The Art Gallery Problem

By Vince Geiger

Time required
45 minutes

ACTIVITIES EXCHANGE **ID: 8300**

Activity Overview

The question of where patrons should stand to enjoy the best view of a painting is one that art gallery curators must consider on a regular basis. For this activity, we will consider a painting that is 2 meters in height that is placed 1 meter above the average person's eye level. At what distance from the wall should the average person stand in order to get the best view?

Concepts

- *Algebraic and geometric modeling*
- *Data representation and interpretation*
- *Angle measurement and inverse trigonometric functions*
- *Optimization using numeric, functional, and differential calculus based approaches*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to measurement of distances and angles and inverse trigonometric functions. Care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing those who may have missed aspects of earlier work the opportunity to build a new and deeper understanding.

- *At the Calculus level, this activity can serve to consolidate earlier work on trigonometric functions. It offers a suitable introduction to exploring trigonometric data, model fitting using inverse trigonometric functions, and interpreting graphs.*
- *The screenshots on pages 68 and 69 demonstrate expected student results. Refer to the screenshots on page 71 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file, go to education.ti.com/exchange and enter "8300" in the quick search box.***

Classroom Management

- *This activity is intended to be a **cooperative problem-solving experience** with students in **small groups**. You should seat your students in pairs so they can work cooperatively on their handhelds. The following pages, in conjunction with the .tns file, may be used to present the material to the class and encourage discussion. Students will engage in the task using their handhelds, although the majority of the ideas and concepts are only presented in this document; be sure to cover all the material necessary for students' total comprehension.*
- *Answers for questions posed in the student .tns file are provided on page 70, as is a suggestion for an optional extension.*

TI-Nspire™ Applications

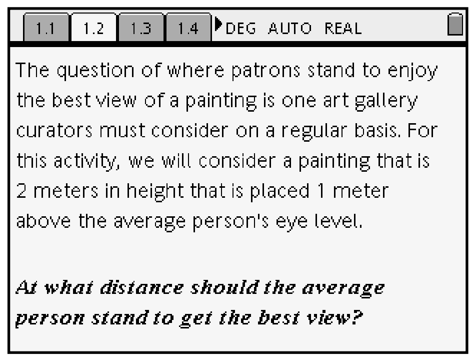
Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes

The problem of determining the best position to view a painting hanging on an art gallery wall has been contemplated since at least the time of Regiomontanus in 1471 (Maor, 1998). The essence of the problem lies in finding the distance, *x*, a person should stand away from the wall that will optimize the viewing angle of the art work. Use of the multi-representational facilities of the TI-Nspire handheld and computer software has the potential to provide students with the opportunity to engage in an authentic mathematical modeling activity.

The mathematics associated with non-contrived applications becomes complex very quickly. TI-Nspire learning technology permits the management of this complexity, allowing students to engage with real-world problems and the “mathematization” of these problems via the process of modeling.

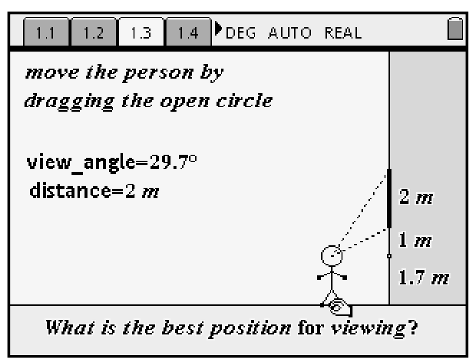
Step 1: After reading the introduction on page 1.2, students should role-play the problem by choosing a suitable piece of artwork around the classroom (or elsewhere in the school) and trying to find the best place to stand by simple trial and error. This experience will allow students to appreciate the fundamental ideas behind the problem they are trying to solve.

After the initial experience described above, students should work in groups to assist each other to fully understand the problem and then restate it in its mathematical context. Make it clear to students that you expect them to solve the problem in more than one way but that you also expect all members of the group to contribute to the development of each solution.

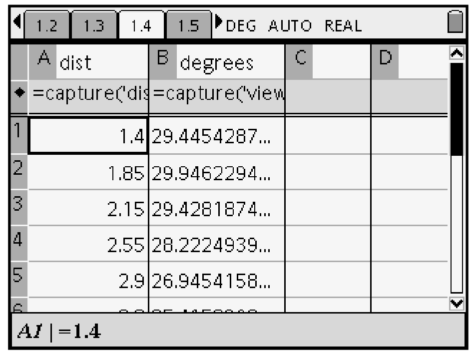


Step 2: After considering some real-world depictions of the problem, students are ready to work with the dynamic geometric model provided on page 1.3 of the student .tns file.

By varying the distance of the person from the wall, students should attempt to describe their initial observations and make conjectures based on their intuitive feel for the situation—they should observe that the viewing angle varies with the distance of the viewer from the wall. In fact, they may notice that it appears to increase to a maximum value as the person approaches a distance of 2 meters from the wall.

