NUMB3RS Activity: Risky Business
Episode: "Waste Not"

Topic: odds ratios and risk ratios **Grade Level:** 9 - 12
Objective: To appreciate different ways of expressing probability data, including the use of natural logarithms, depending on the application
Materials: A scientific calculator
Time: 10 - 15 minutes

Introduction

Agent Reeves believes that the children in the area of a collapsed playground seem to have an unusually high incidence of rashes, tumors, and cancer. In Charlie's office, we get a brief glimpse of a spreadsheet on Amita's computer. Millie enters the room and asks for an explanation of Charlie and Amita's use of the school's computer time. Amita replies, "We're working with odds ratios to conduct an analysis of childhood cancers, birth defects, and neurological deficits related to a potentially toxic material."

While the audience does not get an explanation of the spreadsheet, it actually consists of odds, log odds, and log-odds ratios. The purpose of this activity is for students to understand the meaning and appropriate use of each of these measures, in addition to another measure called a risk ratio.

Discuss with Students

This activity is best implemented as a teacher-directed exploration. A detailed introduction to odds ratios is available in the *NUMB3RS* activity "Odds-on Favorite" for the season 3 episode "Longshot." The "Odds-on Favorite" activity can be downloaded by going to **http://education.ti.com/exchange** and searching for "7589." For the purpose of this activity, students first need to understand how odds differ from probability. The probability of an event is the ratio of the number of *favorable* outcomes (the ones being counted, whether they are actually good or not) to the number of *possible* outcomes. The odds in favor of an event are the ratio of the number of *favorable* outcomes to the number of *unfavorable* outcomes. Finally, the *odds ratio* is the ratio of the odds of an event occurring in one group to the odds of the same event occurring in another group.

Odds ratios and risk ratios are just different ways of measuring the same thing. Since risk ratios use probabilities, the values are less extreme than odds ratios. For both measures, equal likelihood between groups is represented by the ratio 1:1, or 1.00. In either case, the range for the ratios is between 0 and infinity, with 1 at the "center." At a much more sophisticated level, the natural logarithm of the odds ratio, called log-odds, is used so that the center (equal likelihood) of the distribution of all possible values is 0, and the distribution then contains both positive and negative values.

Student Page Answers:

1. *AD/BC* **2.** *The symptoms are equally likely in both groups.* **3.** *409,249/19,278 ≈ 21.23* **4.** *The odds are 21.23 times as great for cancer in children who use the playground.*

5.

	Cancer	No Cancer	Total
Uses Playground	0.1173	0.8827	1.00
Does Not Use Playground	0.0062	0.9938	1.00

6. $\frac{61}{61+459} \div \frac{42}{42+6709} \approx 18.8558$ **7.** *The probability is almost 19 times as great for cancer in children who use the playground.* **8.** *ln(409249/19278) ≈ 3.06*

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Name: _____ Date: _____

NUMB3RS Activity: Risky Business

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This activity is meant to help in understanding odds ratios, risk ratios (also called relative risk), and log-odds ratios. The data used here are not based on actual data; they are meant for illustrative purposes only.

Suppose that a study is conducted to verify the "discovery" by Agent Reeves. The results could look like this:

	Cancer	**No Cancer**	**Total**
Uses Playground	61	459	520
Does Not Use Playground	42	6709	6751
Total	103	7168	7271

Generally, the results of a study can be summarized in a table like this one:

	Symptoms	**No Symptoms**	**Row Total**
Experimental Group	A	B	A + B
Control Group	C	D	C + D
Column total	A + C	B + D	A + B + C + D

1. The odds ratio for the two groups is the ratio of the two odds: Odds Ratio = $\frac{A/B}{C/D}$.

Simplify this expression.

2. What does an odds ratio of 1 mean?

3. What is the odds ratio for the cancer study?

4. What is the interpretation of this result?

In medical studies, the odds ratio is the preferred measure for studies designed to measure results based on data of exposure that has already happened. This is called a *retrospective* study.

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Study data can also be summarized using probabilities, like this:

	Symptoms	No Symptoms	Row Total
Experimental Group	A/(A + B)	B/(A + B)	1.00
Control Group	C/(C + D)	D/(C + D)	1.00

The *risk ratio* for symptoms, or relative risk, is the ratio of the probabilities:
Risk Ratio = $\frac{A/(A+B)}{C/(C+D)}$. As a comparison of probabilities, it is more direct than an odds ratio and less sensitive to variation.

5. Complete the risk ratio table for the cancer study:

	Cancer	No Cancer	Total
Uses Playground			1.00
Does Not Use Playground			1.00

6. Based on these data, what is the risk ratio for cancer?

7. What is the interpretation of this result?

In medical studies, the risk ratio is the preferred measure for studies designed to predict results, as in a randomized clinical trial of a new medicine. This is called a *prospective* study.

Finally, as in Charlie and Amita's analysis, there is the *log-odds ratio*. The logarithm of a number y with respect to a base b is the exponent to which we have to raise b to obtain y. In algebraic notation: $b^x = y \leftrightarrow x = \log_b y$. Log-odds ratios use *natural logarithms* (base e). The notation is $\log_e y = \ln y$, or $e^x = y$. Note that if $x = 0$, then $y = 1$, and vice versa. This is the basis for using log-odds ratios.

The log-odds ratio is the natural logarithm of the odds ratio:
Log-Odds Ratio = $\ln\left(\frac{A/B}{C/D}\right) = \ln\left(\frac{AD}{BC}\right)$

Looking back to Question 2, an odds ratio of 1 produces a log-odds ratio of 0. The properties of log-odds ratios include that they are centered at 0 (equal likelihood) and can be positive (more likelihood in the experimental group) or negative (more likelihood in the control group).

8. Compute the log-odds ratio for the cancer study.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Odds, odds ratios, and risk ratios are relatively straightforward to compute and interpret. Log-odds ratios, while still simple to compute, are actually rather sophisticated in their application and mathematical interpretation. Statistical analysis and tests at this level are generally even beyond the scope of a high school AP® statistics course. However, students interested should not hesitate to explore a topic such as this.

Related Topic

Another name for the natural logarithm of odds expressed as a decimal is the *logit* function. This topic was introduced in the "Extensions" of an earlier *NUMB3RS* activity, listed in the "Additional Resources" below. The mathematical progression of sophistication goes from odds, to odds ratios, to the logit function, to logistic regression. If interested, the UCLA site below explains this progression, with the additional context of the use of computer software.

Additional Resources

The *NUMB3RS* activity "Odds-On Favorite" on odds ratios that can serve as an introduction to this activity, with the logit function in the "Extensions." To download this activity, go to **http://education.ti.com/exchange** and search for "7589."

A detailed mathematical discussion of the differences between odds ratios and log-odds ratio can be found at: **http://planetmath.org/encyclopedia/LogOddsRatio.html**

For the mathematical development that goes from odds to logistic regression, see: **http://www.ats.ucla.edu/stat/Stata/faq/oratio.htm**

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