Exploring Space Through MATH
Applications in Geometry

EDUCATOR EDITION

An Astronaut in Motion

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will

- apply translations in the coordinate plane; and
- apply reflections over the *y*-axis in the coordinate plane.

Prerequisites

Students should have prior knowledge of the coordinate plane and basic navigation of the TI-Nspire™ handheld.

Background

This problem is part of a series that applies mathematical principles in NASA's human spaceflight.

Maneuvering around or performing a task while in the reduced microgravity environment of space can be a difficult feat. Doing these things while wearing a bulky space suit may present additional and unique challenges. To better understand how astronauts function in their suits and their environment, researchers at NASA Johnson Space Center's Anthropometry and Biomechanics Facility (ABF) are studying motor control and evaluating human physical measurement, variation, and movement. The results of these studies are helping astronauts to perform tasks more efficiently in space and complete mission goals with optimal performance.

The ABF is one of few facilities in the world that has gathered both suited and unsuited human strength data relevant to Earth, lunar, and Martian gravitational environments. It is uniquely equipped to conduct ground-based biomechanics and ergonomics research studies, including human motion research – which deals with potential movement issues astronauts may encounter while living, working, and exploring in space.

For example, researchers may study how crews perform work procedures while wearing space suits to learn if the suits allow for proper movement while performing tasks related to a spacewalk. Astronauts' movements and their interactions with equipment within a space vehicle might be tested, or even simulated. The data collected and analyzed would be used to evaluate human performance issues. Ultimately, the research conducted at

Key Concepts
Transformations on the coordinate plane

Problem Duration
65 minutes

Technology
Computer with projector, TI-Nspire™ handheld

Materials
- *An Astronaut in Motion* Student Edition
- VICON video
- Graph paper

Skills
Translations, reflections

NCTM Standards
- Geometry
- Problem Solving
- Communication
- Connections
- Representation

the ABF supports projects for evaluating space suit and human performance data for future exploration missions. This research is used to improve crew living and working conditions in order to enhance productivity and operational efficiency.

Figure 1: Surface Airlock Hatch Opening Evaluation Analysis

Figure 2: Advanced Crew Escape Suit (ACES) Seated Motion Analysis

To study human movement, ABF researchers use motion capture technology. One system is a camera-based or "optical" system called VICON®. Using VICON, a series of cameras track a set of retro-reflective markers attached to a subject. Light-emitting diodes (LEDs) are placed around the perimeter of each camera's lens that produce light, which is reflected off the markers and back into the camera. Each camera can determine where a marker is in its own view, and the cameras are adjusted (or calibrated) in the VICON software, so that cameras know their positions relative to each other. The position of the camera system relative to the floor is also calibrated. This allows a marker to be tracked in three-dimensional (3-D) space whenever it is in view of at least two cameras.

By using the VICON motion capture technology, a segment of the body (e.g. the lower arm) can be tracked in 3-D as it moves. This data is used to determine the range of joint motion required to complete a task. The joint motion data can then be applied to an avatar (a computer model of the astronaut) to identify potential issues that could occur when an astronaut performs a specific task.

The avatar of the astronaut can also be placed into a model of a space vehicle to see if an astronaut would have difficulty moving naturally within the confined space, or to detect potential collision issues. This same avatar can also be scaled up and down in size, to get an idea of what problems a smaller or larger-sized astronaut could have while doing the same task. Conducting ground-based research with this technology is helping NASA designers and engineers develop efficient tools and equipment, non-restrictive space suits, and easily accessible controls on space vehicles.

NCTM Principles and Standards

Geometry

- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices
- Use various representations to help understand the effects of simple transformations and their compositions
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Problem Solving

- Solve problems that arise in mathematics and in other contexts

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Use the language of mathematics to express mathematical ideas precisely

Connections

- Recognize and apply mathematics in contexts outside of mathematics

Representation

- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

The questions in this activity are embedded in the TI-Nspire file: *Astronaut_Motion.tns*. Some screenshots have been provided in the solution key to show what the students will see on their handhelds.

