

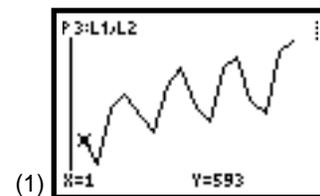
EXPLORATIONS

Activity 12

Forecasting Making forecasts from data requires many tools, including regression that was covered in Activity 11. In Activity 12, we will look at two techniques: Topic 54 covers *exponential smoothing* using some of the spreadsheet capabilities of the TI-83. Topic 55 covers program **FORECAST**, which uses a multiplicative time-series model with trend, seasonal, and irregular components.

The data for this activity is given below along with a plot in screen 1. The data represents the number of customers for a small business over the past four years.

Year	Quarter	x in L1 Period	y in L2 Count
1	1	1	593
	2	2	512
	3	3	705
	4	4	756
2	1	5	680
	2	6	623
	3	7	785
	4	8	846
3	1	9	708
	2	10	662
	3	11	853
	4	12	884
4	1	13	736
	2	14	692
	3	15	908
	4	16	945



Put the time period in list L1 and the customer count in L2, and then continue with Activity 12.

Activity 12, Forecasting (cont.)

Topic 54—Exponential Smoothing

We will use the model $F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$. This model uses the data for this period (Y_t) and the forecast for this period (F_t) to forecast for the next period (F_{t+1}). α is the smoothing constant and can be a value from 0 to 1.

Setup

1. With the data in **L1** and **L2**, as depicted in the table at the beginning of Activity 12, set up **Plot1** for an **xyLine** plot, as shown in Topic 9 and in screen 2.

Press **ZOOM** **9:ZoomStat** [TRACE] for the plot shown in screen 1.

2. Store a copy of **L1** in **L5** with **L1** **STO** **L5**, and store a copy of **L2** in **L3** (screen 3).
3. Press **STAT** **1:Edit** **DEL** to delete the first value of both **L3** and **L5**, and store the deleted value of **L3** in the first row of **L4**.

The lists should look those in screen 4.

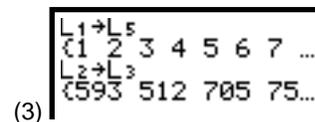
First Try ($\alpha = 0.2$)

1. Type **0** **.** **2** **STO** **A** : **1** **STO** **R** (do not forget the colon) **ENTER** for the first two lines in screen 5.
2. Type **A** **x** **L3** **[]** **R** **]** **+** **[]** **1** **-** **A** **]** **x** **L4** **[]** **R** **]** **STO** **L4** **[]** **R** **+** **1** **]** : **R** **+** **1** **STO** **R**. Then press **ENTER** until you run out of data (about 16 times, one for each point) and get the error message **ERR:INVALID DIM**, as shown in screen 7.
3. Select **1:Quit**, and then press **STAT** **1:Edit**, and use the cursor control keys to go to the last value of **L4** for the forecast for the next period:

$F_{17} = 808.976$ or 809 customers (screen 8).

4. Use **DEL** to delete the last value in **L4** so that **L3**, **L4**, and **L5** are all the same length (15 rows).

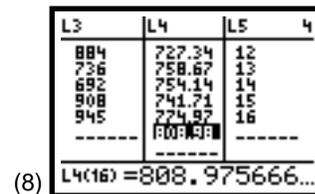
Measures of Fit will use the 15 values in **L3** and **L4**. The first period value was dropped because it had no forecast, but was used as the forecast for the second period.



(4)

L1	L2	L3	L4	L5	4
1	593	512	593		
2	512	705			
3	705	756			
4	756	680			
5	680	623			
6	623	785			
7	785	846			

L1(1)=1 L4(2)=



Note: If you get a dimension error in step 4, you forgot to delete the extra value in **L4**.

Activity 12, Forecasting (cont.)

