Building Concepts: Equivalent Fractions

Lesson Overview

This TI-Nspire™ lesson helps students understand the concept of equivalent fractions. Two fractions are equivalent (or equal) if they are located at the same point on the number line.

Two fractions are equivalent (equal) if they are the same size or are located at the same point on a number line.

Learning Goals

1. Understand and explain that two fractions are equivalent (equal) if they are the same size or the same point on a number line.
2. Recognize and generate simple equivalent fractions.
3. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers;
4. Compare two fractions referring to the same whole (with the same numerator or the same denominator) by reasoning about their size;
5. Express the relationships between two fractions using the symbols <, =, or > and justify the conclusions by using a visual fraction model**;**
6. Understand that fractions greater than one are called improper fractions.

Prerequisite Knowledge

Equivalent Fractions is the second lesson in a series of lessons that explore fractions. Each lesson builds on the knowledge presented in the previous lesson. Prior to working on this lesson, students should:

- be familiar with the terms *numerator*, *denominator*, and *unit fraction.*
- have completed the lesson *What is a Fraction?*

Vocabulary

- **equivalent fractions**: fractions that are located at the same point on the number line.
- **improper fraction**: a fraction with a value that is greater than one.

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Equivalent Fractions_Student.pdf
- Equivalent Fractions_Student.doc
- Equivalent Fractions.tns
- Equivalent Fractions_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

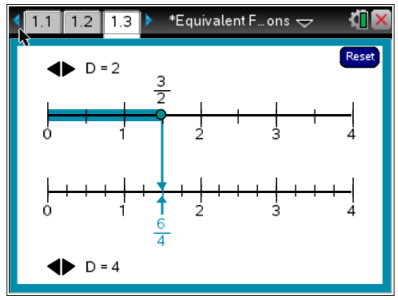
This TI-Nspire™ lesson helps students to understand the concept of equivalent fractions. Once students understand equivalence, they can begin to order fractions according to their size. Students recognize that, as with whole numbers, when ordering fractions the larger of two fractions is located farther to the right on the number line. Students should be familiar with the terms *numerator, denominator*, and *unit fraction* and have worked with the concepts in *What is a Fraction?*

The lesson can also be used to introduce the concept of improper fractions; however, it is not necessary to do so at this time. This lesson addresses improper fractions only in terms of describing fractions greater than 1. Converting improper fractions into mixed numbers does not appear until after students have become familiar with adding fractions.

Part 1, Page 1.3

Focus: Students will develop the concept of equivalence and comparing fractions. This activity can be extended to include a discussion about improper fractions.

On page 1.3, the black arrows are used to set the denominators (number of equal partitions) on each of the number lines. Dragging the dot on the top number line changes the numerator in the fraction. When the fraction on the top number line is equivalent to a fraction on the bottom number line, both fractions are displayed.



TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

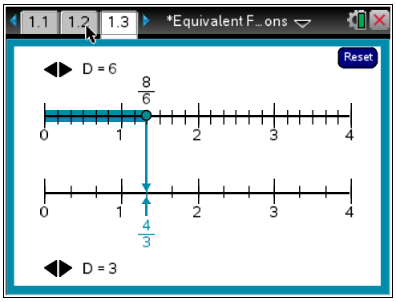
To reset the page, select **Reset** in the upper right corner.

For example, when each unit on the top number line is partitioned into four equal pieces (D = 4) and the dot is moved to show 6 copies of the $\frac{1}{4}$ pieces, the result is the fraction $\frac{6}{4}$. When the bottom arrows are used to select a denominator of 2 (D = 2)—to partition each unit into 2 pieces—the equivalent fraction $\frac{3}{2}$ is displayed on the bottom number line. The discussion should focus on why 3 copies of $\frac{1}{2}$ is the same as 6 copies of $\frac{1}{4}$.

Teacher Tip: Instruction and discussion for this lesson should help students focus on how to identify the unit fraction, the number of copies of that unit fraction and where that locates the fraction on a number line.

Part 1, Page 1.3 (continued)

It might take more than one day for the concept of equivalent fractions to make sense to students. Give students time to repeat the activity several times before or after asking them a focused set of questions. This technique can help them understand the concepts.



> **Teacher Tip:** Ask students to make up a question that they could answer using the number lines on page 1.3. Have students share their questions with each other, explaining their reasoning for solving their problems.

