Login to Logarithms

Teacher Notes

7 8 9 10 **10A** **11** 12

TI-Nspire CAS **Investigation** **Student** **60 min**

Aim:

To discover the meaning of "logarithms" and to make use of this understanding to develop laws of addition and subtraction of logarithms.

Number & Algebra – Year 10: Real Numbers

Use the definition of a logarithm to establish and apply the laws of logarithms (ACMNA265).

Equipment:

For this activity you will need:

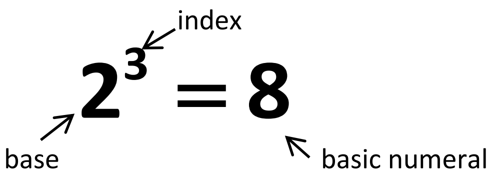
- TI-Nspire CAS

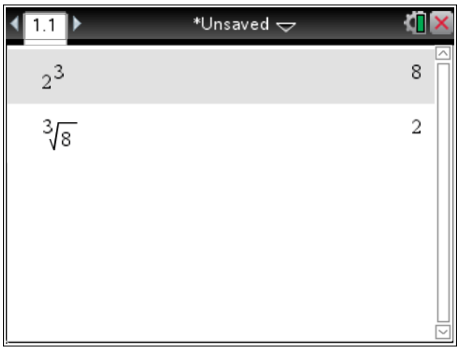
Technology:

Start a new document and open a **Calculator** page. The main function that we will be using for this activity is the **log** function. To access **log**, press **ctrl + 10^x**.

Initial Exploration and definition of logarithms

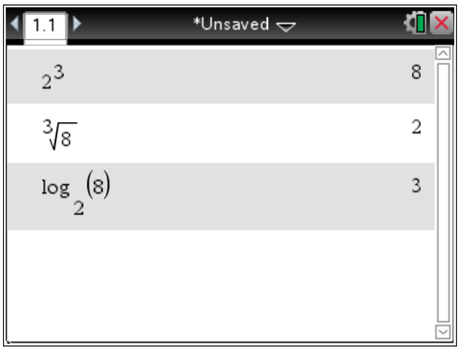
Step: 1. Input $\mathbf{2^3}$ and the result is **8**. For the relationship $\mathbf{2^3 = 8}$, the **2** is the *base*, the **3** is the *index* and the **8** is referred to as the *basic numeral*.



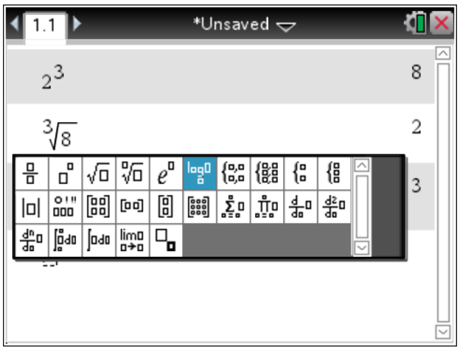


Step: 2. Input $\sqrt[3]{8}$ and the result is **2**. Notice that the relationship $\sqrt[3]{8} = 2$ uses the same three numbers.

Step: 3. Is there a way to rearrange the numbers to isolate the **3**? The answer is "Yes". This can be done by pressing **ctrl + 10^x** to access the **log** function. Input **2** for the base of the log and then input **8** on the main line. Notice that the relationship $\log_2 8 = 3$ uses the same three numbers.



Step: 4. The template $\log_{\square}(\square)$ can also be used. Calculate a few values using both the **log** key and the **template** and compare your answers.



"Log" is an abbreviation for "Logarithm", which in effect means the "index". A literal translation of the log equation $\log_2 8 = 3$ is "The logarithm to the base 2 of the basic numeral 8 is equal to 3".

It is a good idea to have students label the base, basic numeral and index in each of the three statements: $2^3 = 8$, $\sqrt[3]{8} = 2$ and $\log_2 8 = 3$.
The purpose of these steps is to show students that a logarithm relates to something that they are already familiar with. The "three-number relationship" is a bit like the way the numbers 2, 3 and 6 can be connected by the equations: 2×3=6, 6÷3=2 and 6÷2=3.

Did you know?

Most scientific calculators only have functions for logarithms to the base 10 and base "*e*". Base 10 logarithms are abbreviated as "log". Base "*e*" logarithms are called natural logarithms and these are abbreviated as "ln". We will not be dealing with natural logarithms in this activity.

If you are using one of these calculators and you need to find a logarithm with a different base, you can use either the "log" or the "ln" function.

For example, to find $\log_2 8$, calculate $\frac{\log 8}{\log 2}$ or $\frac{\ln 8}{\ln 2}$. Both answers are equal to 3.

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Practice using the log template

Use the following examples to practise using the log function. Try to predict what the result will be before you press **enter**. (e.g. "the logarithm to the base 2 of 16 is _4_")

a. $\log_2 16 =$ 4

b. $\log_5 25 =$ 2

c. $\log_5 125 =$ 3

d. $\log_5 625 =$ 4

e. $\log_{10} 1000 =$ 3

f. $\log_{10} 1000000 =$ 6

g. $\log_{10} 10 =$ 1

Summary		
base	**basic numeral**	**index**
2	**16**	**4**
5	**25**	**2**
5	**125**	**3**
5	**625**	**4**
10	**1000**	**3**
10	**1000000**	**6**
10	**10**	**1**

Encourage students to think of the logarithm function as asking them "what index is required?" For example, $\log_2 16 =?$ asks "what index must 2 be raised to in order to get 16?"
A quick review of indices or repeated multiplication may also be necessary.

e.g. 2×2=4 → 2, 4, 8, 16
4×2=8
8×2=16

2×2×2×2=16, so the index is 4.