- Use $\boxed{2\text{nd}}$ [QUIT] to return to the home screen, and then press $\boxed{2\text{nd}}$ [LIST] <MATH> 5:sum(([L3] - [L4]) $\boxed{x^2}$) $\boxed{15}$ [ENTER] for the first two lines in screen 9.

This is the *Mean of the Squared Errors*, or MSE = **14284.7**.

- Press $\boxed{2\text{nd}}$ [LIST] <MATH> 5:sum([MATH] <NUM> 1:abs(L3 - L4)) $\boxed{15}$ [ENTER] for the last three lines in screen 9.

This is the *Mean of the Absolute Values of the Deviations*, or MAD = **102.358**.

- With **Plot1** still set up as shown in screen 2, set up **Plot2**, as shown in screen 10.

Press [TRACE] for the plot of the original data with the exponentially smoothed values through it (see screen 11).

Second Try

To see if we can get a better fit, we will try other values of α . We want to follow the more current values a bit closer so we will increase α to $\alpha = 0.3$.

- Press $\boxed{2\text{nd}}$ [ENTRY] four times (or until your last line is like the first line in screen 5).

As in the first line of screen 12, change **0.2** to **0.3**, and then press [ENTER] for the **1** in the second line of screen 12.

- Press $\boxed{2\text{nd}}$ [ENTRY] a few times until you have the next three lines shown in screen 12. Press [ENTER] until you get an error message (this may take about 16 presses).

- Select **1:Quit**, and then press [STAT] **1:Edit** and use the cursor control keys to go to the last value of **L4** for the forecast for the next period:

$F_{17} = 841.67$ or 842 customers.

- Use [DEL] to delete the last value in **L4** so that **L3**, **L4**, and **L5** are all the same length (15 rows).

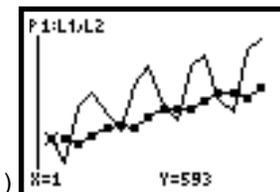
- Use $\boxed{2\text{nd}}$ [QUIT] to return to the home screen, and then press $\boxed{2\text{nd}}$ [ENTRY] a few times until you recall the line necessary to calculate the MSE, as shown in screen 14.

```
sum((L3-L4)^2)/15
      14284.74795
sum(abs(L3-L4))/
15      102.3583677
```

(9)

```
Plot1: 2003 Plot3
Off
Type: L: [ ] [ ]
[ ] [ ] [ ]
Xlist:L5
Ylist:L4
Mark: [ ] + .
```

(10)



(11)

```
0.3→A: 1→R
A*L3(R)+(1-A)*L4      1
(R)→L4(R+1):R+1→    2
R                      3
```

(12)

L3	L4	L5	4
884	751.83	12	
736	791.48	13	
692	774.83	14	
908	749.98	15	
945	787.39	16	
-----	FC17=841.67	-----	

(13)

```
sum((L3-L4)^2)/15
      12767.36071
sum(abs(L3-L4))/
15      102.3619034
```

(14)

Activity 12, Forecasting (cont.)

6. Press **[ENTER]** for MSE = 12767.

With a similar use of **[2nd]** **[ENTRY]**, calculate MAD, as shown in screen 14 for MAD = 102.362.

MSE has been reduced from 14284 to 12767, and MAD basically is unchanged from 102.358 to 102.362.

7. Press **[TRACE]** for the plot in screen 15 that responds more to the changes in sales.

Further Tries

Seeking further improvement, we proceed as in the Second Try, but with $\alpha = 0.4$. MSE is reduced further to 12494, but MAD is increased to 103.34.

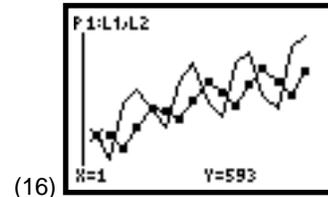
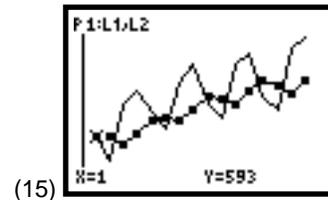
With $\alpha = 0.5$, MSE increases to 12695, and MAD increases to 105.68.

