

Extrema Using Derivatives

ID: 9414

Time required
45 minutes

Activity Overview

In this activity, students will learn how to find and identify extrema using first and second derivatives. They will use **Graph Trace**, **Analyze Graph**, **solve**, **derivative**, **fMin**, and **fMax** to verify computed answers and find the critical points. Then they will inspect a graph and determine confirm the extrema.

Topic: Derivatives of Higher Order

- Identify the conditions on $f'(x)$ and $f''(x)$ for a local maximum, minimum, and point of inflection.
- Find when $f'(x) = 0$ or undefined to find the critical points of $f(x)$.
- Evaluate the critical points of $f(x)$ to classify them as extrema or points of inflection.
- Use **Graph Trace** and **Analyze Graph** to approximate local extrema and inflection points.
- Use **fMin**, **fMax**, **solve**, and **derivative** to verify the manual computation of local extrema. This is a CAS active activity.

Teacher Preparation and Notes

- This investigation uses **fMin** and **fMax** to verify answers. Students will have to restrict the domain to get the desired result. They will use the vertical bar, “such that,” that can be found by pressing $\boxed{\text{ctrl}} \boxed{=}$. Students will also have to use both the math palette and the Greek letter palette. Students should be able to find the Calculus and Algebra menu options like **Derivative** and **solve** commands and also be able to graph functions and use the **Graph Trace** and **Analyze Graph** tool.
- The handheld should be set in radian mode as its General Setting and Graph Setting for when they evaluate and graph trigonometric functions.
- The TI-Nspire CAS or TI-Nspire CX CAS is required for many pages.
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter for “9414” in the keyword search box.**

Associated Materials

- *Extrema_Student.doc*
- *Extrema.tns*
- *Extrema_Soln.tns*

Suggested Related Activities

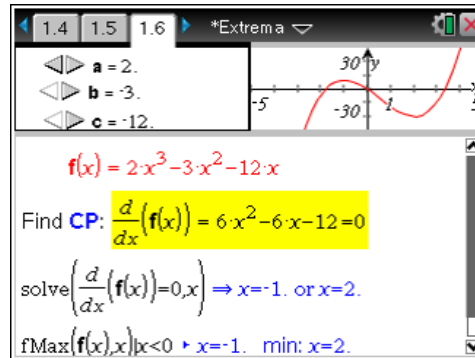
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Application of Critical Points (TI-Nspire technology) — 13449*
- *First Derivative Test (TI-Nspire technology) — 16073*
- *Critical Points (TI-Nspire technology) — 16072*

Problem 1 – The extrema of $y = 2x^3 - 3x^2 - 12x$

Students will first graph the function and answer questions about a function based on their observations. In this calculus, visual inspection is not enough to identify extrema – local maxima and minima. Students need to use calculus.

Critical points (CP) can be found by taking the first derivative and setting it equal to zero. Differentiate again to help find the point of inflection, where the graph changes concavity. This is explored on pages 1.6–1.7.



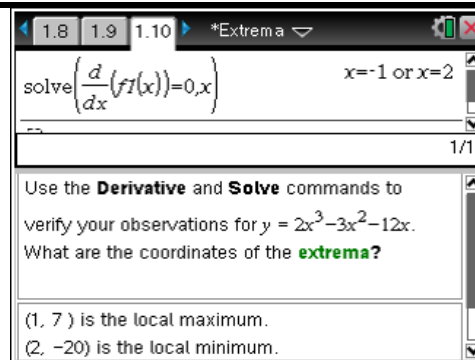
TI-Nspire Navigator Opportunity: Quick Poll and Screen Capture

See Note 1 at the end of this lesson.

On page 1.10, students should find the first derivative of the function. (**MENU > Calculus > Derivative**). They should see that there is a common factor of 6. This should be factored.

$$6x^2 - 6x - 12 = 0 \rightarrow 6(x - 2)(x + 1) = 0$$

Then they can use the **solve** command (**MENU > Algebra > Solve**) to find the two critical points ($x = 2$ and $x = -1$).



TI-Nspire Navigator Opportunity: Live Presenter

See Note 2 at the end of this lesson.

