi}{4}\right)$	$\left(3,\frac{7\pi}{4}\right)$	$\left(6,\frac{2\pi}{3}\right)$	$\left(1,\frac{7\pi}{6}\right)$	$\left(-2,\frac{5\pi}{4}\right)$	$\left(\frac{3}{4},\frac{17\pi}{6}\right)$
(r_2,θ_2)	$\left(2,\frac{9\pi}{4}\right)$	$\left(3,-\frac{\pi}{4}\right)$	$\left(6,\frac{8\pi}{3}\right)$	$\left(1,\frac{19\pi}{6}\right)$	$\left(2,\frac{\pi}{4}\right)$	$\left(\frac{3}{4},\frac{5\pi}{6}\right)$
(r_3,θ_3)	$\left(-2,\frac{5\pi}{4}\right)$	$\left(3,\frac{15\pi}{4}\right)$	$\left(-6,\frac{5\pi}{3}\right)$	$\left(-1,\frac{\pi}{6}\right)$	$\left(2,-\frac{7\pi}{4}\right)$	$\left(-\frac{3}{4},-\frac{\pi}{6}\right)$

b. For each point given in polar coordinates below, determine the rectangular coordinates.

Sample Solutions:

(r,θ)	$\left(3,\frac{7\pi}{3}\right)$	$\left(1,\frac{\pi}{6}\right)$	$\left(-2,-\frac{4\pi}{3}\right)$	$\left(\sqrt{5},-\frac{3\pi}{2}\right)$	$\left(-8,\frac{3\pi}{4}\right)$	$\left(\frac{13}{4},-\frac{\pi}{3}\right)$
x	$\frac{3}{2}$	$\frac{\sqrt{3}}{2}$	1	0	$4\sqrt{2}$	$\frac{13}{8}$
y	$\frac{3\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$\sqrt{5}$	$-4\sqrt{2}$	$-\frac{13\sqrt{3}}{8}$

c. For each point given in rectangular coordinates below, determine two representations in polar coordinates.

Sample Solutions:

(x,y)	$(3,4)$	$(-\sqrt{2},2)$	$(4,-7)$	$(-\sqrt{3},-1)$	$(-5,5)$	$(7,24)$
r_1	5	$\sqrt{6}$	$\sqrt{65}$	2	$5\sqrt{2}$	25
θ_1	0.927	2.186	-1.052	$-\frac{5\pi}{6}$	$\frac{3\pi}{4}$	1.287
r_2	-5	$-\sqrt{6}$	$-\sqrt{65}$	-2	$-5\sqrt{2}$	-25
θ_2	4.069	5.328	2.090	$\frac{\pi}{6}$	$\frac{7\pi}{4}$	4.429

TI-Nspire Navigator Opportunity: ***Class Capture.*** **See Note 2 at the end of this lesson.**

Problem 2 – Graphing Polar Functions

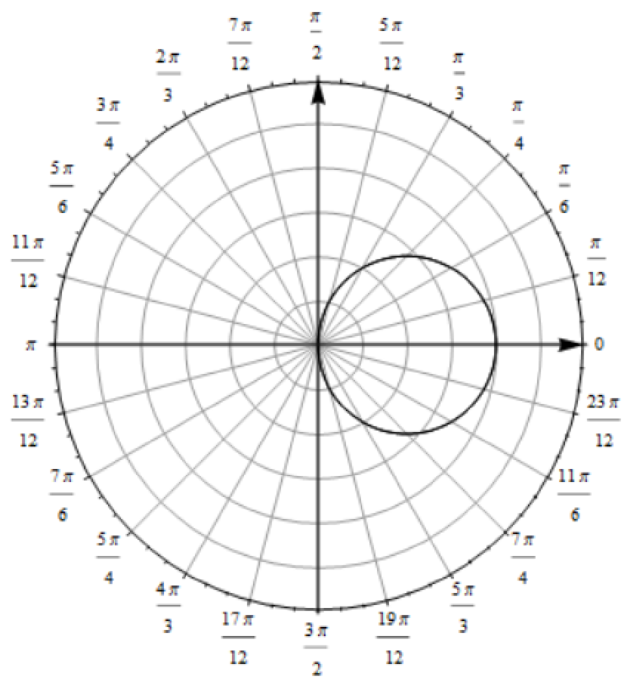
3. **Move to page 3.2**. Over the next two worksheet pages, carefully sketch a complete graph of each given polar equation. Create a table of values, search for patterns, and sketch the graph on the axes provided. Check your results on the handheld and sketch the graph in the right work area of Page 3.2.

Note: Make sure the Graph Type is set to Polar. Use the clicker in the left panel to step through specific points on the curve in polar and rectangular form.

a. $r = 4\cos\theta$.