The plot in screen 16 shows that the forecast might be responding too quickly to the most recent value of data.

$\alpha = 0.39$ gave the minimum MSE = 12493 with a forecast of 863 customers for the period 17, or the first quarter of the fifth year.

$\alpha = 0.22$ gave the minimum MAD = 101.80 with a forecast of 817 customers.

A program called **EXSMOOTH** automates the above procedure. This program listing is shown at right.



PROGRAM EXSMOOTH

```

FnOff :ClrHome
PlotsOff
PlotsOn 3
Ø→Ysc1
Disp "TIME(1,2,3..N)"
Disp "IN L1, Y IN L2."
Pause :ClrHome
Menu("CONT OR
QUIT", "CONTINUE", A, "QUIT", B)
Lb1 B
ClrHome:Stop
Lb1 A:dim(L1→N
ClrDraw
Plot3(xyLine, L1, L2, .)
ZoomStat:Trace
Xmax+1→Xmax
Input "ALPHA=", A: L2→L4
L2(1)→L4(2)
For(I, 2, N-1)
L2(I)*A+(1-A)*L4(I)→L4(I+1)
End
A*L2(N)+(1-A)*L4(N)→P
PlotsOn 2
Plot2(xyLine, L1, L4, □)
DispGraph
Pt-On(N+1, P, 2)
Trace
Output(3, 1, "FORCAST AT T=")
Output(3, 14, N+1)
Output(4, 5, P)
L2-L4→L5
sum((L5²)/(N-1)→D
sum((abs(L5)/(N-1)→E
Output(6, 1, "MSE=")
Output(6, 5, D)
Output(7, 1, "MAD=")
Output(7, 5, E)
Pause :ClrHome
Return
    
```

Activity 12, Forecasting (cont.)

Topic 55—Multiplicative Model (Program FORECAST)

For the multiplicative time-series model, we assume the Y data value is made up of three components: $Y = T * S * I$, where T , S , and I stand for the *trend*, *seasonal*, and *irregular* components. (See *Statistics for Business and Economics 6th Edition*, Anderson, Sweeney, and Williams, West Publishing Company.) Other textbooks include a C or a “*cyclical*” component. (See *Business Statistics* by Triola and Franklin, Addison Wesley Publishing Company.)

Because the calculations are tedious, we will be using program **FORECAST**. This program is available from Texas Instruments over the internet (www.ti.com) or on disk (1-800-TI-CARES) and can be transferred to your TI-83 with TI-GRAPH LINK. (The program listing is in Appendix B.)

We will use the data for customer count given at the beginning of this activity with the period stored in L_1 and the count in L_2 . We must use quarterly or monthly data for complete years for the program to work.

1. Press **[PRGM]** **<EXEC>**, highlight program **FORECAST**, and then press **[ENTER]** to paste the program name to the home screen, as shown in screen 17.

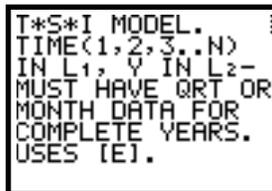
Press **[ENTER]** again for the instructions shown in screen 18.

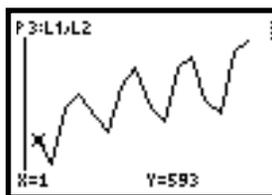
2. Press **[ENTER]** to continue.

The data is automatically plotted with the period, or time, on the x-axis and the sales on the y-axis, as shown in screen 19.

You are in Trace mode, so you can use the cursor control keys to go from point to point.

(17) 

(18) 

(19) 

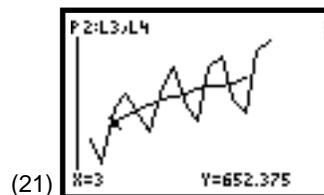
(20) 

Activity 12, Forecasting (cont.)