The second derivative can be found by taking the derivative of $6x^2 - 6x - 12$ or use nested derivatives $\frac{d}{dx}\left(\frac{d}{dx}2x^3 - 3x^2 - 12x\right)$. The result is $12x - 6$. When students evaluate the critical points, they will get the following:

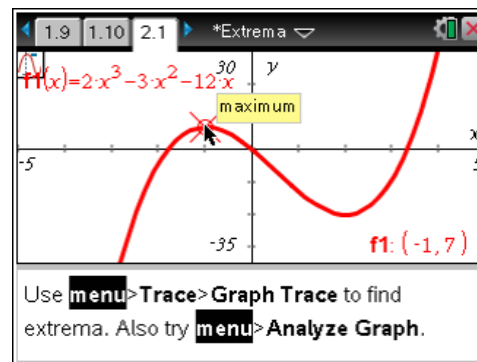
$$12(2) - 6 = 18 > 0 \text{ so } x = 2 \text{ is a minimum}$$

$$12(-1) - 6 = -18 < 0 \text{ so } x = -1 \text{ is a maximum}$$

Solving $12x - 6 = 0$, $x = 0.5$. So $x = 0.5$ is a point of inflection.

Student worksheet solutions

1. One maximum and one minimum
2. The point of inflection is the point where the graph changes concavity.
3. (-1,7) is the local maximum
(2, -20) is the local minimum
4. $y' = 12x - 6 = 0 \quad x = \frac{1}{2}$
5. It is concave up if the second derivative is positive;
concave down if negative.
6. Point of inflection is (0.5, -6.5)
7. Yes
8. Yes



TI-Nspire Navigator Opportunity: Live Presenter

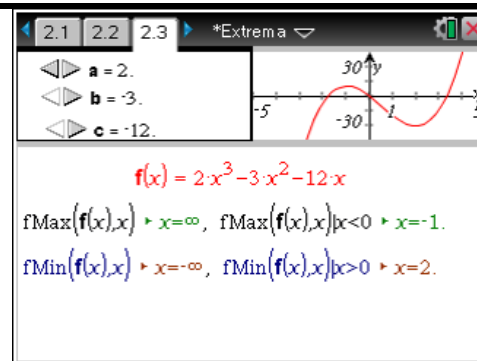
See Note 3 at the end of this lesson.

Problem 2 – fMin and fMax

Students should then use the **fMin** and **fMax** commands from the Calculus menu to verify the maximum and minimum.

If students use **fMin** on the function without any qualifiers on x , they will get $-\infty$. So since we know that the minimum is greater than 0, they need to append $|x>0$ to the end of the command.

If students use **fMax** on the function without any qualifiers on x , they will get ∞ . So since we know that the maximum is less than 0, they need to append $|x<0$ to the end of the command.

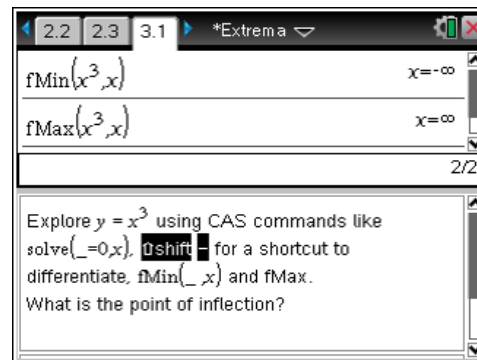


Problem 3 – The Extrema of $y = x^3$

On page 3.2, students should notice that there are no extrema to be seen on the graph. However, the function changes concavity at $x = 0$. Therefore, there is a point of inflection but no extrema.

The first derivative is $3x^2$.

$3x^2 = 0$ means that $x = 0$ which is a point of inflection.