Class Discussion

Set the denominator on the top number line to 3. Drag the point on the top number line to set the fraction $\frac{2}{3}$. Use the bottom arrow to choose different numbers of parts on the lower number line.

- ***Name three fractions that are at the same point as $\frac{2}{3}$ on the number line. These are equivalent (or equal) fractions. Explain what you think it means for fractions to be equivalent.***

  Answer: $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots$; Fractions are equivalent if they are located at the same point on the number line.

  > **Teacher Tip:** Before working on the next question you might take the opportunity to review multiplication patterns with students. Remind them that a **common factor** is a factor of two or more numbers. Have students identify the common factor of the numerators and denominators in the equivalent fractions.

- ***Describe any pattern you found in the fractions that are equivalent to $\frac{2}{3}$.***

  Possible answer: When fractions are equivalent, the numerator and denominator of one fraction share a common factor with the numerator and denominator of the other fraction.

- ***Is $\frac{2}{3}$ equivalent to a fraction with a denominator of 4? Why or why not?***

  Answer: No, because the only fractions equivalent to one with a denominator of 4 are ones with denominators that are multiples of 4; like 8, 12, 16. Also, $\frac{2}{3}$ is bigger than $\frac{2}{4}$ and smaller than $\frac{3}{4}$, so there are no other fractions with a denominator of 4 that could be equivalent to $\frac{2}{3}$.

Class Discussion (continued)

✓ ***Write the fraction equivalent to*** $\frac{3}{4}$ ***whose***

a. ***denominator is 8.***

Answer: $\frac{6}{8}$

b. ***numerator is 9.***

Answer: $\frac{9}{12}$

(Question #1 on the Student Activity sheet.)

> **Teacher Tip:** Have students make predictions about the size of the unknown numerator or denominator based on the relationship of the given number to $\frac{3}{4}$. Then have students use the interactive number line to find the answer.

List the equivalent fractions with denominators up to 12 that are also equivalent to each of the following. Then tell whether your list is a complete set. Explain your answer.

a. $\frac{6}{12}$

Answer: $\mathbf{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}}$

b. $\frac{2}{3}$

Answer: $\mathbf{\frac{4}{6}, \frac{6}{9}, \frac{8}{12}}$

c. $\frac{0}{4}$

Answer: 0 over denominators from 1 to 12

d. $\frac{3}{7}$

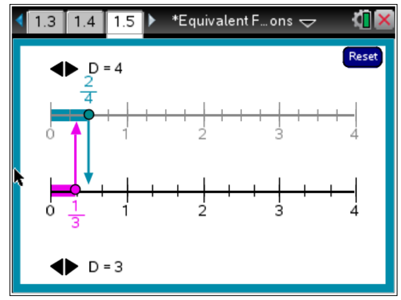
Answer: none

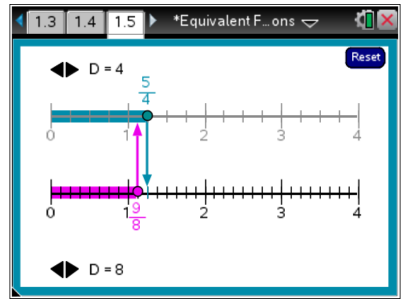
These are not the complete sets because many more fractions are equivalent to $\mathbf{\frac{1}{2}}$ like $\mathbf{\frac{12}{24}}$.

Part d might also be equivalent to a fraction with a denominator larger than 12.

Part 2, Page 1.5

Focus: Students will further explore equivalent fractions and compare fractions on a number line.





This activity can be used to either continue the discussion of equivalent fractions or to focus on comparison of fractions by observing their location on the number line. One fraction is larger than another when it is located to the right of that fraction on a number line ($\frac{a}{b} > \frac{c}{d}$ if $\frac{a}{b}$ is to the right or "above" $\frac{c}{d}$); a fraction is less than another when it is located to the left of that fraction on a number line ($\frac{a}{b} < \frac{c}{d}$ if $\frac{a}{b}$ is to the left or "below" $\frac{c}{d}$). Students can also reason about the size of two fractions by observing the length of each unit fraction on the number line. Students should be able to write a comparison statement ordering two fractions using the symbols < , =, or >.