1 – Engage (10 minutes)

- With students in small groups of three to four, ask them to review and discuss the main points of the Background section for several minutes to be sure that they understand the material. Circulate to help facilitate discussion in small groups. Ask if any group needs clarification.
- Play the video (4.5 min.), "VICON: Motion Capture and Motion Builder Demo". Access the video by following this link, and scrolling down the page slightly. http://www.vicon.com/applications/animation.html
- Encourage student discussion of the Background and video. Some suggestions for teacher-led questions are:
    - *What are the main components of the system?*
    - *On what are the actor's movements being displayed?*
    - *What are some other possible applications for the use of this software for NASA?*

2 – Explore (10 minutes)

- Allow students to remain in their groups.
- Distribute the TI-Nspire file: *Astronaut_Motion.tns* to the students' handhelds.
- Have students read the problem setup and directions, and then answer the questions on pages 1.2-3.6.

3 – Explain (15 minutes)

- Allow students to remain in their groups to read directions and answer questions on pages 4.1-4.5.
- Call on students to give their answers and discuss.

4 – Extend (15 minutes)

- Still in groups, have students read the directions and answer the questions on pages 5.1-5.8.
- Encourage student discussion and ask if there are any questions.

5 – Evaluate (15 minutes)

- Have students work independently to read the directions and answer questions on page 6.1-6.6.
- Distribute a piece of graph paper to students to complete the question on page 6.7.This may be done *in class or assigned* as homework.

An Astronaut in Motion

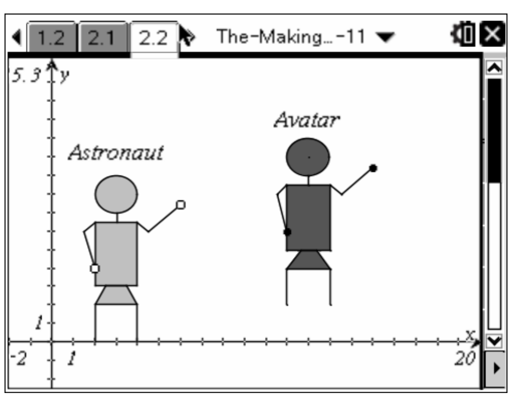
Solution Key

Problem

Students are given the following problem information within the TI-Nspire document, Astronaut_Motion.tns, with the questions embedded within the document.

The VICON technology used by the ABF tracks movement in three-dimensions using coordinates (*x, y, z*). Using these coordinates, avatars can be translated to other areas in a coordinate plane. The motion of the avatars can be mimicked and placed in various situations. We will study this technique in two-dimensions using coordinates (*x, y*).

On TI-Nspire page 2.2, you will find a two-dimensional figure representing an astronaut. The ranges of motion for the astronaut's arms have been captured. To move the astronaut's arm, grab the open point and move it around.

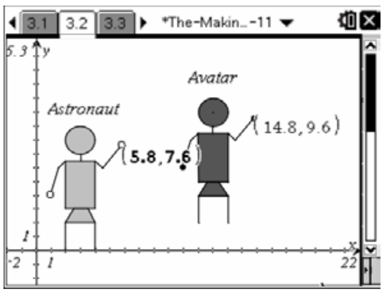


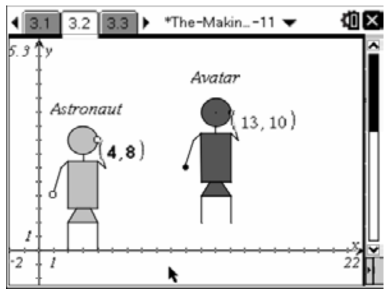
2.3 What is moving besides the astronaut's arm?

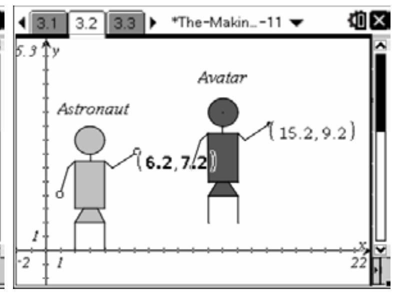
The corresponding avatar's arm is also moving. The movements of the avatar are mimicking the movements of the astronaut.

