

Student Worksheet 1 Solutions

TI-15 Explorer™: Prime Factors

WS1

1.

| Number to try dividing into 60 | Answer | Factor? Yes or No 'Gozinter' | Any factors found |
|--------------------------------|----------|------------------------------|-------------------|
| 1 | 60 | Yes | 1, 60 |
| 2 | 30 | Yes | 2, 30 |
| 3 | 20 | Yes | 3, 20 |
| 4 | 15 | Yes | 4, 15 |
| 5 | 12 | Yes | 5, 12 |
| 6 | 10 | Yes | 6, 10 |
| 7 | 8.571... | No | |
| 8 | 7.5 | No | |
| 9 | 6.66... | No | |
| No more to try | | | |

The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

2. 12

3. The factors of 60 are **1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60**

The factors in bold are not prime. 2, 3 and 5 are prime factors of 60.

4.

| Number | List of factors Circle any that are prime numbers | How many factors does this number have altogether? | Any PRIME factors found | $\sqrt{\text{Number}}$ from your TI-15 |
|--------------------------|--|--|-------------------------|---|
| 24 | 1, 2, 3, 4, 6, 8, 12, 24 Only 2 and 3 are prime | 8 | 2, 3 | $\sqrt{24} = 4.89$ |
| 17 | 1, 17 | 2 | 17 | $\sqrt{17} = 4.12$ |
| 36 | 1, 2, 3, 4, 6, 9, 12, 18, 36 | 9 | 2, 3 | $\sqrt{36} = 6$ |
| 25 | 1, 5, 25 | 3 | 5 | $\sqrt{25} = 5$ |
| 50 | 1, 2, 5, 10, 25, 50 | 6 | 2, 5 | $\sqrt{50} = 7.07$ |
| 64 | 1, 2, 4, 8, 16, 32, 64 | 7 | 2 | $\sqrt{64} = 8$ |
| 72 | 1, 2, 3, 4, 6, 8, 9, 12, 18, 36, 72 | 12 | 2, 3 | $\sqrt{72} = 8.48$ |
| Add other numbers to try | | | | |

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5. Perfect squares have an ODD number of factors because the 'middle' factor is repeated.
6. Any perfect square.
7. You can stop when the numbers start to repeat.

Student Worksheet 2 Solutions

TI-15 Explorer™: Prime Factors

WS2

1. a) $1 \times 72, 2 \times 36, 3 \times 24, 4 \times 18, 6 \times 12, 8 \times 9$.

The next number to try would be 9 and we have it already, so we can stop.

- b) $1 \times 2 \times 36, 1 \times 3 \times 24, 1 \times 4 \times 18, 1 \times 6 \times 12, 1 \times 8 \times 9$.

$2 \times 2 \times 18$ not all different, $2 \times 3 \times 12, 2 \times 4 \times 9, 2 \times 6 \times 6$ – not different, $3 \times 4 \times 6$.

Encourage students to work systematically.

- c) If 1 is allowed as a factor, then adding in a multiplier of 1 to any of the above with 3 different not including 1 gives $1 \times 2 \times 3 \times 12, 1 \times 2 \times 4 \times 9, 1 \times 3 \times 4 \times 6$. If 1 is not included, it is not possible to have 4 different factors, nor 5 of course.

- d)

| Number | As the product of PRIME factors | How many factors does this number have altogether? |
|--------|--|--|
| 24 | $2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$ | 8 |
| 17 | 17^1 – Remember 1 is not prime | 2 |
| 36 | $2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$ | 9 |
| 25 | $5 \times 5 = 5^2$ | 3 |
| 50 | $2 \times 5 \times 5 = 2^1 \times 5^2$ | 6 |
| 64 | $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$ | 7 |
| 72 | $2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$ | 12 |

There is a pattern to see here to find the total number of factors....

Add 1 to each exponent and then multiply, the answer is the number of factors.

e.g. if the number is $2^a \times 3^b$ then the number has $(a + 1) \times (b + 1)$ factors.

The same applies if there are more than two primes in the factorisation.

2. a) 16

- b) 36

- c) 90

- d) 4 200

5. a) $30 = 2 \times 3 \times 5$

- b) $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

- c) $67 = 1 \times 67$ and is prime

6. $2^5 = 32$ which has 6 factors. They are 1, 2, 4, 8, 16 and 32. Note the exponent $5 + 1 = 6$

3^5 or any prime to the power 5 will have 6 factors.

Any factorisation with a prime to the power 2 multiplied by any other prime to the power 1 will have $(2 + 1) \times (1 + 1) = 6$ factors. Students may use trial and error of course.

7. Any number which is the product of any 6 different primes.
e.g. $2 \times 3 \times 5 \times 7 \times 11 \times 13$ etc

8. The smallest number with 6 factors is $2^2 \times 3^1$ which is 12. $2^1 \times 3^2 = 18$ etc.

9. This is quite difficult but since $20 = 4 \times 5$ we could try $2^{(5-1)} \times 3^{(4-1)} = 2^4 \times 3^3 = 16 \times 27 = 432$, but is there a smaller number? Since $20 = 5 \times 2 \times 2$, we could try $2^{(5-1)} \times 3^1 \times 5^1$ which is $16 \times 3 \times 5 = 240$ and this is the smallest one with 20 factors.

1. 1 is not prime by definition, 4 is not because it can be divided by 2 as well as itself and 1, 15 is not (factors of 3 and 5), 23 is, 6.5 is not a whole number, 133 is not because it can be divided by 7.
2. $2^2 \times 3^4$
3. $3^5 \times 7^2 = 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 = 11\,907$
4. 4 is not a prime number.
5. $30 = 2 \times 3 \times 5$
 $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$
6. $24 = 2^3 \times 3$ and it has 8 factors which are 1, 2, 3, 4, 6, 8, 12 and 24
 $2^1 \times 3^3 = 54$ has 24 factors