Arithmetic Sequences & Series

ID: 8638

Time required
40 minutes

Activity Overview

This activity begins by displaying four sequences in a spreadsheet. Students use formulas to find the differences of consecutive terms, plot a scatter plot, and determine that sequences with common differences (called arithmetic sequences) have scatter plots whose points form a straight line. They then write explicit formulas for the first n terms and find the sum of the first n terms.

Topic: Sequences, Series, & Functions

- *Given several terms of a sequence, write an algebraic expression that generates the nth term.*
- *Graph the first n terms of a sequence.*
- *Derive and apply a formula for the first n terms of an arithmetic sequence.*

Teacher Preparation and Notes

- *This activity is designed to be used in an Algebra 2 classroom.*
- *Students should begin this activity knowing that a sequence is an ordered list of numbers that follows a pattern and that a series is an indicated sum of the terms in a sequence. For example, 1, 2, 3, 4 is a sequence and 1 + 2 + 3 + 4 is a series.*
- *This activity is intended to be mainly teacher-led, with breaks for individual student work.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "8638" in the keyword search box.***

Associated Materials

- *ArithSeqSeries_Student.doc*
- *ArithSeqSeries.tns*
- *ArithSeqSeries_Soln.tns*

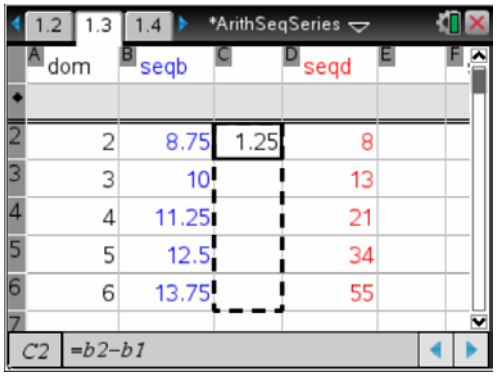
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Sequence Investigation (TI-Nspire technology) — 11062*
- *Sum of Sequences (TI-Nspire technology) — 11135*
- *Sequences and Series—Introductory Quiz (TI-84 Plus and TI-Navigator) — 10486*
- *Sequences (TI-84 Plus) — 6429*

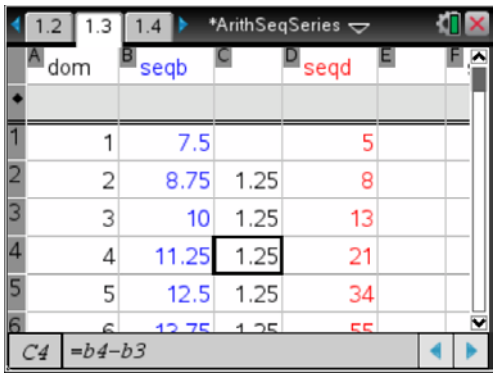
Problem 1 – Sequences and scatter plots

Step 1 There are four finite sequences on page 1.3. They appear in Columns B, D, F, and H. Each has six terms. Instruct students to find the differences between consecutive terms by writing formulas.



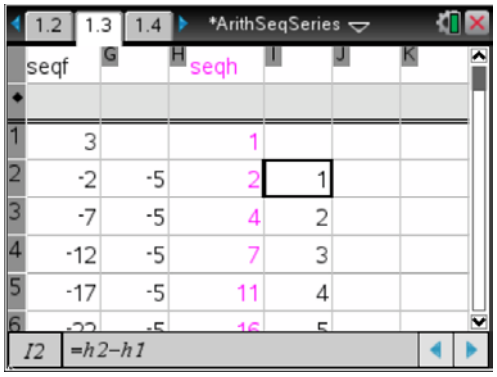
In Cell C2, have students type **=b2–b1** to find the difference between the first two terms in sequence B. The cell displays 1.25; but notice that when the cell is active, the formula appears at the bottom of the screen.

To copy this formula to find the differences between the remaining terms of the sequence, tell students to go to Cell C2 and press **MENU > Data > Fill**. Then, arrow down to highlight Cells C2 through C6, and press enter.



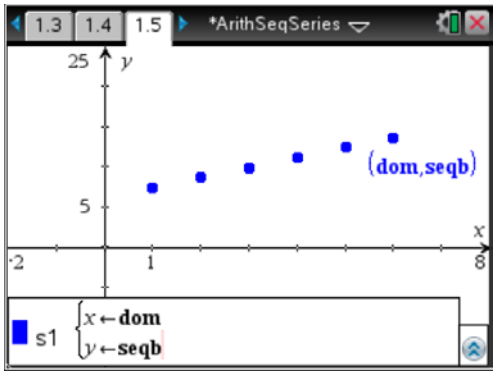
Students should now arrow down to examine each cell, looking at the formulas used to determine the value of that cell.

Step 2 Repeat the process of finding differences between consecutive terms for the sequences in Columns D, F, and H.