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Using CAS to develop logarithm laws

There are a couple of clear advantages of using TI-Nspire for this process. One is that you can use the template to easily specify the base of the logarithms you want to work with. Another main advantage is that CAS is a *Computer Algebra System*, which allows you to perform computations with algebraic variables. Use CAS to work through the following questions to discover some special relationships about logarithms. These are logarithm laws and are very similar to the index laws which you have already learnt.

A review of index laws may be useful at this point:

$$\boldsymbol{a^1 = a,\ a^0 = 1,\ a^x \times a^y = a^{x+y},\ a^x \div a^y = a^{x-y}}, \text{ and } \boldsymbol{(a^x)^y = a^{xy}}.$$

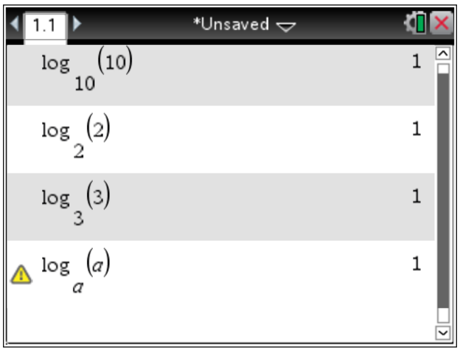
1. Find:

a. $\log_{10} 10 =$ **1**

b. $\log_2 2 =$ **1**

c. $\log_3 3 =$ **1**

d. $\log_a a =$ **1**



(This is a rearrangement of the index law $\mathbf{a^1 = a}$.)

2. Now that you have identified the pattern and given what you now understand about the definition of logarithms, does this general result surprise you? Describe this general result in your own words.

A logarithm with the same base and basic numeral is always equal to 1.

3. Find:

a. $\log_{10} 10 + \log_{10} 100 =$ 3

b. $\log_{10} 1000 =$ 3

c. $\log_2 8 + \log_2 16 =$ 7

d. $\log_2 128 =$ 7

e. $\log_3 9 + \log_3 27 + \log_3 81 =$ 9

f. $\log_3(9 \times 27 \times 81) =$ 9

g. $\log_3 4 + \log_3 5 =$ $\log_3 20$

h. $\log_a 4 + \log_a 5 =$ $\log_a 20$

Students should discover how these questions are related in pairs a-b, c-d, e-f, g-h.
For example, in the c-d pair, 8×16=128. Examples g-h show that this idea works for any base.

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4. Without using your calculator, complete the general rule:

$$\log_a x + \log_a y = \log_a(x \cdot y)$$

If students do not see the pattern, encourage them to complete more examples until they do. Alternatively, point out how this log law relates to the idea of $a^x \times a^y = a^{x+y}$.

5. State this rule in your own words.

When two logarithms of the same base are added together, the result is one logarithm (to that base) of a basic numeral that is the product of the two original basic numerals.

6. Find:

a. $\log_{10} 1000000 - \log_{10} 100 =$ 4

b. $\log_{10} 10000 =$ 4

c. $\log_2 128 - \log_2 16 =$ 3

d. $\log_2 8 =$ 3

e. $\log_3 35 - \log_3 5 =$ $\log_3 7$

f. $\log_2 7 =$ $\log_3 7$

g. $\log_a 35 - \log_a 5 =$ $\log_a 7$

Students should discover how these questions are related in pairs a-b, c-d, e-f, and note that 1000000÷100=10000, 128÷16=8, 35÷5=7. Part g shows that this idea works for any base.

7. Without using your calculator, complete the general rule:

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

If students do not see the pattern, encourage them to complete more examples until they do.

8. State this rule in your own words.

When one logarithm is subtracted from another logarithm of the same base, the result is one logarithm (to that base) of a basic numeral that is the quotient of the two original basic numerals.

Ensure that you engage students in the correct and precise language of mathematics. While it is important to have them articulate mathematical patterns and rules in their own words, you cannot compromise on the correctness of language. "Mathematical literacy" is the key to helping them develop their understanding. If they are not familiar with the vocabulary, then it must be explicitly taught.

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9. Find:

a. $\log_5 8 =$ $3 \cdot \log_5 2$

b. $\log_2 27 =$ $3 \cdot \log_2 3$

c. $\log_{17} 1000 =$ $3 \cdot \log_{17} 10$

d. $\log_a 8 =$ $3 \cdot \log_a 2$

e. $\log_a 27 =$ $3 \cdot \log_a 3$

f. $\log_a 1000 =$ $3 \cdot \log_a 10$

g. $\log_a 64 =$ $6 \cdot \log_a 2$

h. $5^6 =$ 15625

i. $\log_a 15625 =$ $6 \cdot \log_a 5$

You may need to ask students additional questions such as:

- Following from 9a, "What is 2^3?"
- Following from 9b, "What is 3^3?"
- Following from 9c, "What is 10^3?"

The purpose of 10 d–i is to show students that this works for any base.

10. Without using your calculator, complete the general rule**:**

$$\log_a(x^y) = y \cdot \log_a(x)$$

If students do not see the pattern, encourage them to complete more examples until they do.

11. State this rule in your own words.

If the basic numeral can be expressed in the form x^y, then the logarithm can be rewritten as *y* multiplied by the log of *x*.

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Conclusion

You can do further investigation of logarithms and their properties. For now, write a summary of the laws that you have discovered in this exercise.

From question 1 d $\log_a a = 1$

From question 4 $\log_a x + \log_a y = \log_a (x \cdot y)$

From question 7 $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

From question 10 $\log_a (x^y) = y \cdot \log_a x$

An appropriate follow-up activity from here would be to have students complete a mixed set of problems similar to those in questions 1, 3, 6 and 9. You can also get students to research applications of logarithms in areas such as decibels, Richter scale, pH and Vernier scales.

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