Gettin' the Swing

ID: 11689

Time Required
40–45 minutes

Activity Overview

In this activity, students will investigate sinusoidal functions, collect data from a swinging pendulum, model the data with a cosine function, and take the derivative to find the velocity and acceleration. Self-check multiple-choice questions help students review and build the foundation for their data collection and mathematical model activity.

Topic: Model Sinusoidal Data

- *Write an equation for a sinusoidal function.*
- *First and second derivative to determine velocity and acceleration*

Teacher Preparation and Notes

- *Students will write their responses directly into the TI-Nspire handheld or on the accompanying handout. On self-check questions, students can then press* (menu) *and select* ***Check Answer*** *(or* (ctrl) *+▲).*
- *Students will need to know that the derivative of position is velocity, and the derivative of velocity is acceleration. For the multiple-choice questions at the end, students are expected to know the derivative of trigonometric functions.*
- *The activity is designed to be a student-centered discovery/review involving data collection. If a motion sensor is not available, then delete Problem 4 of the TI-Nspire document and have students disregard this portion of the worksheet.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11689" in the quick search box.***

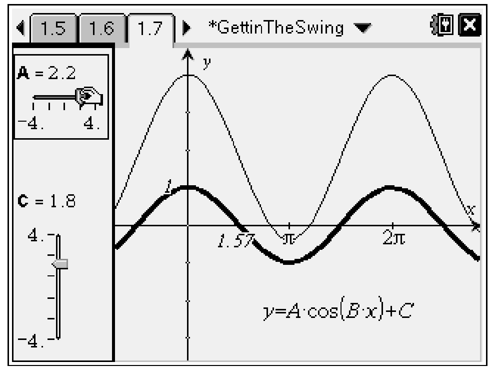
Associated Materials

- *GettinTheSwing_Student.doc*
- *GettinTheSwing.tns*
- *GettinTheSwing_Soln.tns*
- *Calculator-Based Ranger 2™ or Vernier Go!™ Motion*
- *pendulum (smooth ball hanging from a string)*
- *meter stick*
- *stop watch*

Suggested Related Activities

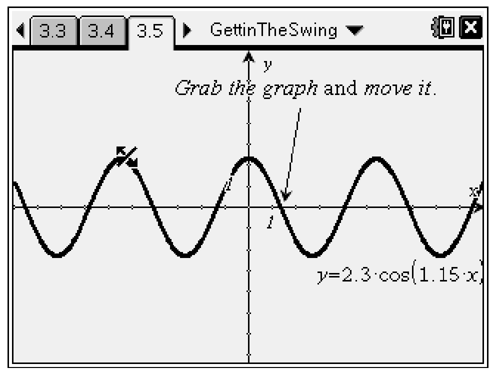
- *Derivatives of Trigonometric Functions (TI-Nspire CAS technology) — 9290*
- *Graphs of Sine and Cosine (TI-Nspire technology) — 9431*
- *Vertical and Phase Shifts (TI-84 Plus) — 9608*

Part 1 – Warm-up



Beginning on page 1.2, students answer self-check questions to help them visualize and review characteristics of sinusoidal functions.

Page 1.7 and 2.2 allow students to explore the effects of varying the parameters of the general sinusoidal functions. Page 3.1 has the students examine more carefully the effect of changing *B*. This data and the graph on page 3.2 help students see that as the *B* increases the period decreases. Students should also see that this relationship involves 2π. On page 3.2, students can check their relationship by pressing **MENU > Analyze > Plot function** or they could use a power regression.



Some kinesthetic learners will appreciate page 3.5 more than the sliders. They can directly see that the changes they make impact the graph. There are two different icons that appear when you hover over the graph. The "cross-hairs" (✣) produce a vertical and horizontal shift. The sloping line with oblique arrows (⤡) changes the amplitude and frequency when it is grabbed and moved. Although this method of grabbing *y* = cos(*x*) easily shows the change, when they are modeling the data in Part 2 they should be encouraged to estimate the amplitude, period, phase shift and vertical shift (how far the pendulum is away from the motion sensor when it is at equilibrium).

