# Meet You at the Intersection: Solving a System of Linear Equations

Many times, the solution to a real-life problem involves solving more than one mathematical equation at the same time. The simplest situation of this type involves a pair of equations with two unknown quantities called a *linear system*. Graphically, the solution to this kind of system represents the point where the graphs of these two lines intersect.

For example, imagine that a person is running to catch up to his friend who is walking ahead of him. These motions can be modeled graphically by plotting distance versus time. The motion graphs are linear if each person is moving at a constant speed. If the plots are made on the same set of axes, the point where the two lines cross represents the physical location where the two friends pass each other.

It is possible to model situations of this type in the classroom using two Motion Detectors. In this activity you will collect and analyze motion data in order to determine the solution to a linear system of equations.
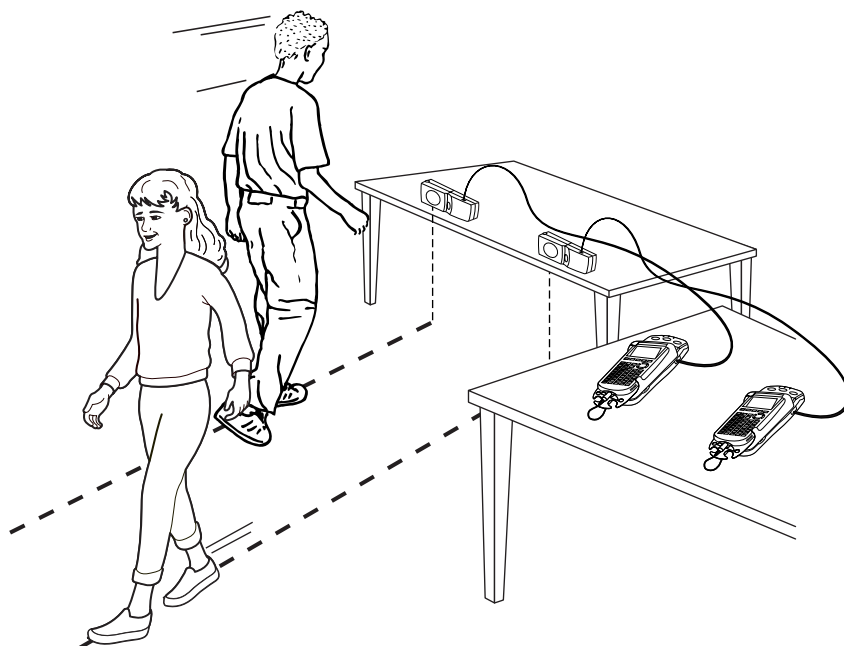
OBJECTIVES

- Record motion data for two walkers.
- Graph both motions on a common axis and find their intersection.
- Find linear equations to model the motions.
- Solve the system of two linear equations to determine the intersection.
- Compare the algebraic solution to the graphical solution.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator(s)*
EasyData application
two CBL 2 and two Motion Detectors
    or
two CBR 2 or Go! Motion with direct calculator cables
    or
one Vernier LabPro and two Motion Detectors
meter stick
stopwatch

*Two calculators are needed if using CBR 2, Go! Motion, or CBL 2. One calculator is needed if using a Vernier LabPro.

PROCEDURE

This activity can be done with a variety of combinations of motion detectors and data-collection interfaces. Determine the combination you will be using and carefully read the instructions for your combination.

1. Set up the Motion Detector and calculator.

   a. Turn on your graphing calculator(s).
   b. Open the pivoting head of the Motion Detectors. If your Motion Detectors have sensitivity switches, set them to Normal as shown.
   c. Connect the Motion Detectors to the calculators(s). (This may require the use of a data-collection interface.)
      - If you are using two CBL 2 data-collection interfaces, connect the Motion Detectors to the DIG/SONIC port of each.
      - If you are using two CBR 2 or Go! Motion with direct calculator cables, connect the Motion Detector directly to the calculator.
      - If you are using a LabPro, connect the Motion Detectors to the DIG/SONIC 1 and DIG/SONIC 2 ports (use Port 1 for the *x*-detector).

2. Set up the Motion Detectors on a table or desk. The detectors should be about two meters apart, aimed parallel to one another and about 1.2 m above the floor.

3. Set up EasyData for data collection. If you are using two calculators, do this on both calculators.

   a. Start the EasyData application, if it is not already running.
   b. Select ⌈**File**⌉ from the Main screen, and then select **New** to reset the application.
   c. Select ⌈**Setup**⌉ from the Main screen, then select **Time Graph…**
   d. Select ⌈**Edit**⌉ on the Time Graph Settings screen.
   e. Enter **0.2** as the time between samples in seconds.
   f. Select ⌈**Next**⌉.
   g. Enter **50** as the number of samples and select ⌈**Next**⌉.
   h. Select ⌈**OK**⌉ to return to the Main screen.