The arrows near the number line on the top of the page determine D, the unit fraction ($\frac{\mathbf{1}}{\boldsymbol{b}}$) – or the denominator of the fraction – on that number line. Students can drag the dot on the number line to specify the number of ***a*** copies of $\frac{1}{b}$ for the fraction $\frac{a}{b}$. The arrows near the number line on the bottom of the page determine the denominator for a unit fraction $\frac{1}{d}$ on the lower number line. Students can drag the dot along the number line to create the number of ***c*** copies of $\frac{1}{d}$ for the fraction $\frac{c}{d}$. To reset the page, select **Reset** in the upper right corner.

As students work with the interactive number line, help them to focus on the relationship between the numerator and the denominator of the given fraction. Discuss how this relationship affects the value of the numerator and denominator in their answers.

Class Discussion

- ***Find a fraction equivalent to*** $\frac{10}{4}$ ***whose***

  a. ***numerator is 25.*** **b.** ***denominator is 8.***

  Answer: $\frac{25}{10}$ Answer: $\frac{20}{8}$

Have students…

- ***Find three fractions with different denominators that are equivalent to*** $\frac{7}{3}$***.***

Look for/Listen for…

Answer: $\mathbf{\frac{14}{6}, \frac{21}{9}, \frac{28}{12}}$

- ***Do you think there are any fractions equivalent to*** $\frac{13}{3}$ ***? Explain your reasoning.***

Answer: Yes, all of the thirds have equivalent fractions, so $\frac{26}{6}$ would be equivalent.

✓ ***Is there a fraction equivalent to*** $\frac{7}{3}$ ***that has a denominator less than 3? Why or why not? (Hint: Think about fractions with consecutive numerators and denominators of 1 or 2.)***

(Question #2 on the Student Activity sheet.)

Answer: the only denominators less than 3 are 2 and 1. Any fraction with a denominator of 1 is a whole number and $\frac{7}{3}$ is not a whole number. The only fractions with a denominator of 2 are $\mathbf{\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}}$ and $\mathbf{\frac{6}{2}}$. None of these are equivalent to $\frac{7}{3}$.

As you increase the number of copies of a unit fraction, the length of the segment covered on the number line by the copies is longer.

- ***Explain why it makes sense to say*** $\frac{5}{3}$ ***is greater than*** $\frac{2}{3}$ $\left(\frac{5}{3} > \frac{2}{3}\right)$ ***or*** $\frac{2}{3}$ ***is less than*** $\frac{5}{3}$ $\left(\frac{2}{3} < \frac{5}{3}\right)$.

Answer: $\frac{5}{3}$ is located farther from 0 on a number line than $\frac{2}{3}$ (by $\frac{3}{3}$ or one whole unit), so $\frac{5}{3}$ is larger than $\frac{2}{3}$. Another way to say it is that $\frac{2}{3}$ is less than $\frac{5}{3}$.

✓ ***Tomas says that*** $\frac{1}{8}$ ***is larger than*** $\frac{1}{3}$ ***because 8 is bigger than 3. What would you tell Tomas? Use the number line to illustrate your answer.***

(Question #3 on the Student Activity sheet.)

Answer: The denominator indicates the number of equal partitions of a unit. So, the larger the denominator in the unit fraction, the smaller the partition of the unit, so $\frac{1}{8}$ is smaller than $\frac{1}{3}$.

Another way to think is that the larger fraction is the one that is located farther to the right on the number line; $\frac{1}{3}$ is farther to the right than $\frac{1}{8}$.

Class Discussion (continued)

- ***Sue says $\frac{5}{3}$ is greater than $\frac{4}{2}$ because 5 is greater than 4 and 3 is greater than 2. What would you tell Sue?***

Answer: Sue is incorrect. $\frac{4}{2}$ is located further right on the number line than $\frac{5}{3}$ which means that $\frac{4}{2}$ is the greater fraction. Another way to think about this is that since $\frac{6}{3}$ is equivalent to $\frac{4}{2}$, and $\frac{5}{3}$ is smaller than $\frac{6}{3}$, then $\frac{5}{3}$ must also be smaller than $\frac{4}{2}$.

For each pair of fractions identify the greater fraction and write a comparison statement using <, =, or >. Explain your reasoning using the number line.

