

NUMB3RS Activity: Exponential Modeling Episode: "Sniper Zero"

Topic: Exponential Modeling

Grade Level: 10 - 12

Objective: In this activity, students will find an exponential model of a data set. This activity can be used to introduce the concept of exponential functions, to identify exponential functions from a data set, and to see how the values of a and b impact the graph of $y = ab^x$.

Time: 25 - 30 minutes

Materials: TI-83 Plus/TI-84 Plus graphing calculators, TI-Navigator™ system, and the following activity settings file: *Exponential.act*
To download this file, go to <http://education.ti.com/exchange> and search for "7724."

Introduction

In "Sniper Zero," Charlie demonstrates that the frequency of sniper victims as a function of time follows an exponential pattern. He uses this information to convince the FBI agents that there is "an epidemic of copycat snipers." In an effort to explain the concept of exponential growth, Charlie asks his colleagues to imagine a neighborhood in which all the houses are painted white. One day, someone paints his house blue. Another neighbor likes the color and decides to color his house blue as well. This influences two more people and the exponential spread of blue houses has begun. Agent Terry Lake chimes saying, "viral behavior...Malcolm's notion of a tipping point...ideas and trends...what's in what's not...it's really just human behavior spreading infectiously."

In Part I of this activity, students will look at a set of data that shows the number of cellular telephone subscribers, in millions, from 1985 to 1995, a trend that is similar to those discussed in the Sniper Zero episode. Students will look at the ratios of consecutive data points to see that the rate of growth from one year to the next is fairly consistent, which will help justify the use of an exponential model.

In Part II, students will experiment with different values of a and b until they find a function of the form $y = ab^x$ that fits the data well. Students will see that the value of b is similar to the value of the ratios found in Part I. In Part III, students will use their calculators to find an exponential regression model to see how it compares with the equations they found in Part II.

Part I: Identifying an Exponential Model


- Students should work individually during this part of the activity.
 - Launch TI-Navigator on the computer and press **Begin Class** to start the session.
 - Have each student log into NavNet on their calculators.
- Load the *Exponential.act* activity settings file into Activity Center and click the 'List – Graph' tab. This will display a scatterplot, along with the data points, for year after 1985 versus the number of cellular telephone subscribers, in millions (*source: CTIA – The Wireless Association*).

- b. Ask students to comment on the data and recommend how it might be modeled. Many students will suggest that the data can be modeled by a quadratic function, especially if you are just introducing the concept of exponential functions. Explain to students that this is a reasonable guess and that a quadratic model will in fact fit the data quite well.
- c. Tell students that you are going to have them try to find an exponential model, of the form $y = ab^x$, to fit the data. To justify this decision, assign any ten students in the classroom a number from 1 to 10. Click **Quick Poll** and select *Open Response*. Also, make sure that the *Numeric* box is checked. Press and tell your ten students to take the number of cell phone users for their year and divide by the number of cell phone users for the previous year (for example, a student assigned to year 5 will divide 5.2 by 3.5). After the responses have come in, press and click the **Poll Summary** tab. You will see a list of numbers ranging from about 1.40 to 2, with most numbers clustered between 1.4 and 1.5. You can also uncheck the *Numeric* box, which will reveal the calculations each individual student made.
- d. Explain to students that, since these ratios between successive terms are similar, it suggests that an exponential model is a good choice. This is a good time to emphasize that exponential functions are characterized by a constant rate of growth (or decay).

Part II: Find an Exponential Model


3. a. Now tell students that they will try to find an exponential function of the form $y = ab^x$ to fit the data. Press and have students enter Activity Center. This activity has been configured for students to **Send** one equation. They also have the option of viewing the scatterplot on their calculators, along with their equations, before sending it into Activity Center. Emphasize to students that they should experiment with different values of a and b until they find an equation that fits the data well. When they are ready, have students press **Send** to submit their graphs to Activity Center.
- b. Press . Click the 'Graph-Equation' tab and discuss the results with the entire class. Point out to students which values of a and b resulted in the model that seemed to best fit the data. In particular, show students that the value of b is similar to the ratios that were calculated in Part I of this activity. You can click on individual equations to highlight specific graphs during this part of the discussion. This is a good time to explain that b is related to the growth rate of the function. That is, a value of $b = 1.5$ relates to a rate of growth of 50% from one year to the next.
- c. Ask students how they think the value of a affects the graph. You can give them a hint by asking them what happens when x equals zero. Make sure that they understand that a corresponds to the y -intercept of the graph of $y = ab^x$.

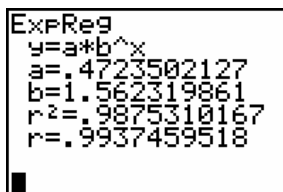
Part III: Find the Regression Equation

4. a. Change *Contribute to Lists* and press . This will automatically send lists L_1 and L_2 to each pair of students. (**Note:** List L_3 will also be sent to students, which is used with the Extension to this activity.)

b. Press .