- Press **ENTER**, and you are given the options shown in screen 20.

Because you have quarterly data, press **1:4 TERM MOVING A**(verage) for screen 21. (See Note.)

We see from the plot in screen 21 that the four-term moving average is a very good smoother for the quarterly data. The graph is in Trace mode, so you can go from point to point on either curve.



(21)

Note: If you wish to fit a trend line to the moving average of the data (as in the Triola and Franklin reference) instead of to the deseasonalized data, make the following modifications to program FORECAST, and then skip to step 5.

- Replace the following line of code `Plot2(xyLine, L1, L5,)Text(0, 36, "DESEASONALIZED")`: Trace with : L3 → A1: L4 → A2.
- Replace L1, L5 with LA1, LA5 after `LinReg(a+bx)`, `QuadReg`, and `ExpReg` in the fourth, sixth, and eighth lines after the line in (i).

- Press **ENTER** for the next plot shown in screen 22.

This last plot (screen 22) is the original data and the deseasonalized data plotted in Trace mode. Looking at the data, we decide to fit a linear least squares fit line to the deseasonalized data as our predictor of the trend.

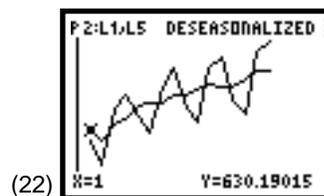
- Press **ENTER** for the menu of choices shown in screen 23.

- Select **1:LinReg(a+bx)** for the plot shown in screen 24. (Notice the other choices available on the menu.) The original data, the deseasonalized data, and the linear trend line are all shown on this plot.

- Press **ENTER** for the next plot shown in screen 25. The plot shows the original data, but this time with black squares so you can compare it with the forecast for the next year that is given as open squares.

- Press **ENTER** for a list of the seasonal indexes given to three decimal places, as shown in screen 26.

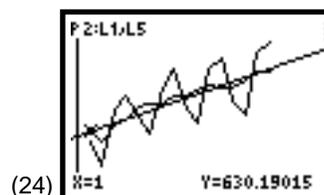
Use **▶** to read the index for the fourth quarter.



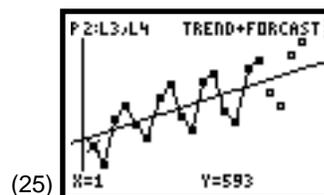
(22)



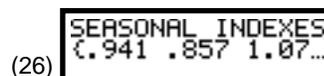
(23)



(24)



(25)



(26)

Activity 12, Forecasting (cont.)

9. Press **ENTER** and you get the forecast values for the next year (screen 27).
10. Press **ENTER** again, and the MSE and MAD are given (screen 27) as discussed in Topic 54.

These values are much smaller for this technique than they were for exponential smoothing because the periodic seasonal effect is more directly involved in this forecast.

11. Press **ENTER** for the instructive information shown in screen 28, which informs you how the calculated values are stored in matrix [E]. The last two lines remind you how to get back to the final graph. (We will investigate screen 28 more later in this topic.)
12. Do as suggested in the last two lines of screen 28 by pressing **TRACE** for screen 29.

13. Use ∇ to get to the Trend line. Type 17, and then press **ENTER** for the coordinates shown in screen 29. For the first quarter of the next year, period 17, the Trend value is **865.2**. Since the seasonal index is less than 1 (0.941), we forecast a value below the line of $865.2 * 0.941 = 814.1532$ or, if we carried all the decimals of the TI-83, we would have **814.14728**, as shown in screen 27.

Notice at the top of screen 29, the $Y_1 = a + b * x$ is turned on. As shown in screen 30, you can paste the a and b values using **[VARS] 5:Statistics.<EQ> 2:a** and **3:b**. The slope is **14.58** or about 15.