- $\frac{1}{2}$ ***and*** $\frac{1}{5}$

  Answer: $\frac{1}{2}$ is greater; $\frac{1}{2} > \frac{1}{5}$

- $\frac{4}{5}$ ***and*** $\frac{3}{4}$

  Answer: $\frac{4}{5}$ is greater; $\frac{3}{4} < \frac{4}{5}$

- $\frac{3}{8}$ ***and*** $\frac{5}{6}$

  Answer: $\frac{5}{6}$ is greater; $\frac{5}{6} > \frac{3}{8}$

  The reason for each is that the larger fraction is located farther to the right on the number line.

Write a fraction that satisfies the conditions below. Explain your reasoning.

- ***greater than*** $\frac{1}{2}$ ***and less than 1***

  Possible answer: $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, …

- ***greater than*** $\frac{1}{4}$ ***and less than*** $\frac{5}{8}$

  Possible answer: $\frac{3}{8}$, $\frac{4}{8}$, …

Class Discussion (continued)

- ***greater than*** $\frac{3}{8}$ ***and less than*** $\frac{4}{8}$

  Possible answer: $\boldsymbol{\frac{3}{7}}$, $\boldsymbol{\frac{4}{9}}$, $\boldsymbol{\frac{5}{11}}$, …

  The reason for each is that the answer is located between the fractions given on the number line.

- ***Describe in words what the comparison*** $\frac{1}{2} < \frac{2}{3} < \frac{5}{4}$ ***means.***

  Answer: It means that $\frac{1}{2}$ is to the left of $\frac{2}{3}$ and $\frac{2}{3}$ is to the left of $\frac{5}{4}$ on the number line. This also means that $\frac{1}{2}$ is the smallest of the 3 fractions and $\frac{5}{4}$ is the greatest of the 3 fractions.

- ***Use*** <, =, or > ***to order the fractions*** $\frac{3}{5}$, $\frac{5}{7}$ and $\frac{2}{3}$ ***from least to greatest. Explain how you thought about the problem.***

  Answer: $\frac{3}{5} < \frac{2}{3} < \frac{5}{7}$. I compared $\frac{5}{7}$ and $\frac{2}{3}$ on a number line to figure out which one was smaller, and then compared that fraction to $\frac{3}{5}$.

For each of the following write a comparison statement that shows two fractions with a denominator of three so that:

- $\frac{1}{2}$ ***is between them***

  Answer: $\frac{1}{3} < \frac{1}{2} < \frac{2}{3}$

- $\frac{5}{2}$ ***is between them***

  Answer: $\frac{7}{3} < \frac{5}{2} < \frac{8}{3}$

- $\frac{7}{2}$ ***is between them***

  Answer: $\frac{10}{3} < \frac{7}{2} < \frac{11}{3}$

Class Discussion (continued)

Predict each of the following and write your prediction in a number sentence. Explain why you think your prediction makes sense and then check your reasoning using the number line in the file.

- ***Which fraction is greater:*** $\frac{2}{3}$ ***or*** $\frac{3}{4}$***?***

  Answer: $\frac{3}{4}$; $\frac{3}{4} > \frac{2}{3}$

- ***Which fraction is greater:*** $\frac{7}{3}$ ***or*** $\frac{13}{4}$***?***

  Answer: $\frac{13}{4}$; $\frac{13}{4} > \frac{7}{3}$

- ***Which fraction is greater:*** $\frac{7}{3}$ ***or*** $\frac{11}{4}$***?***

  Answer: $\frac{11}{4}$; $\frac{11}{4} > \frac{7}{3}$

- ***For which two pairs was it harder to predict the largest fraction:*** $\frac{7}{3}$ ***and*** $\frac{13}{4}$ ***or*** $\frac{7}{3}$ ***and*** $\frac{11}{4}$***?***

  Explain your reasoning.

  Answer: $\frac{13}{4}$ is past 3 on the number line and $\frac{7}{3}$ is only past 2 so $\frac{13}{4}$ is larger. $\frac{7}{3}$ and $\frac{11}{4}$ were both past 2 but had to figure out which one was farther to the right.

- ***Which of the fractions below are improper fractions? Justify your reasoning in terms of the number line.***

  a. $\frac{7}{5}$ b. $\frac{3}{2}$ c. $\frac{5}{7}$ d. $\frac{1}{12}$ e. $\frac{13}{5}$ f. $\frac{4}{1}$

  Answer: a, b, e, and f are improper fractions because they are located to the right of 1 on the number line.

