Arithmetic Sequences & Series

TEACHER NOTES

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Math Objectives

- Students will explore arithmetic sequences and series.
- Students will consider the effect of the value for the common difference and first term using Lists and Spreadsheets on the TI-Nspire CX II.
- Students will graphically analyze the arithmetic series using graphs.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Arithmetic SequenceArithmetic Series
- Common DifferenceExplicit Formula
- About the Lesson
- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 1 Algebra:
 - **1.2a** Arithmetic sequences and series
 - **1.2b** Use of the formula for the n^{th} term and the sum of the first n terms of the sequence

1.2d Applications

• As a result, students will: Apply this information to real world situations

≰TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <u>http://education.ti.com/calculat</u> <u>ors/pd/US/Online-</u> <u>Learning/Tutorials</u>

Lesson Files:

Student Activity Arithmetic_Sequences_and_Serie s_Student-Nspire.pdf Arithmetic_Sequences_and_Serie s_Student-Nspire.doc Arithmetic_Sequences_and_Serie s.tns

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Activity Materials

Compatible TI Technologies: III TI-Nspire™ CX Handhelds,
 TI-Nspire™ Apps for iPad®, II-Nspire™ Software

Part 1 – Sequences and Scatter Plots

Open the file *Arithmetic_Sequences_and_Series.tns*. Move to **page 1.2**.

- Column A, titled n, shows a finite sequence with six terms.
- Column B, titled seq1, shows the term numbers.

Find the differences between consecutive terms of the sequence in Column B and record them in Column C, title it diff.

For the first difference, in the first row of Column C, subtract the second term of Column 2 from the first by typing =b2 – b1. Do this for the next four rows:
 =b3 – b2, b4 – b3, etc.

Now enter the following data into the first 6 rows of Column D: 5, 8, 13, 21, 34, 55. Title it seq2.

- Column 4 shows a finite sequence with six terms.
- L1 shows the term numbers for this sequence.

Find the consecutive differences for the Column D sequence and record them in Column E using the same method from before finding the differences in Column B.

Move to **page 1.3** and graph the sequences in Columns B and D.

Press the **var** button and select **n**. Move down to the y input. Press the **var** button a second time and select **seq1**, press **enter**.

1	1 1.2 🕨	Arithmeti	c…ies	RA	• 🛯 🗡
	An	B seq1	С	D	1
=					
1	1.	7.5			
2	2.	8.75			
3	3.	10.			
4	4.	11.25			
5	5.	12.5			-
Α	n				4 F

◀ 1.	1 1.2 🕨	Arithmeti	c…ies	RAD 📘	×
	An	B seq1	⊂ diff	D	
=					
1	1.	7.5	=b2-b1		
2	2.	8.75			
З	3.	10.			
4	4.	11.25			
5	5.	12.5			•
C1	=b2-b1			•	۲

◀ 2.	4 2.1 2.2 3.1 ▶ Arithmetic F					
	An	B seq1	⊂ diff	D seq2	E	1
=						
1	1.	7.5	1.25	5.		
2	2.	8.75	1.25	8.		
З	З.	10.	1.25	13.		
4	4.	11.25	1.25	21.		
5	5.	12.5	1.25	34.		•
Α	n				•	•

1.1 1.2 1.3 ► Arith	metic RAD 📋 📈
-10	
-6.67	





Press tab and repeat for **seq2** ($x \leftarrow n \text{ and } y \leftarrow seq2$), then press **enter**.

Once both are graphed, press **menu**, **4 Window/Zoom**, **9 Zoom – Data**.

2.1	2.2	2.3 🕨	Arithmetic	rad 📘 🔀
36.9 y			(n,seq 2)	0
• • •			٥	
а а а			0	
• 0.5		8	•	(n,seq1) 6.5

- 1. For each sequence, write the differences between the consecutive terms and give a description of the scatter plot.
 - a. Column B: seq1

Answer: 1.25, 1.25, 1.25, 1.25, 1.25. Possible answer: The points of the scatter plot form a straight line that slants up to the right.

b. Column D: seq2

Answer: 3, 5, 8, 13, 21. Possible answer: The points of the scatter plot form a curve.

c. Study the graphs and the differences you found in Column C and Column E. Make a conjecture.

