## Similar Signs

Teacher Notes

## $\begin{array}{llllll}7 & 8 & 9 & 10 & 11 & 12\end{array}$


TI-Nspire CAS

Investigation

Student

60min

## Aim

The aim of this investigation is to use similarity to explore ratios in right angled triangles.
National Curriculum Statement: Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (ACMMG223)

## Equipment

For this activity you will need:

- TI-nspire CAS
- TI-nspire CAS document - Similar Signs


## Introduction - Setting up the calculations

During this activity, and the "Just cos" activity, there is no reference to sine, cosine or tangent. Some students may have already seen or used these ratios, however the purpose of this activity is to have students using the ratios through their existing or constructed knowledge of similar triangles. (Constructivist Theory) Students can use their tables constructed in this investigation to help with basic length and angle computation without the direct use of sine, cosine or tangent.

The "Similar Signs" TI-nspire file can be distributed using TI-Navigator, the TI-nspire docking station or the teacher/student software. To distribute the file using the Teacher software, use the Tools menu and select the Transfer Tool. Locate the "Similar Signs" TI-nspire file on your computer and then start the transfer. Once the file is transferred to the first handheld, unplug the handheld and continue plugging in each student's handheld device. Once all the students have the file, stop the transfer. Note that students can also transfer the file from one handheld device to another from within the My Documents folder.

This activity requires access to the "Similar Signs" TI-nspire document. This document should be loaded on your device before proceeding.

Once the document is on your handheld, press [Home] and select My Documents. Locate the document and press [Enter] to open.

The location of the file depends on the selected location during the file transfer.


Navigate to page 1.3 and use the spinners to adjust the angle ( $\alpha$ ) and scale factor (grow) to:

$$
\alpha=30^{\circ}
$$

$$
\text { Grow = } 1.5
$$

Use the mouse to click on the spinner, the measurements and diagram change accordingly.

4. The first question in this investigation requires calculations to be performed. The Scratchpad is a place where calculations can be computed and then discarded. To access the Scratchpad press [Home] and select Scratchpad or press $\mathbf{A}$.

To return to the current document, press [Esc] followed by Current (4).


1. Copy and complete the table below.

| Angle | Grow | $\mathrm{B}^{\prime} \mathrm{C}^{\prime}$ | $\mathrm{AC}^{\prime}$ | $\mathrm{B}^{\prime} \mathrm{C}^{\prime} / \mathrm{AC}$ |
| :---: | :---: | :---: | :---: | :---: |
| $30^{\circ}$ | 1.5 | 6 | 12 | 0.5 |
| $30^{\circ}$ | 2.0 | 8 | 16 | 0.5 |
| $30^{\circ}$ | 2.5 | 10 | 20 | 0.5 |
| $30^{\circ}$ | 3.0 | 12 | 24 | 0.5 |

Change the angle ( $\alpha$ ) and scale factor (grow) to:

$$
\alpha=50^{\circ}
$$

Grow $=1.5$

2. Copy and complete the table below.

Some students may not identify the 'constant' ratio due to the small rounding variation in the answers. It should be highlighted to students that the original side lengths as stated with only 3 digit accuracy, therefore the ratios will only be accurate to two decimal places.

| Angle | Grow | $\mathrm{B}^{\prime} \mathrm{C}^{\prime}$ | $\mathrm{AC}^{\prime}$ | $\mathrm{B}^{\prime} \mathrm{C}^{\prime} / A C^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| $50^{\circ}$ | 1.5 | 12.4 | 16.2 | 0.765 |
| $50^{\circ}$ | 2.0 | 16.5 | 21.5 | 0.767 |


| $50^{\circ}$ | 2.5 | 20.6 | 26.9 | 0.766 |
| :--- | :--- | :--- | :--- | :--- |
| $50^{\circ}$ | 3.0 | 24.8 | 32.3 | 0.767 |

3. Complete the following statement:
"For a given angle, the ratio $\mathrm{B}^{\prime} \mathrm{C}^{\prime} / \mathrm{AC}$ ' remains constant when the scale (grow) is changed."
This is the FOCUS of the activity! Ensure each student understands this concept.
It is possible to have the calculator perform computations on this Geometry application directly. Move the mouse to an clear space and use a right mouse click [CTRL] + [Menu] and select text.

Type the following text:

```
opp / hyp
```

where $\div$ is displayed as / then press [Enter]
This could have been included earlier, however if too much is done automatically, students can miss the detail or the detail may seem too trivial. The Spreadsheet application can also be used to automatically capture and graph the relationship. This is included in later activities.