Sample Solution:

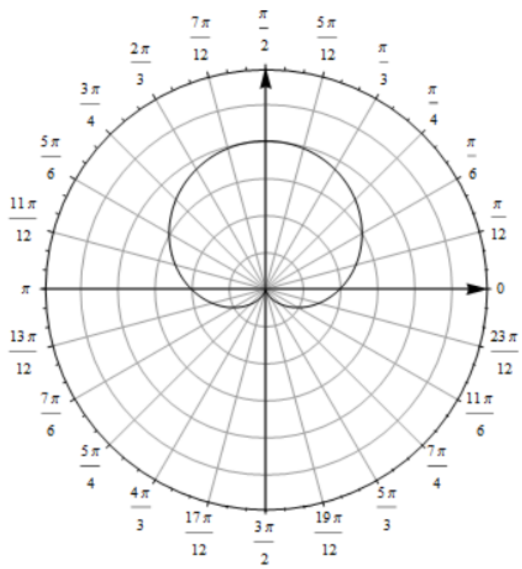
θ	r
0	4
$\frac{\pi}{6}$	$2\sqrt{3}$
$\frac{\pi}{4}$	$2\sqrt{2}$
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-2
$\frac{3\pi}{4}$	$-2\sqrt{2}$
$\frac{5\pi}{6}$	$-2\sqrt{3}$
π	-4



b. $r = 2 + 2\sin\theta$.

Sample Solution:

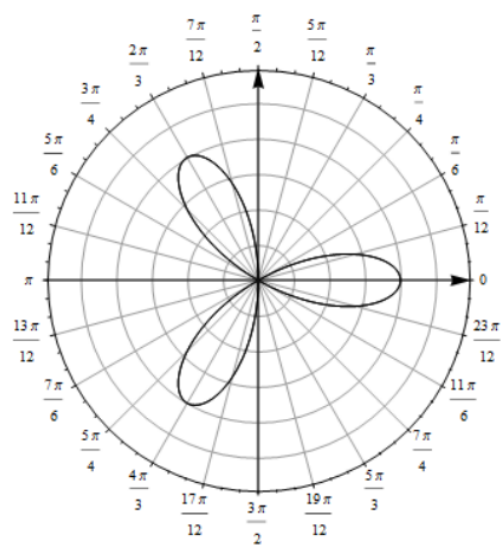
θ	r
0	2
$\frac{\pi}{6}$	3
$\frac{\pi}{4}$	$2+\sqrt{2}$
$\frac{\pi}{3}$	$2+\sqrt{3}$
$\frac{\pi}{2}$	4
$\frac{2\pi}{3}$	$2+\sqrt{3}$
$\frac{3\pi}{4}$	$2+\sqrt{2}$
$\frac{5\pi}{6}$	3
π	2



c. $r = 4\cos 3\theta$.

Sample Solution:

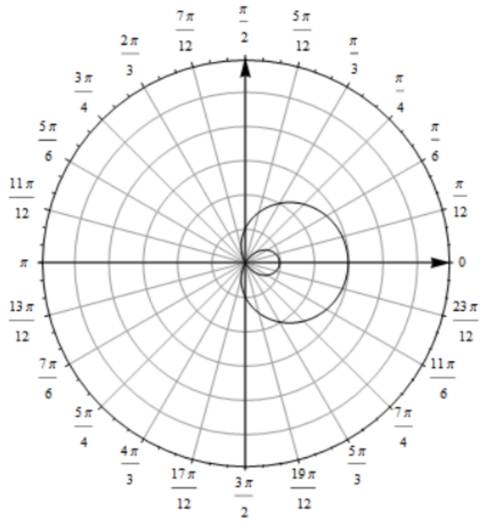
θ	r
0	4
$\frac{\pi}{6}$	0
$\frac{\pi}{4}$	$-2\sqrt{2}$
$\frac{\pi}{3}$	-4
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	4
$\frac{3\pi}{4}$	$2\sqrt{2}$
$\frac{5\pi}{6}$	0
π	-4



d. $r = 1 + 2\cos\theta$.

Sample Solutions:

θ	r
0	3
$\frac{\pi}{6}$	$1+\sqrt{3}$
$\frac{\pi}{4}$	$1+\sqrt{2}$
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	0
$\frac{3\pi}{4}$	$1-\sqrt{2}$
$\frac{5\pi}{6}$	$1-\sqrt{3}$
π	-1



Problem 4 – Comparing Trigonometric Functions in the Polar and Rectangular Systems

Teacher Tip: Problem 4 makes a connection to the rectangular function equation of the four previous graphs. There is another document you can use with your students to prepare them for this problem, entitled *Special Types of Polar Functions*. Part (b) for each question might require a lot of group discussion and teacher guidance.

In this problem, you will compare the four polar graphs from **Problem 3** with their rectangular system counterparts. **Move to page 4.1**. Graph each of the following on the **Graphs** page provided and answer each corresponding question.

