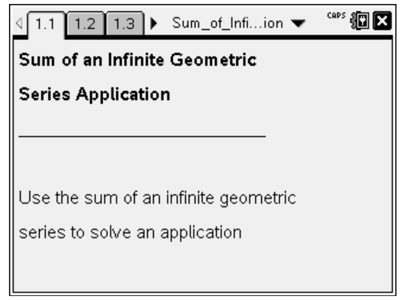


Math Objectives

- Students will use an infinite geometric series to find altitudes of a hot air balloon.
- Students will determine what makes the application problem an infinite geometric series.
- Students will use a formula and sigma notation to find partial sums (altitude of the hot air balloon at a given minute).
- Students will use a formula and sigma notation to find the maximum altitude of the hot air balloon.

Vocabulary

- geometric sequence
- infinite geometric series
- partial sum
- sigma notation

About the Lesson

- This lesson involves finding the altitude of a hot air balloon to which a probe has been attached to collect data regarding air quality.
- Students will use an up/down arrow slider to view the first five minutes of the balloon's flight.
- Students will use formulas and sigma notation to find partial sums regarding the altitude of the balloon at a given time and the finite sum regarding the maximum altitude of the balloon.

TI-Nspire Navigator™ System

- Use Screen Capture to share students' formulas.
- Use Teacher Edition computer software to review student documents.
- Utilize Class Analysis to display students' answers.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use up/down arrow slider

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- In the *Graphs & Geometry* application, you can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Sum_of_Infinite_Geo_Series_Application_Student.pdf
Sum_of_Infinite_Geo_Series_Application_Student.doc

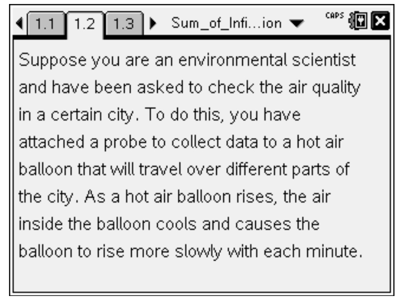
TI-Nspire document
Sum_of_Infinite_Geo_Series_Application.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

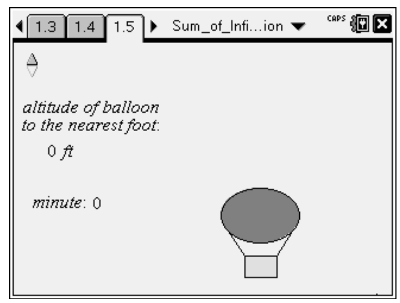
Discussion Points and Possible Answers

Tech Tip: Press (esc) to hide the entry line if students accidentally click the chevron.

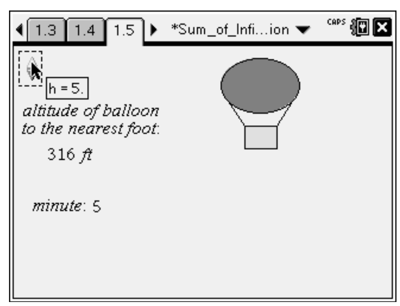
Move to page 1.2.



Suppose you are an environmental scientist and have been asked to check the air quality in a certain city. To do this, you have attached a probe to collect data to a hot air balloon that will travel over different parts of the city. As a hot air balloon rises, the air inside the balloon cools and causes the balloon to rise more slowly with each minute. Assuming air resistance is negligible, suppose the balloon rose 114 feet the first minute. For each minute after the first minute, the hot air balloon rises 70% as far as it rose the previous minute. You will need to know the altitude when analyzing the data taken by the probe. What will be the balloon's maximum altitude?



Move to page 1.5.



1. Use the up/down arrows on page 1.5 to see the altitudes for the first five minutes of the balloon's flight. Calculate the distance the balloon travels for each of these minutes. Show your work in the table below.