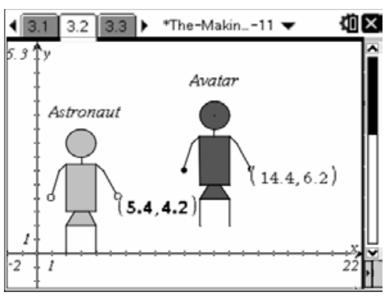
3.1 On page 3.2, the same two-dimensional figures are shown with the coordinates denoting the location of each right arm movement. Move the astronaut's arm again by grabbing the open point. Record five different positions of the astronaut's right arm along with the corresponding position of the avatar's right arm on page 3.3.

Positions students choose will vary.









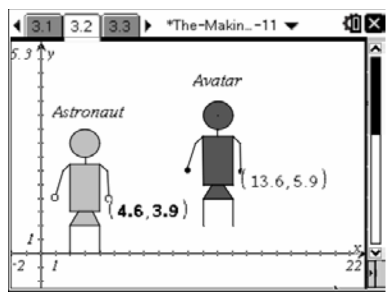


Table 1: Example of students findings on TI-Nspire page 3.3

Position #	Astronaut's Coordinates (pre-image)	Avatar's Coordinates (image)
1	**(5.8, 7.6)**	**(14.8, 9.6)**
2	**(4, 8)**	**(13, 10)**
3	**(6.2, 4.8)**	**(15.2, 6.8)**
4	**(5.4, 4.1)**	**(14.4, 6.1)**
5	**(4.6, 3.9)**	**(13.6, 5.9)**

3.4 What is the relationship between the astronaut's coordinates and the avatar's coordinates?

The avatar's x-coordinate is nine units larger than the astronaut's x-coordinate and the avatar's y-coordinate is two units larger than the astronaut's y-coordinate. The translation can also be written as $T_{9,2}$

3.5 What type of transformation could be used to describe the relationship between the two?

The avatar is a translation of the astronaut.

3.6 If the astronaut's position is (-4, -2), predict the avatar's position (*x, y*) using the translation used on page 3.3. Explain.

The avatar's position would be (5, 0). The avatar's position is found by adding nine units to the astronaut's x-coordinate and adding two units to the astronaut's y-coordinate.

Page 4.2 shows another two-dimensional figure representing an astronaut. This time, the astronaut is in a space suit. The space suit limits the range of motion of the arms. Move the right arm by grabbing the open circle.

4.3 Why does a space suit limit the range of motion in this way?

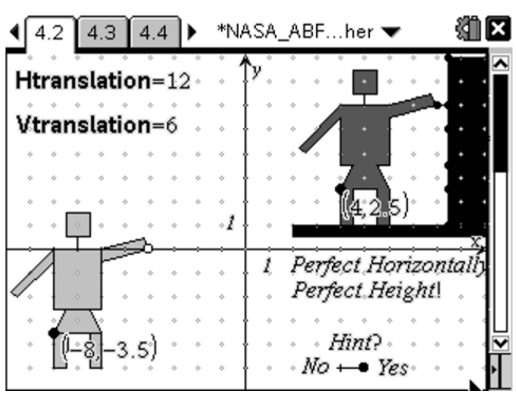
Student answers may vary. The space suit has various layers to protect the astronaut and it is pressurized, making it difficult for the astronaut to have a normal range of motion.

4.4 Edit the values for both the horizontal translation **(Htranslation)** and the vertical translation **(Vtranslation)** on page 4.2 in order to determine what translation is needed to move the astronaut's avatar to the platform, so that the avatar's arm can reach the button on the control panel. Record your answers here.

HTRANSLATION = 12, VTRANSLATION = 6. The translation can also be written as $T_{12,6}$.

4.5 On page 4.2, move the hint slider to the yes position. The hint slider shows a point on the astronaut and the corresponding point on the avatar. How do these values help you to predict/verify the translation values for the avatar?

The hint gives a point on the astronaut (-8, -3.5) and a target point for the avatar's corresponding location of (4, 2.5). In order to reach the targeted position, the astronaut's x-coordinate must be increased by twelve units and the y-coordinate must be increased by six units.



