EXPLORATIONS

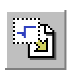
Activity 14

Discovering the Derivative of the Sine and Cosine Functions

The slope of the tangent to a curve at a point is defined to be the derivative. By calculating the derivative of a curve at many points, a new function can be obtained. By finding the equation that will fit the points, the derivative of *f(x) = sin(x)* and *g(x) = cos(x)* can be discovered.

Exploration

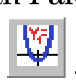
1. Open a new TI InterActive! document. Title this document **Derivatives of the Sine and Cosine Functions**. Add your name and the date to this document

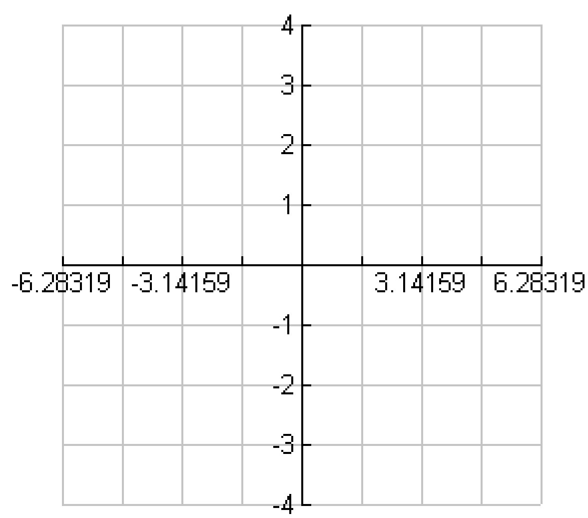
2. Select Math Box  and define f(x): = sin(x).

3. In a math box, define x1: = {1}. This will give *x* a value of 1.

4. In a math box, store **x1→L1** .

5. In a math box, calculate the numerical derivative of *f(x)* at *x*1 = 1 and store it in L2 using the syntax **nDeriv(f(x1), x1)→L2**. Close the Math Palette.

6. Select Graph . Define y1(x): = f(x). Click on the tab Stat Plots. Enter **L1** in the first field and **L2** in the second field.



7. Click on Zoom Trig . Sketch f(x) and the scatter plot L1, L2 on the provided grid. Click on Save to Document .

8. Double-click on x1: = {1} and change this list to x1: = {0, 1, 2, 3, 4, 5, 6}. Add the new slopes to your scatter plot on the same grid as question 7.

9. Double-click on x1: = {0, 1, 2, 3, 4, 5, 6} and change the list to x1: = {-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6}. Add the new slopes to your scatter plot on the same grid as question 7.

10. Double-click on x1: = {-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6} and change to x1: = seq(x, x, $^{-}2\pi$, 2π, $\pi/12$). This defines *x* to be the arithmetic sequence of numbers from $^{-}2\pi$ to 2π in steps of $\pi/12$.

    Note: π can be found under .

Analysis

1. Double-click on the graph. Define y2(x): = *your guess* for the function that would connect the data in the scatter plot . Click the checkbox to the left of y2 to select the equation. When you are satisfied with your graph, record your function.

   y2(x) = ____

2. Define y3(x):= nDeriv(f(x), x). Click in the box to the left of *y*3 to select the equation. Does *y*3 match *y*2?

   ____

3. Click on Save to Document .

4. When $f(x) = \sin(x)$, $f'(x) =$ ____

5. If $f(x) = \cos(x)$, predict $f'(x)$. ____

6. Double-click on f(x): = sin(x), and redefine f(x): = cos(x). How does this change your graph?

   ____

7. Double-click on the graph. Define y2(x): = *your guess* for the function that would connect the data in the scatter plot. When you are satisfied with your graph, click Save to Document. Record your function.

   y2(x) = ____

8. When $f(x) = \cos(x)$, $f'(x) =$ ____

9. Save this Document as **derivatives.tii**. Print a copy of this document.

Additional Exercises

Use the previous steps to discover the derivative of each of the following functions.

1. Define g(x): = sin(2x).
2. What is your guess for the derivative?

   $y2(x)$ = ________________________________
3. When $g(x) = \sin(2x)$, $g'(x)$ = ________________________________
4. Define g(x): = sin(3x).
5. What is your guess for the derivative?

   $y2(x)$ = ________________________________
6. When $g(x)$: = $\sin(3x)$, $g'(x)$ = ________________________________
7. Define g(x): = sin(5x).
8. What is your guess for the derivative?

   $y2(x)$ = ________________________________
9. When $g(x) = \sin(5x)$, $g'(x)$ = ________________________________
10. Define g(x): = cos(2x).
11. What is your guess for the derivative?

    $y2(x)$ = ________________________________
12. When $g(x) = \cos(2x)$, $g'(x)$ = ________________________________
13. Define g(x): = cos(3x).
14. What is your guess for the derivative?

    $y2(x)$ = ________________________________
15. When $g(x) = \cos(3x)$, $g'(x)$ = ________________________________
16. Define g(x): = cos(5x).
17. What is your guess for the derivative?

    $y2(x)$ = ________________________________

18. When $g(x) = \cos(5x)$, $g'(x) =$ ____________________

19. If $f(x) = \sin(n * x)$, $f'(x) =$ ____________________

20. If $f(x) = \cos(n * x)$, $f'(x) =$ ____________________