Answer: Students should make conjectures about the scatter plot of a sequence and the differences between the consecutive terms. They should conjecture that for sequences with a common difference, the points form a straight line.

Move back to **page 1.2** and clear the data from Columns B, C, D, and E. Leave the natural numbers in Column A. To clear, select the equals box below each heading, press **menu**, **3 Data**, **4 Clear Data**.

• Enter the following sequences into Column B and Column D.

Column B: 3, -2, -7, -12, -17, -22

Column D: 1, 2, 4, 7, 11, 16

• Recalculate the differences between consecutive terms and record them in Columns C and E.

Move to page 1.3 and press menu, 4 Window/Zoom, 9 Zoom – Data.

1 .	2 1.3	2 .1) /	Arithmetic		RAD 📘	×
	An	B seq1	⊂ diff	D seq2	⊑ diff2	1
=						-
1	1.	3.	-5.	1.	1.	
2	2.	-2.	-5.	2.	2.	
З	з.	-7.	-5.	4.	3.	
4	4.	-12.	-5.	7.	4.	
5	5.	-17.	-5.	11.	5.	•
Α	n				•	Þ



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- **2.** For each sequence, write the differences between the consecutive terms and give a description of the scatter plot.
 - a. New Column B Sequence

Answer: -5, -5, -5, -5, -5. Possible answer: The points of the scatter plot form a straight line that slants down to the right.

b. New Column D Sequence

Answer: 1, 2, 3, 4, 5. Possible answer: The points of the scatter plot form a curve.

- c. With a classmate, discuss how your observations affect your conjecture about the scatter plot of a sequence and the differences between the consecutive terms. Share your discussion with the class.
 - **Answer**: Students should find that the new data reinforces their conjecture that for sequences with a common difference, the points form a straight line.

Part 2 – Explicit Formulas and Sums

An **arithmetic sequence** is formed by adding a fixed number, called a **common difference** (d), to each previous term (this number can be positive or negative).

The explicit formula for the *n*th term in an arithmetic sequence is

 $u_n = u_1 + (n-1) \cdot d,$

- u_n is any term of a sequence
- *n* is the term number
- u_1 is the first term
- *d* is the common difference

You can use this formula to calculate any term in an arithmetic sequence.

Move to **page 2.1** and generate a sequence in Column B to display the first 30 terms of

 $u_n = 7.5 + (n-1) \cdot 1.25.$

- Title Column B as seq3.
- Move down to the equal row below the heading seq3. Press menu, 3 Data, 1 Generate Sequence.
- Enter **7.5** + (n–1)*1.25, n0 is **1**, nMax is **30**, and the step is **1**. Press enter or select OK.



<u>Note</u>: There are two rows that are optional to fill. You can enter your initial term as 7.5 or leave it blank and you do not have to enter a ceiling value since we are only looking at the first 30 terms.

1.	Sequence	Arithmetic RAI	
-	Formula: u(n)=	7.5+(n-1)*1.25	
-	- Initial Terms:		
1	. n0:	1]
2	. nMax:	30	j
3	nStep:	1	i
4	Ceiling Value:		i I
5		OK Cancel	•
В	зецэ	OK	

- **3.** Simplify the formula $u_n = 7.5 + (n 1) \cdot 1.25$ by distributing and combining like terms. Use this formula in the sequence command (equal row below the heading Column C) to generate 30 terms of this sequence in Column C.
 - Note: If the handheld asks if this is a column reference or a variable reference due to a **Conflict Detected**, select **Variable Reference**.