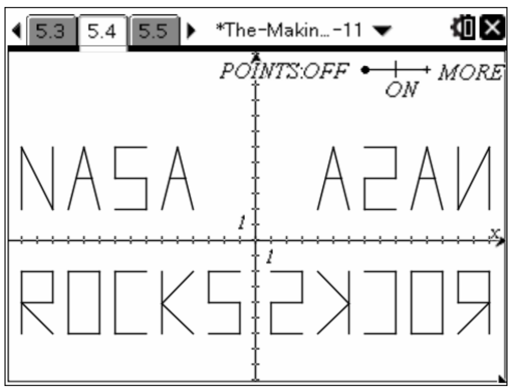
Now let's look at another transformation that involves the limited motion of an astronaut. An astronaut's space suit not only limits the range of motion for the arm, but it also limits the movement of the astronaut's head. Restricted head motion limits the astronaut's ability to read instructions and labels that are placed on the front of the space suit. To assist the astronaut, a mirror (sewn into the astronaut's glove) is used to read the labels. But the labels must be written in such a way that the mirror reflects the instructions in a readable direction.

5.2 How do you think the word "NASA" needs to be written so the mirror reflects the word properly?

It should be written as a horizontal reflection or $R_{y\text{-}axis}$.

5.3 On page 5.4, the words "NASA ROCKS" have been written on the left side of the screen and reflected over the *y*-axis to the right side of the screen. Will the reflection (the words on the right side of the screen) read properly using a mirror?

Yes.



Note: If a student isn't convinced that the words on the right side of the screen are similar to the astronaut's labels, have him place the TI-Nspire handheld in front of a mirror with the left side of the screen covered. He should be able to read "NASA ROCKS" clearly in the mirror.

5.5 On page 5.4, turn on the coordinates of a few of the points by sliding the "Points" switch to ON and complete the table on the next page.

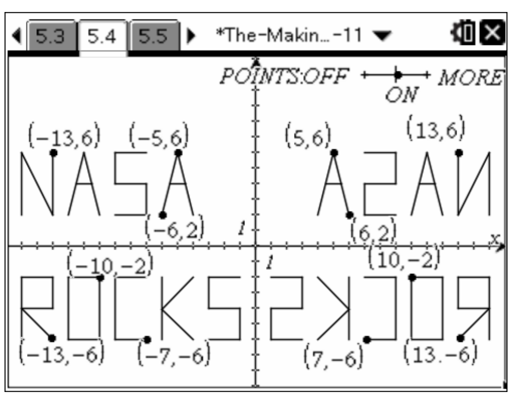


Table 2: Student results on TI-Nspire page 5.6

Coordinates of a point (pre-image)	Coordinates of the point reflected over the *y*-axis (image)
(-13, 6)	**(13, 6)**
(-5, 6)	**(5, 6)**
(-6, 2)	(6, 2)
(-10, -2)	**(10, -2)**
(-13, -6)	**(13, -6)**
(-7, -6)	(7, -6)

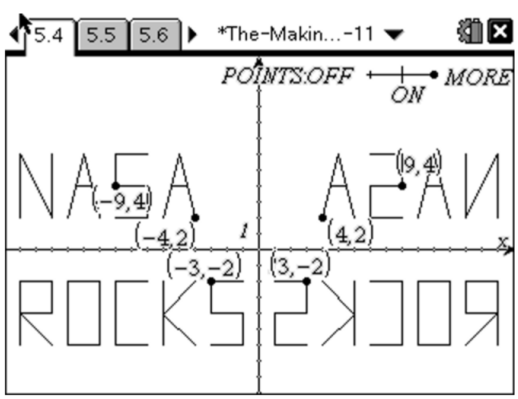
5.7 Using the table on page 5.6, explain the relationship between the coordinates of a point and its reflection over the *y*-axis.

When a point is reflected over the y-axis, the x-coordinate of the reflected point is the opposite sign and the y-coordinate does not change.

5.8 Predict the coordinates of the points given in the table if they are reflected over the *y*-axis.

Coordinates of a point (pre-image)	Coordinates of the point reflected over the *y*-axis (image)
(-9, 4)	**(9, 4)**
(-4, 2)	(4, 2)
(-3, -2)	(3, -2)

Check your answers in the table by moving the “Points” switch to MORE on page 5.4.