4. You will need two people to walk, a third (the timer) to operate the stopwatch, and a fourth (the marker) to mark the place where the two walkers pass each other.

   Practice walking in the following manner:
     • Walker one stands about two meters from the first detector, and prepares to walk directly toward that detector.
     • Walker two stands just over half a meter from the second detector, and prepares to walk directly away from that detector.
     • Once the timer has started data collection, timing must also be started.
     • At the moment the two walkers pass one another, the marker must note the position on the floor, while the timer stops the stopwatch. The walkers then continue to walk at a slow but uniform pace until data collection ends. Walker one must never get closer than 0.5 m to the Motion Detector.

   Practice the procedure several times.

5. Select ⟨Start⟩ (on both calculators simultaneously if you are using two) to begin data collection. Walk as you practiced. Data collection will run for ten seconds.

6. When data collection is complete, a graph of distance versus time will be displayed on the calculator screen(s). Examine the distance versus time graphs. Walker one's graph should be a nearly linear and uniformly decreasing function. Walker two's graph should be a nearly linear and uniformly increasing function. If the final few seconds of data are not useful, do not be concerned.

   Check with the teacher if you are not sure whether you need to repeat the data collection. To repeat data collection, select ⟨Main⟩ and repeat Step 5.

7. Once you are satisfied with the graph, select ⟨Main⟩ to return to the Main screen. Exit EasyData by selecting ⟨Quit⟩ from the Main screen and then selecting ⟨OK⟩.

8. After a good run, have the marker use the meter stick to measure the distance from the point where the two walkers passed one another to the midpoint of the line between the two Motion Detectors.

   Record this directly measured distance in the Data Table on the *Data Collection and Analysis* sheet. (Round this and all other data to three significant figures.) Record the directly measured time interval determined by the timer in the Data Table.

ANALYSIS

1. (Skip this step if you are using LabPro with one calculator.)

   To complete the analysis, you need to move the distance data from one calculator to another. In order to avoid overwriting data, you will first copy the data to a new location in the calculator, and then send it to the receiving calculator. Use walker one's calculator as the sending calculator, and walker two's as the receiving calculator. Be sure you know which is which!

   a. Disconnect the calculators from the data-collection interfaces, and directly connect the two calculators with a black link cable. Firmly press in the cable ends.
   b. On the sending calculator, copy the distance data to a new list name. To do this, press ⟨2nd⟩ [L6] ⟨STO▸⟩ ⟨2nd⟩ [L2].

c. You will now have the expression L6 → L2 on the home screen. Press ⌧ENTER⌧ to copy the distance data in L6 to L2.

d. Press ⌧2nd⌧ [LINK].

e. Press the number adjacent to List... to select a data list.

f. Press ⌄ until L2 is highlighted. Press ⌧ENTER⌧ to select it.

g. Press ⌲ to highlight TRANSMIT.

h. On the receiving calculator, press ⌧2nd⌧ [LINK].

i. Press ⌲ to highlight RECEIVE, and press ⌧ENTER⌧. The receiving calculator will show Waiting...

j. On the sending calculator, press ⌧ENTER⌧ to actually transmit the list.

k. The receiving calculator may display a message screen headed DuplicateName. If it does, press the number adjacent to Overwrite. The receiving calculator will show Done.

2. Perform the following to display the two distance graphs together. Use the receiving calculator if you completed Step 1.

a. Press ⌧2nd⌧ [STAT PLOT] and press ⌧ENTER⌧ to select Plot 1.

b. Change the Plot1 settings to match the screen shown here. Press ⌧ENTER⌧ to select any of the settings you change.

c. Press ⌧2nd⌧ [STAT PLOT] and press ⌄ ⌧ENTER⌧ to select Plot 2.

d. Change the Plot2 settings to match the screen shown here. Press ⌧ENTER⌧ to select any of the settings you change.
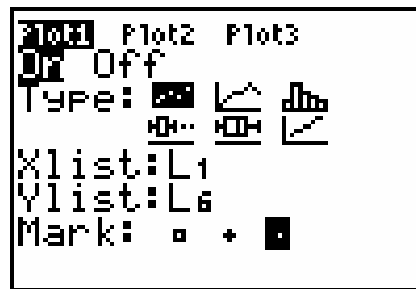
(If you are using LabPro with one calculator, use L9 instead of L2. To enter L9, press ⌧2nd⌧ [LIST], scroll to L9, and press ⌧ENTER⌧ to paste it to the stat plot screen.)