5. Students should then exit NavNet and return to the calculator's home screen.

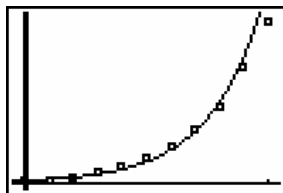
6. Have students perform an exponential regression for L_1 versus L_2 (press , go to the **CALC** menu, and select **0:ExpReg**). Students should enter the command **ExpReg Y_1** . This will perform the exponential regression and save the regression as equation Y_1 . Use **Screen Capture** to view the student results, which should match the screen shown below.



```
ExpReg
y=a*b^x
a=.4723502127
b=1.562319861
r^2=.9875310167
r=.9937459518
```

Ask students to compare this regression equation with the equations found by them in Part II. Emphasize the fact that the regression equation indicates that the number of cell phone users is growing at a rate of 56% during this 10-year period.



7. Next, turn on **Plot 1** for L_1 versus L_2 , and press  to view the scatter plot along with the regression equation. Use **Screen Capture** again to view the results, which should match the screen shown below.

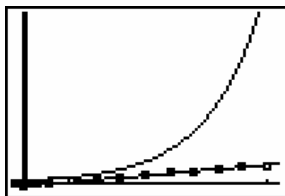


Ask students if they think the regression equation is a good fit for the data. You may also want to ask them if they think it can be used to make predictions beyond 1995. They should recognize that the equation would most likely produce an overestimate. In fact, students should be cautioned that extrapolation of the data beyond 1995 will produce erroneous results (In 1996 there were 44.04 million cell phone subscribers. The regression equation predicts that there were 63.93 million subscribers.)

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

- This extension can be used if students have learned the product and power rules for logarithms, $\log_a bc = \log_a b + \log_a c$ and $\log_a b^d = d \log_a b$.
- The equations submitted to Activity Center in Part II of this activity should still be displayed in Activity Center. Click the 'List – Graph' tab and turn on 'Plot 2'. This will reveal a second plot, which is L_1 versus L_3 . Explain to students that L_3 is a result of taking the natural log of each element contained in L_2 . Explain to students that, if the resulting data points are linear, it confirms that the original data set is in fact exponential.
- Press  and have students submit their linear equations of the form $y = \ln a + x \ln b$. Students should see that their transformed equations are a good match for the second data set. Highlight a particularly good fit and the corresponding exponential equation from Part II (hold down the 'Ctrl' key) to show students that both the exponential model and the linear model fit the data well.
- Press .
- Have students exit NavNet. From Part I of this activity, students will still have the lists L_1 , L_2 , and L_3 on their calculators. They should also have the exponential regression equation for L_1 and L_2 in Y_1 . Have students perform a linear regression for L_1 versus L_3 (press **STAT**, go to the **CALC** menu, and select **4:LinReg(ax+b)**). Students should enter the command **LinReg(ax+b) L1,L3,Y2**. This will perform the linear regression and save the regression as equation Y_2 . Next, turn on **Plot 2** for L_1 versus L_3 , and press **GRAPH** to view the scatter plot along with the regression equation. Use **Screen Capture** again to view the results, which should match the screen shown below.
- Use **Screen Capture** to view the student results, which should match the screen below.



Because the least squares regression line seems to be a good fit, students should be convinced that the original data set is in fact a good exponential model.

- In fact, when the graphing calculator determines the exponential model, it uses the linearized data (L_1 and L_3) to find the least squares regression line, then converts it to an exponential function. This can be verified by turning the calculator diagnostics on as follows:
 - Press $\boxed{2\text{nd}}$ [CATALOG]. This will access the **Catalog** menu and jump down to the functions beginning with **D**.
 - Move the cursor to **DiagnosticOn** and press $\boxed{\text{ENTER}}$. This will ensure that the correlation coefficient **r** will be displayed when a regression analysis is performed.
- Have students redo the exponential regression of L_1 and L_2 and the linear regression of L_1 and L_3 . As seen in the screens below, the correlation coefficients **r** are identical.

```
ExpReg
y=a*b^x
a=.4723502127
b=1.562319861
r^2=.9875310167
r=.9937459518
```

```
LinReg
y=ax+b
a=.446171807
b=-.7500345925
r^2=.9875310167
r=.9937459518
```

Additional Resources

- An additional *NUMB3RS* activity for "Sniper Zero" can be downloaded for free by going to <http://education.ti.com/exchange> and searching for "7723."
- If you would like to learn more about TI-Navigator, visit <http://education.ti.com/navigator>.