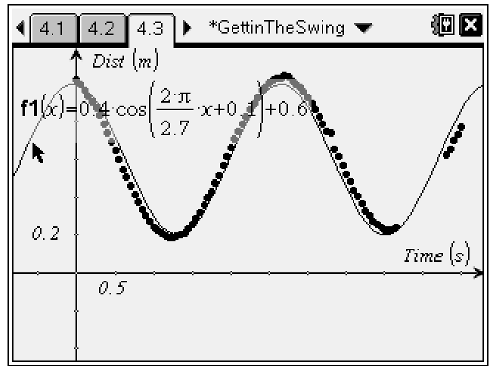
Further Discussion

- *Page 3.4 can be used to ask the students what the derivative of y = cos(x) is. When the A = 1, B = 1, C = 0, and the D is set at $\pi/2$, the graph looks likes y = –sin(x).*

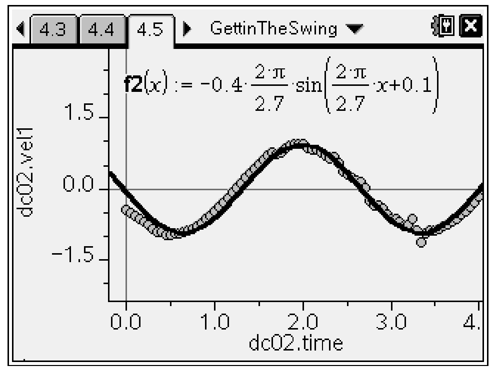
Student Solutions

1. (a) range is from –1 to 1 (b) amplitude *A* = 1 (c) period $T = 2\pi$
2. *A* makes the amplitude bigger, *C* produces a vertical shift, *B* changes the frequency
3. $B = \dfrac{2\pi}{T}$; The inverse function graphed on 3.2 confirms this.
4. $y = A \cdot \cos(B \cdot (x - D)) + C$
5. On page 3.4, when the *D* increases the graph moves to the left, therefore, as shown on the graph $y = A \cdot \cos(B \cdot (x + D)) + C$.
6. Moving up and to the right makes a negative *D* and a positive *C*.

Part 2 – Collect & Analyze Data



This part of the activity begins with students plugging in the CBR 2™ or Go!™ Motion. Auto Launch displays a Data Collection Console. When the play button is clicked, data is collected in real time. If there is a wall, chair, person or some other object that "distracts" the motion detector, the data may not look as good as depicted on the right. Pressing play again can replace the previous data. Have student repeat the data collection until they are pleased with the shape of the graph.



Be sure to use a smooth ball so that the sonic pulse is reflected off the surface well.

Before passing the CBR 2 on to the next student, or before tabbing over to close the Data Collection Console, students should select **MENU > Experiment > Display Data In > New Data & Statistics** and **MENU > Experiment > Display Data In > Lists & Spreadsheet**.

Students can see the connection to real life more clearly if meter sticks and stop watches are available. Students can measure the period with the stop watch. With the meter stick they can see how far the pendulum is from the motion detector when the ball is at rest. This will be their *C*. Also they can measure how far they pull the pendulum. This is the amplitude. The phase shift is related to when they pressed start to collect the data.

Student Solutions

The graphs and equations for the experiment will vary. The following equations correlate to the data in Problem 5.

$$y = 0.12\cos\left(\frac{2\pi}{1.1}\cdot(x-0.38)\right)+0.35$$

$$v = -0.12\left(\frac{2\pi}{1.1}\right)\sin\left(\frac{2\pi}{1.1}\cdot(x-0.38)\right)$$

$$a = -0.12\left(\frac{2\pi}{1.1}\right)^2\cos\left(\frac{2\pi}{1.1}\cdot(x-0.38)\right)$$