Maclaurin Polynomials

MaclaurinPolynomials.tns

Name ______________________

Class ______________________

Functions that are continuous and differentiable at a center can be approximated by polynomials. One such way of doing this is to generate a Maclaurin polynomial for a function.

An approximating polynomial is to be expanded about the center *c* which is in the domain of a function *f*. If this *c* has the same value in a polynomial *P* and function *f* then $P(c) = f(c)$. Graphically, $P(c) = f(c)$ means that the graph of *P* passes through the point $(c, f(c))$.

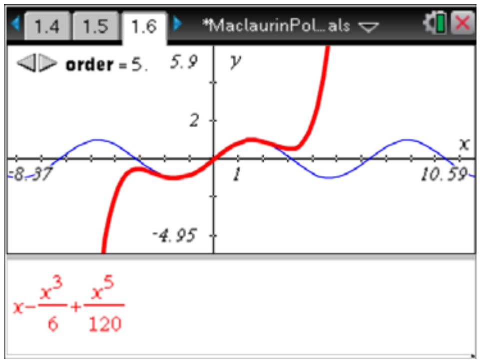
A Maclaurin polynomial is a polynomial that is based upon a function's derivatives at $c = 0$.

Specifically, the *n*th Maclaurin polynomial is defined as

$$P_n(x) = \frac{f(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

Problem 1 – Maclaurin polynomial for *f*(*x*) = sin(*x*)



In generating the third degree Maclaurin Polynomial for $f(x) = \sin(x)$, we compute

$$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3$$

$f(0) =$ ____ $\qquad f''(0) =$ ____

$f'(0) =$ ____ $\qquad f'''(0) =$ ____

Then substitute the values into the Maclaurin polynomial. This results in:

$P_3(x) =$

Click the arrows by the order and observe the graph.

What do you notice when the order is 1 and 2.

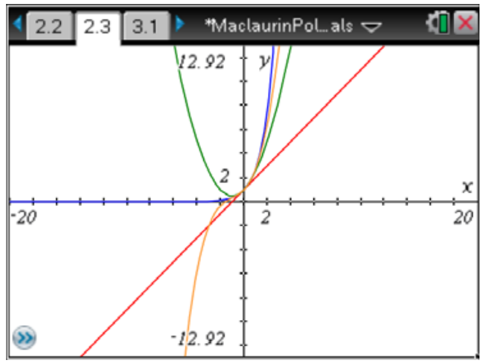
Why do you think this is true?

What do you notice when the order is 3 and 4.

Why do you think this is true?

Problem 2 – Maclaurin polynomial for $f(x) = e^x$

1. Write $P_1(x)$, $P_2(x)$, and $P_3(x)$ for $f(x) = e^x$.



2. Graph $f(x)$, $P_1(x)$, $P_2(x)$, and $P_3(x)$.

What do you notice?

Problem 3 – Maclaurin polynomial for $f(x) = \cos(x)$

1. Find $P_8(x)$ for $f(x) = \cos(x)$.

2. What do you notice about the value of each derivative after 0 has been substituted?

3. What do you notice about this approximation?

4. Write two expressions to describe your findings in the previous question, when differentiating $\cos(x)$, in terms of n.

5. Graph $P_8(x)$ and $f(x) = \cos(x)$. What do you notice?