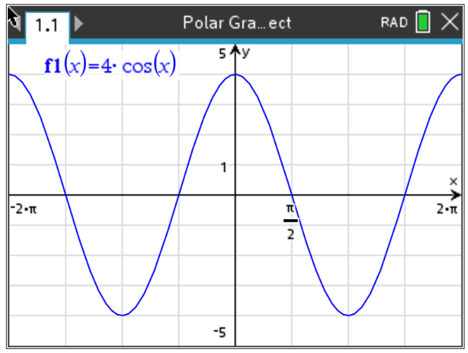
4. $f(x) = 4\cos(x)$

(a) What shape was created on the polar graph from question 3 (a)?

Solution: Circle along the positive x-axis.

(b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.

Possible Solution:



Since this is a cosine curve, and cosine is represented by x in the unit circle, the shape is a circle along the x-axis. The amplitude of the cosine curve is 4, this also is the distance from the pole to the farthest ring, which is also the diameter of the circle of the polar graph. The period of the cosine function is 2π, therefore the circle will be completed twice as it only took a distance of π to create it on the polar graph.

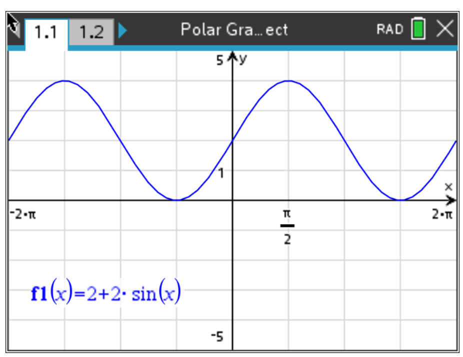
5. $f(x) = 2 + 2\sin(x)$

   (a) What shape was created on the polar graph from question 3(b)?

Solution: Cardioid (Limaçon) along the positive y-axis.

   (b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude, period, and vertical shift in the explanation.

Possible Solution:



Since this is a sine curve, and sine is represented by y in the unit circle, the shape is a limaçon (cardioid) along the y-axis. The amplitude of the sine curve is 2 and the vertical translation is 2, therefore their sum is also the distance from the pole to the farthest ring of the polar graph. The period of the sine function is 2π, therefore the cardioid will be completed once as it took a distance of 2π to create it on the polar graph.

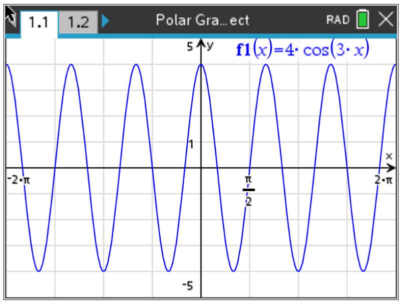
6. $f(x) = 4\cos(3x)$

(a) What shape was created on the polar graph from question 3(c)?

Solution: Rose curve with 3 petals, one centered on the positive x-axis.

(b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.

Possible Solution:



Since this is a cosine curve, and cosine is represented by x in the unit circle, the shape is a rose curve with 3 petals, one centered along the x-axis. The amplitude of the cosine curve is 4, this also is the distance from the pole to the farthest end of each petal. The period of the cosine function is $\frac{2\pi}{3}$ (3 complete cycles from $0\ to\ 2\pi$), therefore the 3 rose petals will be completed twice on the polar graph.

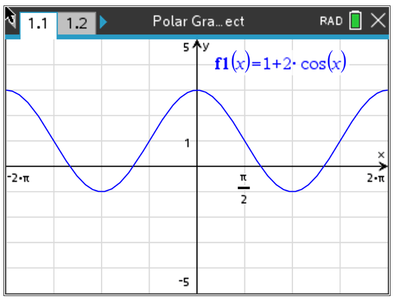
7. $f(x) = 1 + 2\cos(x)$

(a) What Shape was created on the polar graph from question 3(d)?

Solution: Limaçon with an inner loop along the positive x-axis.

(b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.

Possible Solution:



Since this is a cosine curve, and cosine is represented by x in the unit circle, the shape is a limaçon with an inner loop along the x-axis. The amplitude of the cosine curve is 2 and the vertical translation is 1, therefore their sum is also the distance from the pole to the farthest ring of the polar graph. The period of the cosine function is 2π, therefore the limaçon will be completed once as it took a distance of 2π to create it on the polar graph.

Extensions

Here are some possible extensions to this activity:

1. Create a list of polar equations and graphs and ask students to match each equation with its corresponding graph.
2. Ask students to convert specific equations from rectangular form to polar form.
3. Ask students to sketch the graphs of other polar equations.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The polar coordinate system and how to plot points given in polar form.
- How to convert points between rectangular and polar form.
- How to sketch the graph of a polar equation.

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Note 1

Question 1, Class Capture and Quick Poll

Ask students to use the sliders to locate the given point. Use Class Capture to compare student responses. Conduct a Quick Poll in which students must select a quadrant: I, II, III, or IV.

Note 2

Question 2, Class Capture

Ask students to plot points on Page 2.1. Take Class Captures to compare student responses.