## Probability Simulations

Time required
35-45 minutes

## Activity Overview

In this activity, students use commands (randlnt and randMat) to simulate probability experiments. They also graph the number of trials and corresponding probabilities to observe the Law of Large Numbers. Simulated experiments involve tossing a coin, spinning a spinner, and observing the sex of children in a family.

In an optional extension, students are challenged to create simulations in any way they find best to predict how many tosses, on average, it takes to toss two heads in a row.

## Topic: Probability Simulations \& Conjectures

- Simulate a probability experiment and calculate the experimental probability of an event.
- Replicate the simulations of a probability experiment to estimate the theoretical probability of an event.


## Teacher Preparation and Notes

- This activity is designed to be used for students studying Statistics and Probability. It can also be used as an enrichment activity in an Algebra 2 class.
- Students should be familiar with finding a simple theoretical probability (number of favorable outcomes divided by total number of outcomes) in order to make comparisons with their simulated experimental probabilities
- Information for an optional extension is provided at the end of this activity, both on the student worksheet and in the .tns file. Should you not wish students to complete the extension, you may delete the extension from the .tns file and have students disregard that portion of the student worksheet.
- Notes for using the TI-Nspire ${ }^{\text {TM }}$ Navigator ${ }^{\text {TM }}$ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9328" in the keyword search box.


## Associated Materials

- ProbabilitySimulations_Student.doc
- ProbabilitySimulations.tns
- ProbabilitySimulations_Soln.tns


## Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Modeling Probabilities (TI-Nspire technology) - 10254
- Probability Distributions (TI-Nspire technology) - 8972
- Probability of Repeated Independent Events (TI-Nspire technology) - 16060
- Compound Events (TI-Nspire technology) - 10136


## Problem 1 - Rolling Heads

On page 1.3, students are to type randInt $(\mathbf{0}, \mathbf{1})$ in the math expression box and press enter. Note that the " $I$ " in integer does not need to be capitalized. They should note what happens if they press enter repeatedly. They can use these results to tell what the command does and how it can be used to simulate tossing a coin. (It gives a random integer from 0 and 1.)

Have students decide whether they want 0 to represent heads or tails.

\section*{| 1.1 | 1.2 | 1.3 |
| :---: | :---: | :---: | <br> }

One way is to use the random integer command. In the math box below, type randlnt( 0,1 ) and press enter.
randInt $(0,1)+1$
Continue to press enter in the box. Regarding a coin toss, what could the resulls represent? What do you think the command does?

## TI-Nspire Navigator Opportunity: Screen Capture

See Note 1 at the end of this lesson.

On page 1.4, students are to type randint $(\mathbf{0}, \mathbf{1}, \mathbf{5})$ and discuss what happens and what the third number in the list does. (It gives the results for that many trials.) The randint command is also found by pressing (catalog) $\boldsymbol{R}$ and scrolling to randint.

The instructions on 1.5 explain that students are to find the results of tossing a coin $1,5,10,15,20,25$, and 30 times. On page 1.6, as students click on the arrows, the desired random integer list is generated.

| 1.2 | 1.3 |
| :--- | :--- | :--- |
| Below, type randint $(0,1,5)$ in the math box |  |
| and press enter. |  |
| randint $(0,1,5)+\{1,0,0,1,0\}$ |  |
| Explain what happened. What does the third |  |
| number in the list do? |  |
|  |  |



```
# n=15.
```

```
randInt(0,1,n)
```

randInt(0,1,n)

+ {1.,1.,0.,1.,0.,0.,1.,0.,1.,1.,1.,0.,1.,0.,1.}

```
+ {1.,1.,0.,1.,0.,0.,1.,0.,1.,1.,1.,0.,1.,0.,1.}
```


## TI-Nspire Navigator Opportunity: Live Presenter <br> See Note 2 at the end of this lesson.

For 25 and 30 trials, students will need to click in the results and use their arrow keys to see some of the results.

An option that will make it easier to view all the results is doing five trials five times and 10 trials three times.

Students will enter their results on page 1.8 and then make a scatter plot of the data.

Most plots will show that the experimental probability of flipping a head becomes closer to 0.5 as the number of trials increase. This is called the Law of Large numbers.

If time permits, students can expand their charts and graphs with even more trials. They should work together and combine results to save time.


## TI-Nspire Navigator Opportunity: Screen Capture

See Note 3 at the end of this lesson.

## Problem 2 - Spinning a "2"

Students are able to work through this problem independently. They will find experimental probabilities for spinning a 2 on a spinner with four equal sections numbered 1 through 4.
Students will click on the arrows to find the results of spinning the spinner $1,5,10,15,20,25$, and 30 times. They will then record the number of times they see the number 2.

Students should see that their results become closer to the theoretical probability of 0.25 as the number of trials increase.

If time permits, have students make more simulations to expand their charts and graphs even further.


## Problem 3 - Exactly Two Girls

Students are to type randInt(-8,8,2) and press enter five times on page 3.1 to see what happens. You can have them guess what will happen first. (It gives two random numbers integers between -8 and 8.)

Ask how this can be used to simulate observing families with two children and recording if the child is a boy or a girl. Note that there are 8 even numbers and 8 odd numbers. Disregard rows that have a 0 .

Students are to use the command to simulate observing 10 families with exactly 5 children each. If rows are disregarded because of 0 , another row should be formed

Students can use either even/odd numbers or positive/ negative numbers to distinguish between boys and girls.

On page 3.3, students are instructed to students work together to observe a larger number of five-child families and predict the theoretical probability of having exactly two girls in five births. (0.3125)

## Extension

This extension challenges students to find a way to use simulation to predict, on average, the number of times one needs to flip a coin before they get two heads in a row.

One possible simulation involves finding random integers by using randint $(\mathbf{0}, \mathbf{1}, \mathbf{1 0})$ five times and using the results as one list of 50 consecutive coin toss results. (A matrix can also be used.) For the results shown at right (1 represents heads), it takes 3 tosses to get the first pair of consecutive heads, 5 to get the next pair, two for the next, and so on.

##  <br> Below, type randint $(-8,8,2$ ) in the math box and press enter. <br> randInt $(-8,8,2)+\{-7,6\}$ <br> Explain what the command appears to do.

\section*{| 4.4 | 3.1 | 3.2 |
| :--- | :--- | :--- | :--- |
|  | *ProbabilityS_ons $\nabla$ |  |}

Use randint in the box below to simulate observing 10 families with five children. rand $\operatorname{Int}(-8,8,5)+\{-4,6,4,6,-3\}$

Use the Scratchpad to calculate the percent that have exactly two girls. What is your experimental probability of having two girls from five births?


The average of these numbers are then taken to predict that it would take, for this experiment, about 4 tosses to get a pair of heads. Note: Students predictions may vary, but the number of tosses should be less than 10.

## TI-Nspire Navigator Opportunities

## Note 1

## Problem 1, Screen Capture

A few students may think the handheld will always generate the same random number list when asked. On page 1.4, a Screen Capture is a great idea to take care of this misconception as all the displayed student screens will have similar, yet different lists of random numbers.

## Note 2

## Problem 1, Live Presenter

On page 1.7. use Live Presenter to demonstrate how to click on the arrows as well as demonstrate how the calculation is executed every time the arrows are clicked. Click back and forth between $n=5$ and $n=6$ to further illustrate this point.

## Note 3

## Problem 1, Screen Capture

On page 2.4, use Screen Capture to demonstrate that even though everyone generated different random numbers, that as $n$ became larger the graphs illustrate that students are getting closer to the theoretical value.

