E X P L O R A T I O N S

Activity 7

Partitions—
How Many Ways Can
We Split A Number?

Objective

♦ To explore multiple representations of positive integers

♦ To develop partitions of positive integers

Materials

♦ TI–73 Calculator

♦ Paper cups

♦ Marbles

Teacher Notes

Introduction

How many ways can we place three marbles in cups? One way is to place all three marbles in one cup. Another way is to place each of the marbles in separate cups. Another way is to place two marbles in one cup and the third marble in another cup. These are illustrated below:

$3 = 3$ $3 = 2 + 1$ $3 = 1 + 2$ $3 = 1 + 1 + 1$
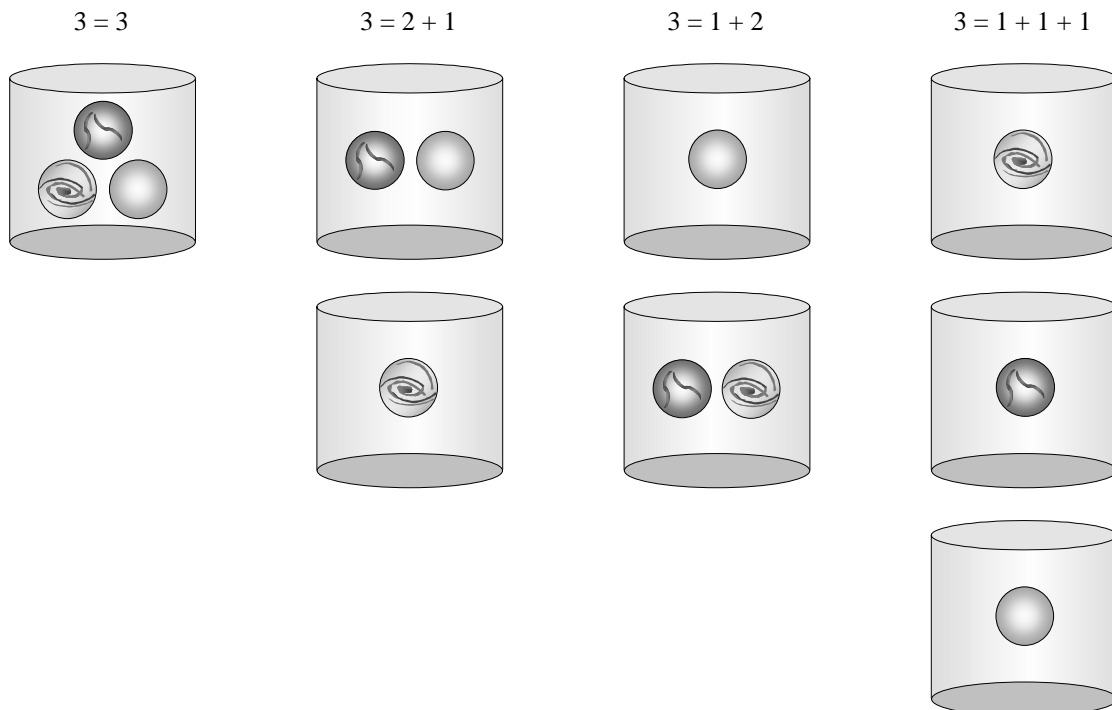
To simplify our counting, we can choose not to include one of the two ways to show $3 = 2 + 1$ and $3 = 1 + 2$. If the cups are indistinguishable, then we need only keep track of one way. Let's choose the first: $3 = 2 + 1$. Thus, there are three different ways to *partition* three marbles into cups.

This activity provides students the opportunity to construct partitions using the Number line Application, paper cups and marbles.

Procedure

1. Demonstrate how to partition 3 marbles into cups.

2. Distribute Student Activity pages with the problem statement. Have students work in pairs using one calculator between them.