Move the mouse over the text and again use a right mouse click, select calculate.


The calculate option checks the text for possible variables and requires the variables to be linked to values on the current screen.
$A C^{\prime}$ represents the hypotenuse (hyp).
The prompt requests "Select hyp ?", move the mouse over the measurement and click on it.
$B^{\prime} C^{\prime}$ represents the opposite side (opp).


Move the mouse over the measurement and click on it. The calculation is complete and the result is 'stuck' to the mouse. Move the mouse adjacent to the text expression and click to place the answer next to the text.

Clicking on the values in the calculate process helps students link
"Opposite" to side $B^{\prime} C^{\prime}$, similarly with hypotenuse and side $A C^{\prime}$.
Adjust the angle and grow variable to:

$$
\begin{aligned}
& \alpha=30^{\circ} \\
& \text { Grow }=2.0
\end{aligned}
$$

Make sure the computation is the same as the original result.

4. Copy and complete the table below.

| Angle | Grow | $\mathrm{B}^{\prime} \mathrm{C}^{\prime}$ | $\mathrm{AC}^{\prime}$ | $\mathrm{B}^{\prime} \mathrm{C}^{\prime} / \mathrm{AC}^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| $10^{\circ}$ | 2.0 | 2.44 | 14.1 | 0.174 |
| $20^{\circ}$ | 2.0 | 5.04 | 14.7 | 0.342 |
| $30^{\circ}$ | 2.0 | 8 | 16 | 0.5 |
| $40^{\circ}$ | 2.0 | 11.6 | 18.1 | 0.643 |
| $50^{\circ}$ | 2.0 | 16.5 | 21.5 | 0.766 |
| $60^{\circ}$ | 2.0 | 24 | 27.7 | 0.866 |
| $70^{\circ}$ | 2.0 | 38.1 | 40.5 | 0.94 |
| $80^{\circ}$ | 2.0 | 79.8 | 0.985 |  |

5. Explain why the ratio $A B^{\prime} / A C^{\prime}$ gets close to 1 (one) when the angle is large, such as $80^{\circ}$.

The opposite side gets closer in length to the hypotenuse therefore the ratio approaches one. Other questions such as 'explain why the ratio gets close to zero when the angle approaches zero can also be asked. Alternatively, investigate small angles and the ratio... the values are almost the same.
6. Does the ratio increase by the same amount each time the angle is changed by $10^{\circ}$ ?

The ratio goes up by a different amount each time, it is not linear.
7. Complete the following table by estimation. Estimate the ratio to 2 (two) decimal places. Check your estimation using the calculator and record the actual ratio correct to 2 (two) decimal places.
Notice in this question the 'Grow' factor changes, students should realise this can be ignored since the ratio is constant regardless of the dilation (grow) factor, from question 3.

| Angle | Grow | Estimate: Opp/Hyp | Actual: Opp/Hyp |
| :---: | :---: | :---: | :---: |
| $5^{\circ}$ | 2.0 | Answers will vary. <br> Responses generally within 0.02 of actual ratio. Larger variation generally indicates student is using a linear | 0.09 |
| $15^{\circ}$ | 2.0 |  | 0.26 |
| $25^{\circ}$ | 2.0 |  | 0.42 |
| $35^{\circ}$ | 2.0 |  | 0.57 |
| $45^{\circ}$ | 1.5 |  | 0.71 |


| $55^{\circ}$ | 2.5 | interpolation. | 0.82 |
| :---: | :---: | :---: | :---: |
| $65^{\circ}$ | 3.0 |  | 0.91 |
| $75^{\circ}$ | 0.6 |  | 0.91 |
| $85^{\circ}$ | 2.4 |  | $1.00(0.996)$ |

8. Complete the following table by estimation. Estimate the ratio to 2 (two) decimal places. Ask your teacher to check your estimates.
These values can't be obtained from the spinner as it is set to 5 degree increments. Students may however recognise the ratio as the sine ration and be able to compute them.

| Angle | Grow | Estimate: Opp/Hyp | Teacher: Opp/Hyp |
| :---: | :---: | :---: | :---: |
| $28^{\circ}$ | 2.2 | Answers will vary | 0.469 |
| $42^{\circ}$ | 1.6 |  | 0.669 |
| $72^{\circ}$ | 2.0 |  | 0.951 |