1.3 1.4	2.1 🕨	Arithme	tic		RAD 📘	\times
An	Вs	eq3	C	۵)	
=	Confl	ict Dete	cted			
1	=1.25*	n +6.25.				
2	n: Co	olumn or \	/ariabl	e?		
3	Varia	ble Refer	ence	•		
4		ОК	Can	cel		
5	5.	12.5				•
C =1.25	n +6.25					•

Explain what you notice about the terms in Columns B and C.

◀ 2.	2 2.3	3.1	Arithm	etic	RAD	
	An		B seq3	С	D	1
=			=seqgen()	=1.25*'n+6		
1		1.	7.5	7.5		
2		2.	8.75	8.75		
З		3.	10.	10.		
4		4.	11.25	11.25		
5		5.	12.5	12.5		-
C1	=7.5					•

Answer: The terms that appear in Columns B and C should be the same.

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Part 3 – Practice Finding the Sum of a Series

The expression consisting of summing the terms in a sequence is called a **series**. To find the **finite** sum of the first *n* terms of an arithmetic sequence algebraically, you will use the formula: $S_n = \frac{n}{2}(2u_1 + (n-1) \cdot d)$ or $S_n = \frac{n}{2}(u_1 + u_n)$

Move to page 2.2. You can use the handheld to check the sum of the 30 terms by hand on the home screen enter **sum(seq3)**.

The Sum command can be found by pressing **menu**, **6 Statistics**, **3 List Math**, **5 Sum of Elements**. Seq3 can be typed in manually or found by pressing the **var** button.

4 2.3 3.1 3.2 ▶ Arithmeti	ic RAD 📘
sum(seq3)	768.75
1	

4. Find the sum of the first 30 terms of this sequence in Column B algebraically. Check your result using the **sum** command.

Answer: 768.75

5. Now, let's look at another sequence. Find the sum of the first 80 terms of the sequence below, using the Data and Statistics page to generate the following sequence as was done in part 2 and the sum() command on page 3.1.

a. Find the explicit formula for this sequence in simplified form.

1.	Sequence	rithmetic ies BAD 🗍 🕅
_	Formula: u(n)=	5n+57
-	- Initial Terms:	62
1	. n0:	1
2	. nMax:	80
3	nStep:	1
4	Ceiling Value:	
5		OK Cancel
В	3547	

<u>Answer</u>: $u_n = 5n + 57$

Ü	Arithmetic Sequences & Series		
b.	Find the sum of the first 80 terms.		
		2.1 3.1 3.2 Arithmetic ies sum(seq 4)	RAD 🗋 🗙

Answer: 20,760

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)* (to review before the Further IB Extension) Find the pattern of the following sequence: 5, -3, -11, -19, ... Tell why the sequence is arithmetic or not.

Further IB Extension

The Clemson Tigers football team play in the multilevel Memorial stadium. The closer you are to the field, the higher the ticket prices. The ticket prices for the first 4 rows of a Tigers football game are as follows:

Row 1: \$120 per ticket; Row 2: \$117 per ticket; Row 3: \$114 per ticket

These ticket prices continue in an arithmetic pattern.

(a) Find the common difference between each consecutive row price. [1 mark]

<u>Answer</u>: d = 114 - 117 = 117 - 120 = -3 A1

The ticket prices go down by \$3 each row.

(b) Calculate the price of a ticket in row 20.

Answer:
$$u_{20} = 120 + (20 - 1)(-3)$$
 (M1)
= $120 + (19)(-3)$
= $120 - 57$
= 63 A1

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[2 marks]



(c)

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Find the total cost of buying 2 tickets in each of the first 20 rows.				[3 marks]
<u>Answer</u> :	$S_{20} = \frac{20}{2} \left(2(120) + (20 - 1) \cdot (-3) \right)$	or	$S_{20} = \frac{20}{2}(120 + 63)$	(M1)
	= 10(240 - 57)		= 10(183)	
	= 10(183)		= 1830	
	= 1830			A1
	Multiplying their sum by 2 (tickets)			
	2(1830)			
	= \$3660			A1

**Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB[™]. IB is a registered trademark owned by the International Baccalaureate Organization.

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