- ***Which statements are true?***

  a. $\frac{5}{12} < \frac{2}{3}$ b. $\frac{7}{12} < \frac{5}{12}$ c. $\frac{12}{5} < \frac{3}{2}$ d. $\frac{12}{7} < \frac{12}{5}$

  Answer: a and d are true.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Check the box for each fraction equivalent to $\frac{7}{4}$.

   □ $\frac{3}{4}$ ✓ $\frac{14}{8}$ □ $\frac{14}{4}$ ✓ $\frac{21}{12}$

2. Find two other fractions equivalent to $\frac{7}{4}$.

   Answer: $\boldsymbol{\frac{28}{16}, \frac{35}{20}}$

3. Insert the symbols <, =, or > to make a true statement.

   a. $\frac{5}{7}\square\frac{8}{9}$

   Answer: $\boldsymbol{\frac{5}{7}\boxed{<}\frac{8}{9}}$

   b. $\frac{13}{4}\square 3$

   Answer: $\boldsymbol{\frac{13}{4}\boxed{>}3}$

   c. $\frac{22}{6}\square\frac{33}{9}$

   Answer: $\boldsymbol{\frac{22}{6}\boxed{=}\frac{33}{9}}$

4. Write a comparison statement to order the fractions from least to greatest.

   $\frac{25}{12}$ $\frac{13}{12}$ $\frac{9}{10}$ $\frac{13}{5}$

   Answer: $\frac{9}{10} < \frac{13}{12} < \frac{25}{12} < \frac{13}{5}$

5. Compare $\frac{14}{12}$ and $\frac{8}{5}$. Identify the true statement.

   a. $\frac{14}{12}$ is greater than $\frac{8}{5}$ because $14 > 8$ and $12 > 5$.

   b. $\frac{8}{5}$ is greater than $\frac{14}{12}$ because $\frac{1}{5}$ is larger than $\frac{1}{12}$

   c. $\frac{8}{5}$ is greater than $\frac{14}{12}$ because $\frac{8}{5}$ is greater than 1.

   d. $\frac{14}{12}$ is greater than $\frac{8}{5}$, because $\frac{14}{12}$ is equal to $\frac{7}{6}$.

   e. $\frac{8}{5}$ is greater than $\frac{14}{12}$, because $\frac{3}{5}$ is larger than $\frac{2}{12}$

   Answer: e

Building Concepts: Equivalent Fractions

TEACHER NOTES

Student Activity solutions

Vocabulary

equivalent fractions:

fractions that are located at the same point on the number line

improper fraction:

a fraction with a value that is greater than one

Students will generate equivalent fractions and compare two fractions referring to the same whole.

1. Write the fraction equivalent to $\frac{3}{4}$ whose

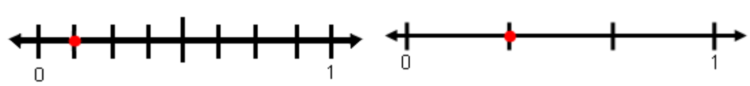
a. denominator is 8. ***Answer:*** $\frac{6}{8}$

b. numerator is 9. ***Answer:*** $\frac{9}{12}$

2. Is there a fraction equivalent to $\frac{7}{3}$ that has a denominator less than 3? Why or why not? (Hint: Think about fractions with consecutive numerators and denominators of 1 or 2.)

Answer: the only denominators less than 3 are 2 and 1. Any fraction with a denominator of 1 is a whole number and $\frac{7}{3}$ ***is not a whole number. The only fractions with a denominator of 2 are*** $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}$ ***and*** $\frac{6}{2}$***. None of these are equivalent to*** $\frac{7}{3}$***.***

3. Tomas says that $\frac{1}{8}$ is greater than $\frac{1}{3}$ because 8 is bigger than 3. What would you tell Tomas? Draw the fractions on the number lines to illustrate your answer.



Answer: The denominator indicates the number of equal partitions of a unit. So the larger the denominator in the unit fraction, the smaller the partition of the unit, so $\frac{1}{8}$ ***is smaller than*** $\frac{1}{3}$***. Another way to think is that the larger fraction***

is the one that is located further to the right on the number line; $\boldsymbol{\frac{1}{3}}$ ***is farther to the right than*** $\boldsymbol{\frac{1}{8}}$***.***

4. Apply what you have learned about equivalent fractions to explain why $\frac{3}{6}$ and $\frac{5}{12}$ are not equivalent.

Answer: The two fractions are not located at the same point on the number line.