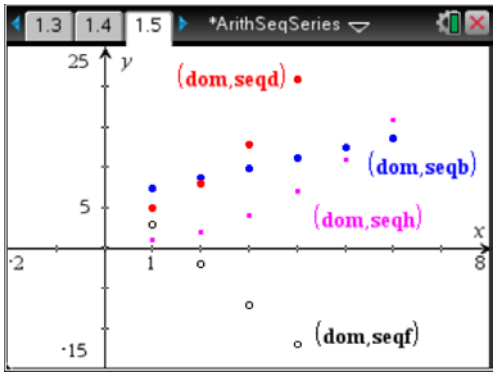
Make sure students always subtract a term ***from*** the term that follows it—not the other way around.

Step 3 On page 1.5, students will make a scatter plot of the domain (the natural numbers in Column A) and the terms in Sequence B. To create a scatter plot, press **MENU > Graph Entry/Edit > Scatter Plot**. Select **dom** for *x* and **seqb** for *y*.



Discuss the differences between the terms in Sequence B and its scatter plot on their worksheet.

Step 4 Instruct students to create scatter plots of the remaining three sequences in the same manner, on the same page.



Step 5 Students should make conjectures about the scatter plot of a sequence and the differences between the consecutive terms. They should conjecture that for sequences with a common difference (B and F), the points form a straight line. Discuss that sequences with a common difference, *d*, are called ***arithmetic sequences*** because you can get from one term to the next by adding the same number (this number can be positive or negative).

TI-Nspire Navigator Opportunity: *Quick Poll* and *Screen Capture*

See Note 1 at the end of this lesson.

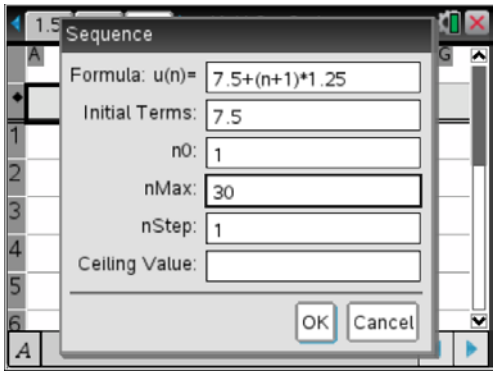
Problem 2 – Explicit formulas and sums

Step 1 Introduce the explicit formula $u_n = u_1 + (n - 1)d$, where u_n is any term of a sequence, n is the term number, u_1 is the first term, and d is the common difference. (You may wish to explain that this formula is *explicit* because finding a given term does not require knowing the previous term.)

Note that many texts use the variable *a* instead of *u*, but the handheld uses *u*, so we will use *u* here.

Step 2 On page 2.2, the spreadsheet should be used to write a formula to generate the first 30 terms of Sequence B in Problem 1 {7.5, 8.75, 10, 11.25, 12.5, 13.75, …}. For now, have students just replace u_1 and d in the formula $u_n = u_1 + (n - 1)d$.

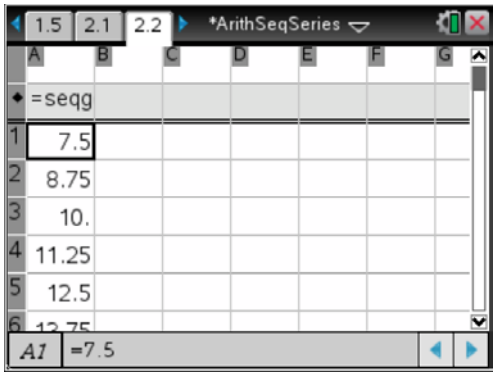
With the gray box at the top of Column A selected, press **MENU > Data > Generate Sequence**. Enter the formula in the first box, the first term in the next box, and the number of terms in the third box. (See the screenshot at right.)



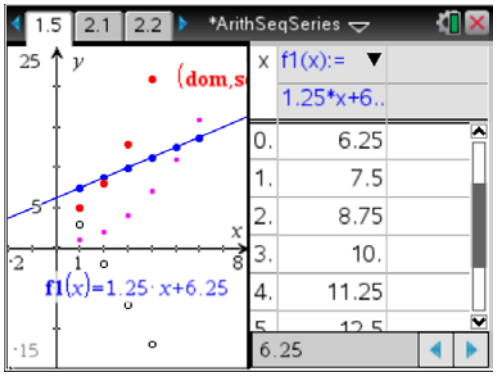
TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 2 at the end of this lesson.

After pressing **OK**, Cells A1 through A30 are populated with the first 30 terms of the sequence.

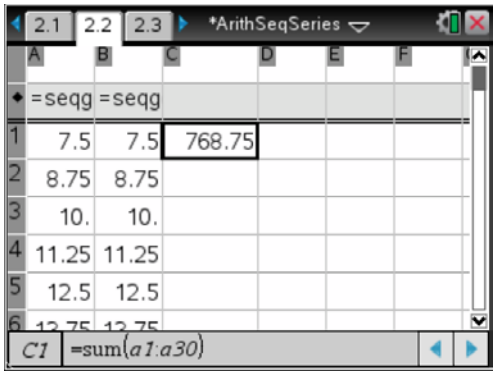


Step 4 If desired, you can have students return to page 1.5 and graph the function $y = 1.25x + 6.25$. The line will pass through the points of Sequence B. Pressing ctrl + T will display a function table showing the terms of the sequence in $f(x)$. Discuss how the simplified explicit formula is related to the graph.