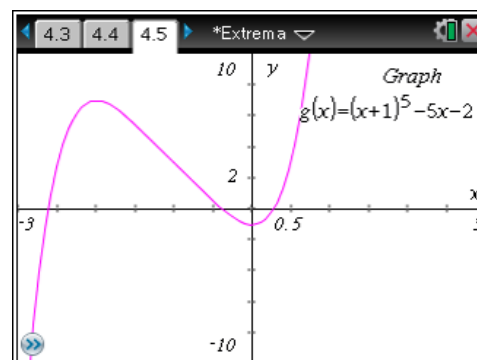
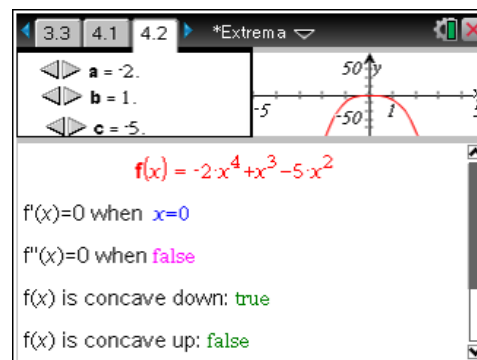
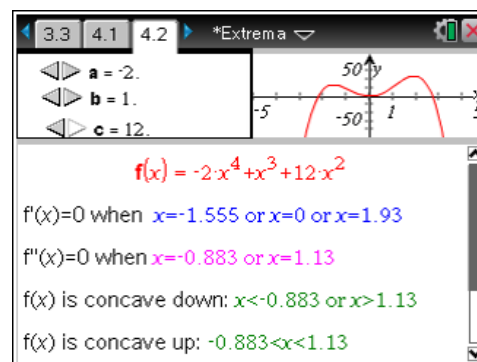
Problem 4 – Extrema for other functions

The first two problems explored cubic functions, on 4.2 students will investigate extrema and concavity for quartic functions. They can compare their observation with cubic functions by letting the coefficient a equal zero.

If the math box to the right of $f''(x)$ displays a result of false, it indicates that the function does not change concavity and there is no point of inflection. If the math box in the last two lines display a result of true, like for concave down, then the function is concave down from $-\infty$ to $+\infty$.

Students are to graph $g(x) = (x+1)^5 - 5x - 2$ on page 4.5. They can use the graph and the **fMin** and **fMax** commands to confirm the extrema they found using calculus.

- minimum: $x = 0$; maximum: $x = -2$
- maximum value: $g(-2)=7$; minimum value: $g(0)=-1$
- inflection point: $(-1, 3)$

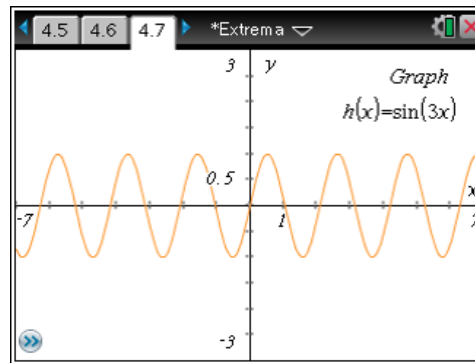


TI-Nspire Navigator Opportunity: Screen Capture

See Note 4 at the end of this lesson.

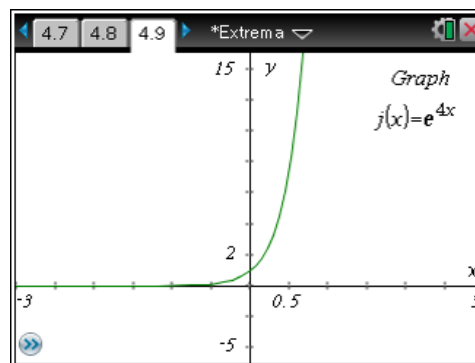
Students graph $h(x) = \sin(3x)$ on page 4.7. There will be multiple minima, maxima, and points of inflection.

- $\sin(3x)$ is positive and concave down on $\left(0, \frac{\pi}{3}\right)$
- $\sin(3x)$ is negative and concave up on $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$
- Minima: $x = \frac{\pi}{2} \pm \frac{2n\pi}{3}$, where n is any integer
- Maxima: $x = \frac{\pi}{6} \pm \frac{2n\pi}{3}$, where n is any integer
- Points of inflection: $x = \frac{n\pi}{3}$, where n is any integer



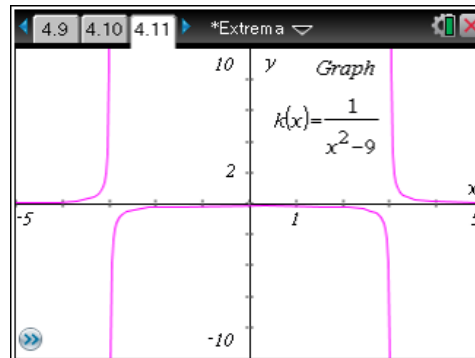
Students will graph $j(x) = e^{4x}$ on page 4.9.

They should see that there are no extrema. There are no critical points, the function is always concave up, and there are no points of inflection.