We revisit the last screen of program **FORECAST** (screen 31). The screen gives the contents of matrix [E], which was used for the previous plots and calculations.

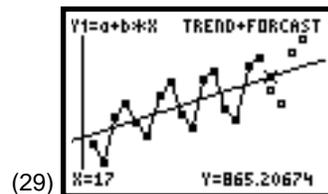
MA in the third line of screen 31 stands for the *centered four-term moving averages* in column 3 of screen 32. Since moving averages cannot be calculated for the first two values, the table shows **1E-8** as a filler or missing value symbol. This is true in other columns as well.

(27)

```
(.941 .857 1.07...)
FORECAST
(814.14728 753...
MSE=
      300.2544911
MAD=
      13.67873875
```

(28)

```
IN [E],T IN C1,
DATA(Y) IN C2,
MA IN C3, Y/MA
IN C4,SI IN C5,
Y/SI IN C6,FIT
C6 N C7,C7*SI-C8
TRACE FOR TREND
(REGEQ)+FORECAST.
```



(30)

```
a      617.3196709
b      14.58159227
```

(31)

```
IN [E],T IN C1,
DATA(Y) IN C2,
MA IN C3, Y/MA
IN C4,SI IN C5,
Y/SI IN C6,FIT
C6 N C7,C7*SI-C8
TRACE FOR TREND
(REGEQ)+FORECAST.
```

(32)

```
MATRIX[E] 20x8
[ 1      593      1E-8      -
[ 2      612      1E-8      -
[ 3      705      652.38      -
[ 4      756      677.13      -
[ 5      680      701      -
[ 6      623      722.25      -
[ 7      785      737      -
```

Activity 12, Forecasting (cont.)

Screen 33 gives the first seven rows of columns 4- 8 of **Matrix [E]**. **SI** in column 5 stands for *seasonal index*. Notice how values in column 5 repeat every four quarters and are calculated from the means of the same quarter data in column 4 (adjusted so all the seasonal indices add to four).

MATRIX[E] 20x8				[E] 20x8			
1E-8	.94099	630.19	-	631.9	1E-8	1	
1E-8	.8565	597.78	-	646.48	1E-8	1	
1.0807	1.0788	653.52	-	661.06	1E-8	1	
1.1165	1.1237	672.76	-	675.65	1E-8	1	
.97004	.94099	722.65	-	690.23	1E-8	1	
.86258	.8565	727.38	-	704.81	1E-8	1	
1.0651	1.0788	727.68	+	719.39	1E-8	1	

(33)

Y÷SI is the deseasonalized data in column 6. A least squares linear regression line was fitted to column 6 data. The resulting equation ($Y = a + bX$) had the periods, or Xs, in column 1 substituted in this equation for the values in column 7. Notice each value in column 7 increases by approximately 15 from the previous value. Fifteen is the slope of the line, as we discovered in step 13 above.

MATRIX[E] 20x8				X[E] 20x8			
.85156	.8565	807.94	+	821.46	1E-8	+	
1E-8	1.0788	841.69	-	836.04	1E-8	1	
1E-8	1.1237	840.95	-	850.63	1E-8	1	
1E-8	.94099	1E-8	-	865.21	1E-8	1	
1E-8	.8565	1E-8	-	879.79	753.54	1	
1E-8	1.0788	1E-8	-	894.37	964.83	1	
1E-8	1.1237	1E-8	-	908.95	1021.4	1	
Z0, 6=1E-8				814.147282...			

(34)

Screen 34 gives rows 14-20 of columns 4-8 of **Matrix [E]**. Notice the period 17 (first quarter of the fifth year) value of the trend forecast in column 7 is **865.21**, as also shown in step 13. If the trend value is multiplied by the seasonal index value in column 5, or **0.94099**, we obtain **865.21 * 0.94099 = 814.15**, as in the forecast in column 8.