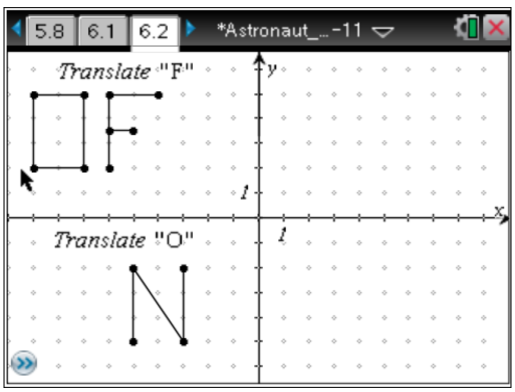


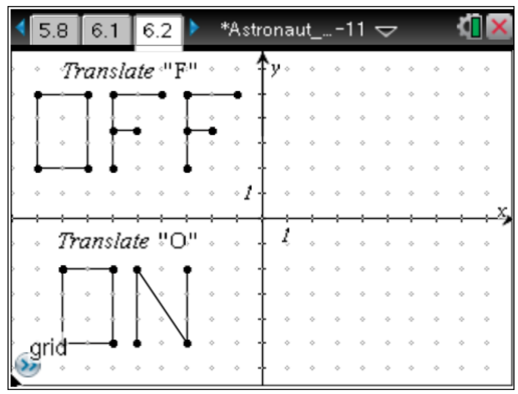
Directions: Complete the following questions independently.

On TI-Nspire page 6.2, you will find partial words, ON and OFF. Use the TI-Nspire transformations tool to complete the words on the left of the screen by translating the “F” and “O”.

To translate an object:

- Press **menu** and select **B: Transformation > 3: Translation**. Select **one** segment by clicking on it once the selector changes to a hand. It will flash when it is selected.
- Select one endpoint of the segment in the same manner.
- Translate the image to its new location by dragging your finger across the touchpad and then clicking once it is in the right location.
- Repeat these steps for each segment.





6.3 Describe the translation(s) of the given letter “F” needed to complete the word OFF. Explain how the coordinates changed from the given “F” to the translated “F”.

Right three units. The x-coordinate increased by three units. The y-coordinate remained the same. The translation can also be written as $T_{3,0}$.

6.4 Describe the translation(s) of the given letter “O” needed to complete the word ON. Explain how the coordinates changed from the given “O” to the translated “O”.

Down seven units and right one unit. The x-coordinate increased by one unit. The y-coordinate decreased by seven units. The translation can also be written $T_{1,-7}$.

Figure 3: Image of an astronaut in a space suit

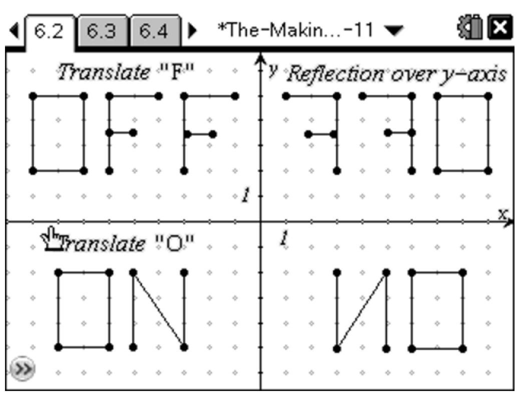
6.5 Use the created image on page 6.2 to design a patch for an astronaut's space suit with the words "ON/OFF". In order for the astronaut to read the patch while inside the suit, what transformation will be used to finish the design? Predict how the coordinates will change from the given "ON/OFF" to the "ON/OFF" patch design. Refer to Figure 3 which shows some patches on a space suit.

The patch will be a reflection of the words across the y-axis. The x-coordinates for the patch will be the opposite of the original and the y-coordinates will not change.

On page 6.2, use the TI-Nspire transformations tool to make the words readable by an astronaut in a space suit and verify your prediction.

To reflect an object:

- Press **menu** and select **B: Transformation > 2: Reflection**.
- Select the line of reflection (*y*-axis) by clicking on it. It will flash when selected.
- Select the segment to be reflected.
- Repeat these steps for each segment.



Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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