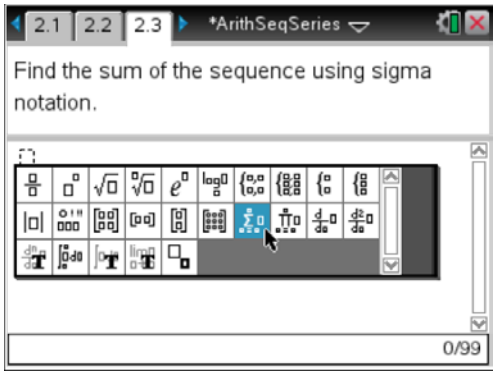
Step 5 Ask students to think about adding all of these terms together. Tell them that the expression consisting of summing the terms of a sequence is called a ***series***. On a spreadsheet, students can move to an open cell, such as C1, and enter the formula **=sum(a1:a30)**. (They could also sum the entries of Column B.) When entering the formula, be sure that they begin the formula with an equal sign and separate the first and last cells to be summed with a colon (?!▸).

To view all the digits in the sum, students can resize the column (**MENU > Actions > Resize > Resize Column Width**).

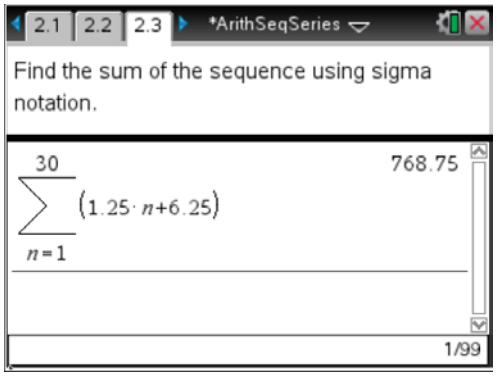


Step 6 On page 2.3, the sum will be found using the *Calculator* application. If needed, explain sigma notation.

The sigma command, Σ, is located by pressing |▫| and selecting the sigma template shown in the figure to the right.



Step 7 Enter the needed information to see that the sum is the same sum found on the spreadsheet.

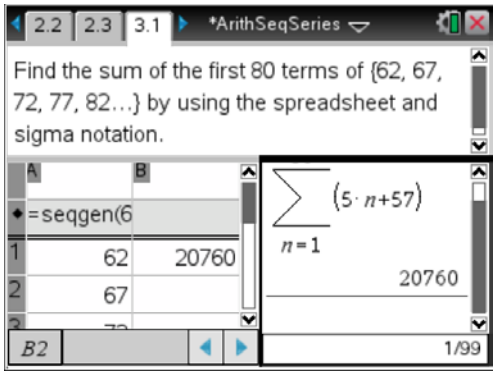


TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 3 at the end of this lesson.

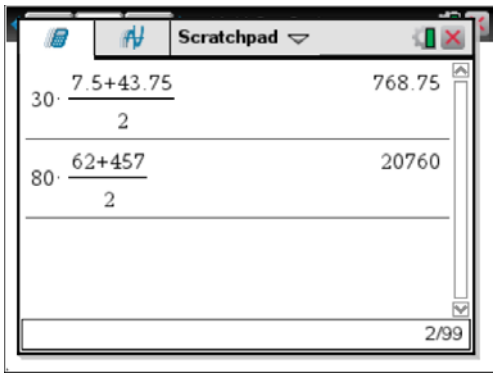
Problem 3 – Practice finding the sum of a series

Allow students to work independently on page 3.1. Using the *Lists & Spreadsheet* and *Calculator* applications, they should determine the explicit formula to be 5*n* + 57 and the sum to be 20,760.



Extension

The sum of the first *n* terms of an arithmetic series can be found by multiplying the number of terms *n* by the average of the first and last terms. Have students use the *Scratchpad* ( ) to show that this holds true for the sums found in Problems 2 and 3.



TI-Nspire Navigator Opportunities

Note 1

Problem 1, *Quick Poll* and *Screen Capture*

Use *Quick Poll* to gather student conjectures about the sequence graphs. Follow this up with by using *Screen Capture* to aide in the discussion of what makes a sequence arithmetic or not. Make use of the display to illustrate the effect the "constant" differences determined on the previous page have on the resulting graphs. Drive the discussion towards connecting "constant difference" to slope of the graph.

Note 2

Problem 2, *Live Presenter*

You may want to use *Live Presenter* to aide students in the use of the Sequence generating wizard.

Note 3

Problem 2, *Screen Capture*

You may choose to use *Screen Capture* to monitor student progress through the rest of the activity, aiding students where needed and promoting discussion where warranted.