Answers:

Minute	Distance traveled per minute (nearest foot)	Total altitude given on page 1.5 (nearest foot)
0	**0**	**0**
1	**114**	**114**
2	**(0.7) * 114 = 80**	**194**
3	**(0.7) * 80 = 56**	**250**
4	**(0.7) * 56 = 39**	**289**
5	**(0.7) * 39 = 27**	**316**

Teacher Tip: You may want to ask students what they notice about the distance traveled each minute and make a conjecture regarding the number of feet as the balloon continues to rise. Having students verbalize that the number of feet the balloon rises each minute is getting smaller may help them understand the idea of a finite sum.

2. If you could press the up arrow again, what would be the balloon's altitude after the next minute (minute 6)? Show your work below.

   Answer: The balloon rose 27 feet during minute 5. In order to find the altitude after six minutes, add 70% of 27 to 316: 316 + (0.7) × 27 = 334.9. Therefore, the altitude after six minutes will be about 335 feet.

3. What makes this problem a geometric sequence? Explain.

   Answer: A geometric sequence occurs when each term after the first is found by multiplying the previous term by a constant called the common ratio r. Since each altitude is found by finding 70% of the previous altitude, that makes this a geometric sequence.

4. What makes this problem an infinite geometric series? What would be the value of the multiplier r?

   Answer: Since we are finding the maximum altitude of the balloon, this would make the geometric series infinite. The common multiplier is the 0.7 or 7/10.

5. How can you tell if this infinite geometric series has a finite sum? Explain.

   Answer: If the absolute value of the common multiplier r is less than 1, the series will have a finite sum.

6. Use the sum of a geometric series formula given below, which will allow you to find partial sums. In this case, you can find the altitude of the balloon for any given minute. Find the altitude at 5, 6, and 10 minutes.

   $S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$, where $r \neq 1$

Answer:

$$S_5 = \frac{114 - 144(0.7)^5}{1 - 0.7} = \frac{94.84002}{0.3} = 316.1334 \approx 316 \text{ ft}$$

$$S_6 = \frac{114 - 144(0.7)^6}{1 - 0.7} = \frac{100.588014}{0.3} = 335.29338 \approx 335 \text{ ft}$$

$$S_{10} = \frac{114 - 144(0.7)^{10}}{1 - 0.7} = \frac{110.7797822}{0.3} = 369.2659405 \approx 369 \text{ ft}$$

Teacher Tip: Remind students that they can check their answer for five minutes on page 1.5. Also, their answer for six minutes should match their answer to Question 2.

7. How would you find the same altitudes as in Question 6 using sigma notation? Show your work below.

Answer:

$$\sum_{n=1}^{5} (114 * (0.7)^{n-1}) \approx 316 \text{ ft}$$

$$\sum_{n=1}^{6} (114 * (0.7)^{n-1}) \approx 335 \text{ ft}$$

$$\sum_{n=1}^{10} (114 * (0.7)^{n-1}) \approx 369 \text{ ft}$$

8. Using the sum of an infinite geometric series formula, $S = \dfrac{a_1}{1 - r}$, find the maximum altitude of the balloon. Show your work below.

Answer: $S = \dfrac{114}{1 - 0.7} = 380 \text{ ft}$

9. How would you find the maximum altitude of the balloon using sigma notation? Show your work below.

Answer: $\displaystyle\sum_{n=1}^{\infty} (114 * (0.7)^{n-1}) \approx 380 \text{ ft}$

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to use an infinite geometric series to find altitudes of the hot air balloon.
- The definition of an infinite geometric series.
- How to find partial sums of a geometric series.
- How to determine if an infinite geometric series is finite.
- How to find the sum of an infinite geometric series.

Assessment

Have students solve a similar hot air balloon application such as: Suppose the hot air balloon rises 110 feet in the first minute of flight. For each minute after the first minute, the balloon rises 80% as far as the previous minute. Complete Questions 1–9 for this new problem.