e. Press ⌧ZOOM⌧ and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.

f. Press ⌧TRACE⌧ to determine the coordinates of a point on the graph using the cursor keys.
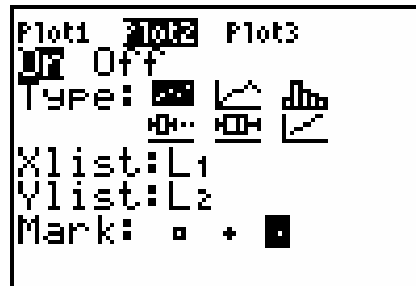
⟹ Answer Question 1 on the *Data Collection and Analysis* sheet.

3. Trace across either line to the place where the two lines cross. Record the coordinates as the Graphical Trace result in the Data Table on the *Data Collection and Analysis* sheet.

4. To create two linear models for the data, you need to record two points from each line. Ignore any short horizontal sections at the beginning or end of the lines. Press ⌄ to move from one line to the other. Trace across the graphs and record the coordinates of two well-separated points from each line. Record these values in the Data Table.

5. Find the slope of each line using the standard slope formula, and enter the slopes in the Data Table.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

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6. The point-slope form of the line equation, or $y - y_1 = m(x - x_1)$, will let you determine the y-intercept from the two points you have already recorded. Here y and x are variables, m is the slope, and x_1 and y_1 are the values of a point on the line. Use this relation to determine the y-intercept of the line fitting each walker's record from the traditional $y = mx + b$ form, and record the intercepts in the Data Table.

7. To graph the two lines, you must enter the equations in the calculator's equation editor.

    a. Press ⬭Y=.
    b. Press ⬭CLEAR to remove any existing equation.
    c. Enter the equation for the first line you determined in the Y1 field. For example, if the expression is $y = 4x + 3$, enter 4*x+3.
    d. Press ⬭ENTER to move to the Y2 field. Press ⬭CLEAR to remove any existing equation.
    e. Enter the equation for the second line in the Y2 field.
    f. Press ⬭GRAPH to see the data with the model graph superimposed.

⟹ Answer Question 2 on the *Data Collection and Analysis* sheet.

8. The calculator can determine the point of intersection of the two lines.

    a. Press ⬭2nd [CALC].
    b. Press the number adjacent to intersect to start the intersection search.
    c. You will see the graphs again, with the prompt First curve? The flashing cursor will be on the Y1 function. Press ⬭ENTER to select the line.
    d. The cursor will jump to the Y2 function. Press ⬭ENTER to select that function.
    e. The calculator will prompt Guess? to ask for your best guess of an intersection. Since there is only one intersection, the calculator will have no problem finding it. Press ⬭ENTER to have the calculator locate the intersection point.
    f. Record the coordinates of the intersection point as the Calculator Intercept Search in the Data Table.

9. Finally, you can find the intersection of the two lines algebraically. Using the slope and intercept of the walker one and walker two lines, write down two linear equations in the form $y = mx + b$. That is, you will have one equation for each line, each in terms of its own intercept and slope.

    What you have is a *system of equations*. There are two equations, each with two unknowns, x and y. Usually (but not always!), when you have two equations and two unknowns, you can solve the system to find specific values for x and y that satisfy both equations simultaneously. In this case, that solution will yield the time and location that both walkers had in common— that is, the time and location when they passed each other.

    Solve the system of equations, and write the solution as the Solve Equation System answer in the Data Table.

⟹ Answer Questions 3–5 on the *Data Collection and Analysis* sheet.

EXTENSION

Repeat the activity, but have the walkers move by starting slowly and then speeding up. The distance versus time plots will be non-linear, so that you can no longer use a linear model. What type of function might be used to model the motion?

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

	Direct Measure	Graphical Trace	Calculator Intercept Search	Solve Equation System
Intersection Time				
Intersection Position				

	x_1, y_1	x_2, y_2	Slope	Intercept
Walker One				
Walker Two				

QUESTIONS

1. How can you identify which trace is walker one and which is walker two? Identify the trace of each walker for future reference.

2. How do the model lines fit the walker data?

3. Compare the values you obtained for the intersection time by all four methods: (1) direct measurement by stopwatch and meter stick; (2) graphical intersection by tracing, (3) the calculator's intersection search, and (4) by solving the system of equations. Are the values consistent?

4. Is it possible for the walkers to move in front of the Motion Detectors so that the resulting plots would not intersect? If so, give an example; if not, explain why.

5. Could the walkers move so that their plots intersect more than once? If so, how?