3. Have students access the Number line application by pressing APPS and pressing ⯆ to select the NUMLINE application. Press ENTER twice to get to the NUM/FRAC line menu and select option 1: Number line. Have the students press ZOOM and select option 2. Have the students change the minimum and maximum values of x in order to view the entire picture. To do this press WINDOW and move to **Min** and enter **0**. Now move down to **Max** and enter **3**. Set the **Scale** to **1** and select **LabelOn**.

4. Students will first investigate how many ways to partition 3 and 6 using the Number line application. They will then find and count the partitions for the numbers 1 through 7. Included are Extra Challenges 8 through 10.

5. Ask the students to share their method for predicting how many partitions there are for the numbers 1 through 7.

Answer Key

6. There are 11 partitions of 6: 6, 5 + 1, 4 + 2, 4 + 1 + 1, 3 + 3, 3 + 2 + 1, 3 + 1 + 1 + 1, 2 + 2 + 2, 2 + 2 + 1 + 1, 2 + 1 + 1 + 1 + 1, and 1 + 1 + 1 + 1 + 1 + 1.

8. Predictions may vary.

Numbers to be partitioned	Predicted number of partitions	Number of partitions
1		1
2		2
3		3
4		5
5		7
6		11
7		15
Extra Challenges		
8		22
9		30
10		42

Activity 7

Partitions—How Many Ways Can We Split a Number?

Objective: *In this activity, you will investigate how to partition integers using the Number Line. You will use the Number Line application on the TI-73 calculator.*

Suppose you have three pieces of hard candy. How many ways could it be distributed? How many ways can you distribute six pieces of candy? Some think that there will be twice as many ways to distribute 6 pieces as 3 pieces because 6 is twice 3. Is this true?

1. Access the Number Line application by pressing ⌷APPS⌷ and pressing ⌷▾⌷ to select the NUMLINE application. Press ⌷ENTER⌷ twice to get to the NUM/FRAC LINE menu and select the Number Line option.

```
APPLICATIONS
1:Link…
2:CBL/CBR
3:NUMLINE
```

2. To capture the entire picture, press ⌷WINDOW⌷. You need to select new minimum and maximum values. All of the sums are positive, so move to **Min=** and enter **0**. Since we are computing sums that equal 3, move down to **Max=** and enter **3**. Set **Scale=1**, **Start=0,** and select **LabelOn**.

3. Press ⌷GRAPH⌷ to return to the Number Line. Press ⌷CLEAR⌷ to clear the expression. Select **Yes**. Now enter **1** ⌷+⌷ **1** ⌷+⌷ **1** and press ⌷ENTER⌷.

4. Clear this expression and show the other ways to get a sum of 3 using positive integers. Draw sketches of your Number Line windows.

1 + 1 + 1	2 + 1	1 + 2	3

Since **2 + 1** and **1 + 2** are similar ways to get a sum of **3**, we will count only one of those ways. Now we have found three distinct ways to split or partition 3.

5. This time find all of the ways to get a sum of 6. What is your prediction for how many ways there are to partition 6?

6. Press ⌷WINDOW⌷ to reset the **Max** to **6**. Press ⌷GRAPH⌷ to return to the Number Line window. Press ⌷CLEAR⌷ to clear the expression. Select **Yes**. Now find as many sums as you can that equal 6.
Draw and label sketches of your expressions on the blank screens below. (There are more screens than you need.)

Note: *The TI-73 Number Line Application can display up to five vectors. For the expression*
1 + 1 + 1 + 1 + 1 + 1 = 6, your sketch should show six vectors of length one, but only five vectors are drawn.

Did you find fewer or more ways to partition than your prediction?

Complete the table below using the Number Line application, cups, and marbles.

Number to be partitioned	Predicted number of partitions	Number of partitions
1		
2		
3		
4		
5		
6		
7		
Extra Challenges:		
8		
9		
10		

Were your predictions close? Mathematicians worked hard to find a way to predict how many ways there were to partition a given number. You can see that the number of partitions grows slowly at first, but then what happens? Explain.