Teacher Notes

Activity 4

Graphical Consequences of Continuity

Objective

- Explore the graphical and numeric consequences of continuity

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 50 minutes

Abstract

In this activity, the continuity of three functions is discussed and explored. The concept of continuity is linked to graphical behavior by performing horizontal zooming: zooming in that leaves the vertical scale unchanged while shortening the horizontal scale. A polynomial function is investigated first, followed by a rational function, and then an oscillating function.

Management Tips and Hints

Prerequisites

- Students should be able to graph functions, generate tables, and navigate the **ZOOM**, **VARS**, and **TEST** menus on the graphing handheld.
- Students should have an intuitive understanding of limits.
- This activity can be done as an introduction to continuity or at any time during its coverage.

Evidence of Learning

- Students will recognize that the graph of a continuous function looks like a horizontal line when you zoom in.
- Students will be able to distinguish between removable and non-removable discontinuities.
- Students will recognize that the function $f(x) = \sin\left(\frac{1}{x}\right)$ has a non-removable discontinuity at $x = 0$, as evidenced by the fact that its graph does not look like a horizontal line when you zoom in horizontally.

Common Student Errors/Misconceptions

- Although the formal definition of limit is not covered in this activity, it nonetheless provides the conceptual underpinnings for the function behaviors students observe.
- Particular emphasis is placed on the exact behavior of the graph upon zooming with unequal horizontal and vertical zoom factors. The fact that function outputs are getting closer together is evidenced by the graph flattening out. It is less obvious that the inputs are simultaneously getting closer together.
- It is important that students correctly position the zoom crosshairs at the indicated coordinates before zooming in. The positioning of the crosshairs is described between Questions **2** and **3**, **11** and **12**, and **16** and **17**.

Teaching Hints

There are some subtle issues in this activity that are worth discussing with students after they have finished it. In particular, the connection between a horizontal line graph and the fact that function outputs are confined to a narrow interval (relative to Δy, the distance between rows of pixels on the graphing handheld screen) is important and could be missed easily.

The activity is actually modeled after the epsilon-delta definition of continuity. You could revise the activity so that the vertical scale is made much smaller at the outset of the zooming in (analogous to choosing a smaller epsilon), and then have students see how many times they need to zoom in to produce a horizontal line graph (which is analogous to finding a delta that is small enough to ensure that the function outputs are very close to the value of $f(c)$). What is important mathematically is that no matter how small the initial vertical scale is, if the function is continuous, it can always be zoomed in horizontally to produce a horizontal line.

The idea of horizontal zooming—in which just the x-scale is changed—is likely to be new to students. It might be necessary to discuss this idea further with them. Questions **3**, **4**, **5**, and **7** attempt to get at the meaning of a horizontal zoom-in.

Extensions

Repeat the activity with a function that has a jump discontinuity, such as

$$f(x) = \begin{cases} \dfrac{3x}{x^2+1}, x < 1 \\ 2 - x^2, x \geq 1 \end{cases}$$

at $x = 1$.

Repeat the activity with a split-defined function that is continuous, such as

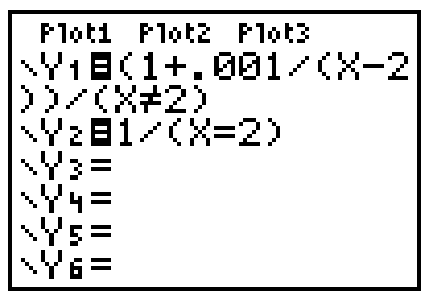
$$f(x) = \begin{cases} \dfrac{2x}{x^2+1}, x < 1 \\ 2 - x^2, x \geq 1 \end{cases}$$

at $x = 1$.

Investigate this function, which has a vertical asymptote at $x = 2$:

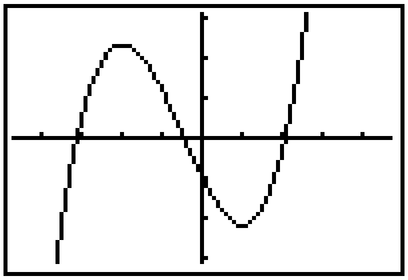
$$g(x) = \begin{cases} 1 + \dfrac{0.001}{x-2}, x \neq 2 \\ 1, x = 2 \end{cases}$$

Enter it into the graphing handheld as shown.



Note that in the **ZDecimal** viewing window, the graph appears horizontal. However, after a few zooms at the point (2,1), the vertical asymptote becomes visible. After a few more zooms, the graph is no longer visible because the outputs are all too large.

Activity Solutions

1. 

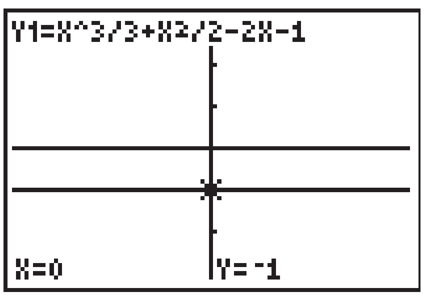
2. ⁻1

3. 0.01

4. 0.01

5. 0.1

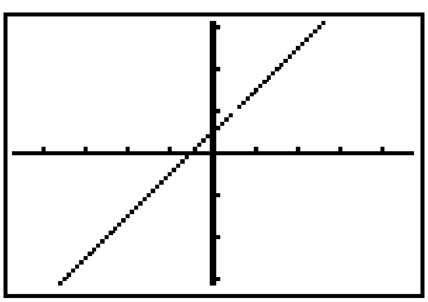
6. The graph looks like a horizontal line:



7. 0.0001

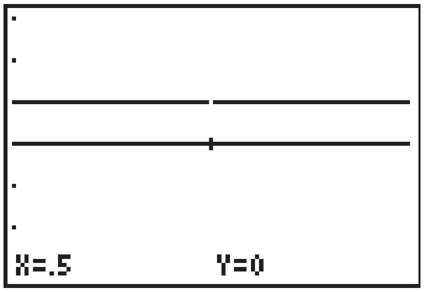
8. ⁻1.009389

9. ⁻0.990589

10. 

11. Undefined

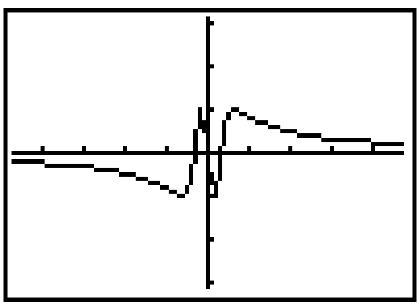
12. The graph looks like a horizontal line with a hole at $x = 0.5$:



13. 1

14. The graph is a horizontal line.

15.



16. Undefined

17. No

18. The graph oscillates wildly between $x = {}^{-}1$ and $x = 1$. In fact, the closer you get to $x = 0$, the faster the oscillations occur.

19. No; the situation is hopeless. The values of the function do not get close to any one number as x gets close to 0. Unlike the previous rational function, the graph does not have a hole that could be filled in at $x = 0$.