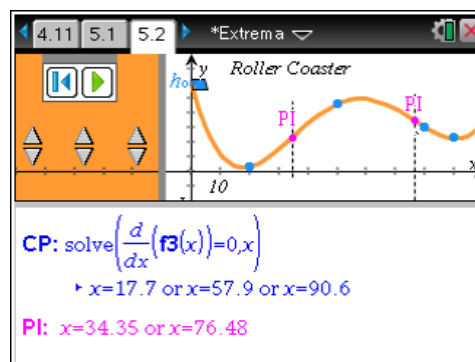
Students should graph $k(x) = \frac{1}{x^2 - 9}$ on page 4.11.

They will find that there is no minimum, but there is a maximum at $x = 0$. The function changes concavity at $x = \pm 3$. Since those values are the vertical asymptotes, there are no points of inflection.



Problem 5 – Evaluate endpoints and critical points

Many functions are defined on a limited domain. The roller coaster function is defined from 0 to 100 meters. Endpoints and critical points of a function should be evaluated. On page 5.2 students will change the profile of the roller coaster by using the arrows or by moving the points. They will record the value of the endpoints and critical points. How does this information help you determine the extrema? *By evaluating the function at each value the maximum and minimum can be determined.*

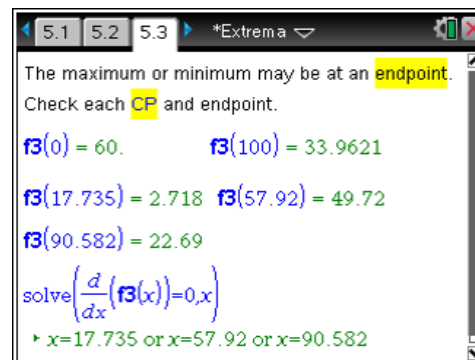


$f_3(0)$ = Answers will vary. See example to the right.

$f_3(100)$ = Answers will vary.

Answers for the endpoints and critical points will vary based on how the students changed the roller coaster function $f_3(x)$. They should evaluate f_3 for each critical point with Math Boxes on page 5.3. The endpoints have been done for them.

TIP: **ctrl M** inserts a Math Box. Press **ctrl enter** to get an decimal approximation.



TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 5 at the end of this lesson.

TI-Nspire Navigator Opportunity: *Collect*

See Note 6 at the end of this lesson.

TI-Nspire Navigator Opportunities**Note 1****Problem 1, Quick Poll, Screen Capture, and Live Presenter**

Page 1.3 can be used as a **Quick Poll**.

Page 1.6 and 1.7 – use **Class Capture** in gallery view. Select two students screen to show side by side with one student on 1.6 and the other on 1.7 to see the entire process. Also compare two graphs of page 1.6 to consider the effect of the coefficients.

Note 2**Problem 1, Live Presenter**

Page 1.10 – Make a student use **Live Presenter** to demonstrate their process.

Note 3**Problem 1, Live Presenter**

Page 2.1 – Have a different student be the **Live Presenter** to demonstrate using **Analyze Graph** to find the maximum, minimum, and point of inflection.

Note 4**Problem 4, Screen Capture**

Page 4.2 – Use **Class Capture** to compare the results when $a = 0$, and when the function is a quartic. Can any student demonstrate what their results when a is positive and there changes concavity.

Page 4.4 and 4.5 – Use **Class Capture** to make sure that students first use calculus to examine $g(x)$. Then use **Live Presenter** to have a student demonstrate that the text $g(x)$ can be grabbed and moved to an axis for a convenient way to graph it. Have them compare their results on page 4.4 with their graph on page 4.5. This can similarly be done with functions $h(x)$, $j(x)$, and $k(x)$.

Note 5**Problem 5, Live Presenter**

Page 5.2 and 5.3 – Have a student be the **Live Presenter** to show how the function can be varied with the arrows of the spinners and the points on the graph. Have them demonstrate how to evaluate $f_3(x)$ at the critical points. If instead of pressing **enter** to evaluate the Math Box, they can press **ctrl enter** to get a decimal approximation.

Note 6

After students have finished the activity, **Collect** the file using the Class workspace on the TI-Nspire Navigator Teacher Software. Save it to the **Portfolio** and review the key points of the lesson in the **Review** workspace.