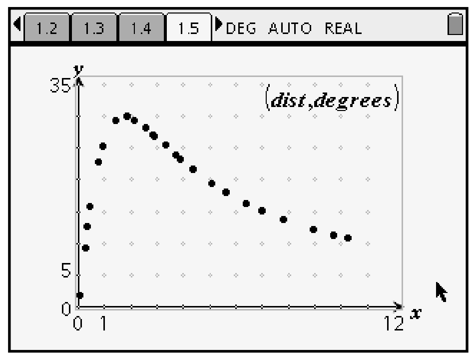


Step 3: A manual data capture has been set up to collect data from the geometric model; namely, the angle measures for different distances from the wall. Have students press ctrl + . at different distances, inspect the captured values in the spreadsheet on page 1.4, and then view the data graphically on page 1.5.



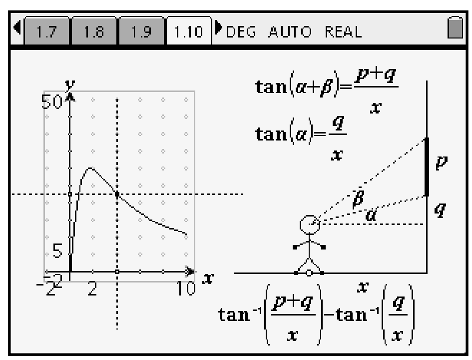
Students should collect a sufficient number of points to form a suitably detailed graph from which to make conjectures and to draw conclusions.

Data collected in this manner will provide enough detail for students to estimate an optimal distance from the wall, but it should point them to the need for an algebraic approach if a more accurate solution is desired.



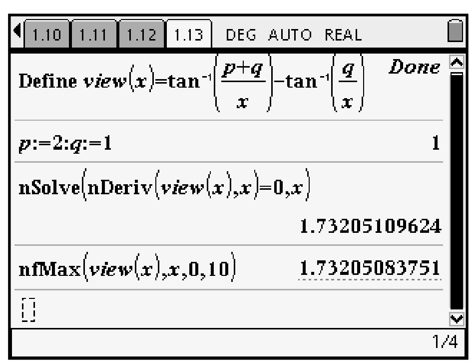
Students may use the geometric model, the spreadsheet, and the scatter plot to answer specific questions and demonstrate their understanding of the model, including an estimate of the best viewing position.

Step 4: Students may then use the labeled geometric model on page 1.10 to write and graph an algebraic function using inverse trigonometric functions that models the described behavior.



The locus may be used to check whether the function is correct. To plot the locus, select the **Locus** tool from the Construction menu. Click first on the point in the "cross hairs" in the graph to the left, and then click on the "draggable" point in the diagram to the right.

Step 5: Finally, students should attempt to compute the optimal position for viewing a work of art using formal or informal calculus techniques.



For an informal approach, the numerical function maximum may be applied to the defined function. For a formal calculus approach, solving to find the value at which the derivative equals zero would be most appropriate.

Student .tns File Solutions

Students should be responsible for the preparation of a report that provides details of how their group solved the problem they investigated. The report should also include, however, details of at least one other approach that was developed by students from another group. This will encourage students to learn from their peers and to be actively engaged in the process of supportive criticism during the presentation of other groups' approaches and solutions.

Scaffolding questions for "The Art Gallery Problem"
[Answers appear in brackets.]

1. What is the angle between the patron's eye and the top and base of the painting when the patron is standing 3 m away? [26.6°]
2. What is the angle between the patron's eye and the top and base of the painting when the patron is standing 1 m away? [26.6°]
3. **a.** Complete this table.

Distance (meters)	0.5	1.0	1.5	2.0	2.5	3.0
Angle (degrees)	[17.1]	[26.6]	[29.7]	[29.7]	[28.4]	[26.6]

   b. Use this table to estimate the best position for the patron to stand in front of the picture. [between 1.5 and 2.0 m]
4. Write a formula for the model that describes the relationship between the distance from the painting and the angle between the patron's eye and the top and bottom of the painting. Use the model on page 1.10 to help.
   [$view(x) = \tan^{-1}\left(\frac{3}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right)$]
5. Graph this function on page 1.10. Then use the graph to determine the *best* distance to stand away from the painting. [about 1.7 m]
6. How might you confirm this result by making use of the calculus facilities of your TI-Nspire handheld? [Solve to find the exact distance at which the derivative of our function equals zero, which correlates to the *x*-value of the maximum point of the function. (The exact distance is $\sqrt{3}$ m.)]

Extension

The growth in popularity of flat-screen television monitors that may be hung on walls lends new relevance to this problem, since students are likely to be very interested in finding the optimal height and position from which to enjoy their wide-screen viewing. In this case, measurements would be made from the preferred viewing position (sofa or armchair) and used to calculate the best height for the screen.

Visit education.ti.com/exchange to download activity files, including the student .tns file CalcAct09_ArtGallery_EN.tns. Enter "8300" in the quick search box.