NUMB3RS Activity: Escape
Episode: “Soft Target”

Topic: Percolation Theory **Grade Level**: 9 - 12
Objective: Introduce Percolation Theory.
Time: 20 minutes

Introduction

Have you ever spilled something on a table cloth and wondered how the liquid traveled? Could you have predicted how the stain would have occurred or is it truly random? Percolation Theory is the study of random flow of fluids in porous media. It is based on geometry and probability, and the application of that theory includes studying how forest fire spreads, how ground water flows, how animals move through their habitats, as well as how criminals navigate through the city to avoid capture. In “Soft Target,” Charlie uses Percolation Theory to speculate how a criminal got past the security checkpoints by examining how a subject navigates through a grid.

A simple neutral (or null) model for percolation can be explored using a grid and a random number generator. Based on the number generated, the appropriate site is marked. This process is repeated until a path of filled or shaded sites extends from the top of the grid to the bottom. This exercise attempts to show that patterns of random movement might occur with minimal assumptions about a subject’s ability to navigate through the grid.

Discuss with Students

***NUMB3RS* Example** In trying to figure out how a criminal slipped past security, Charlie uses percolation theory to model the criminal’s movement. The situation is visualized using a 10 x 10 numbered grid. A random number generator (use **randInt(1,100)** on a TI-84 Plus or a table of random values) is used to fill in the sites. For example, as random numbers, 48, 6, 16, 25, 33, 99, 55, 77, 29, 45, 79, 28, 67, 34, 53, 65, 35, 3, 58, 38, 97, 36, 43, 64,100, 2, 19, 49, 12, 63, 66, 84, 69, 26, 83, 87, are generated, the sites containing the numbers are shaded or filled. In this situation, the blank sites represent areas blocked by security and the filled sites represent parts of a potential path of escape for the criminal.

Sites sharing a common side are *connected sites*. Connected sites that are filled within the grid represent patches through which the subject can move undetected. Percolation out of the grid is achieved when the subject can move through the landscape using connected sites and patches. In this example, the criminal has at least one possible escape path starting at site 6, passing through the connected sites and leaving through site 97. The ratio of number of the connected sites in the largest patch to the total number of filled sites is $\frac{20}{36}$ or about 0.556.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

If we start out at the top of a grid with no sites shaded, and randomly shade sites to create connections, many sites need to be shaded before the criminal can find a path out from the top to the bottom. The criminal would like the path to appear as soon as possible, but the FBI wants it to take longer, since then they will have more time to catch the criminal.

Student page answers:* *1. *Grids will vary* ***2.*** *Answers will vary.* ***3****. Answers will vary.* ***4.*** *Fewer, because each site has more sites connecting to it, so a path can be made more easily.* ***5-6*** *Answers will vary.*

Beth Glassman, Cedar Park High School, TX

Name ____ Date _____________

NUMB3RS Activity: Escape

A criminal is on the loose and trying to escape the FBI. The criminal is injured and disoriented, so his choices are going to be treated as random as he makes his way through a landscape (the grid below). The criminal can only move through filled sites that are adjacent (immediately next to – *not diagonal*). Sites in the middle have 4 adjacent sites, those on the edges have 3, and the corners have only 2. He starts at the top (sites 1-10) and can only escape through the bottom (sites 91–100).

1. Find random numbers to fill in the grid until the criminal has a path from the top to the bottom. Use the TI-84 Plus command **`randInt(1,100)`** or a random number chart to select the sites to fill in. Remember to check for a complete path after you fill in each site. Stop when the criminal has a path from the top to the bottom.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**

2. **a**. When the criminal is able to escape, how many sites will be filled? ___________

b. A group of connected sites that are filled is called a *patch*. How many sites are in your largest patch? ____

3. Find the ratio of the number of filled sites in the largest patch to the total number of filled sites. __________ Compare your answer to the answers of your classmates.

Now assume that the criminal can move diagonally between sites.

4. Would you expect to have to generate more or fewer random numbers to create an escape path? Why? ____

5. Use this new set of rules to find a path through another 10 **x** 10 grid.

6. When the criminal is able to escape, how many sites will be filled? __________
Find the number of filled sites in the largest patch. ______________________
Find the ratio of the number of connected sites in the largest patch to the total number of filled sites. ____________ Compare your answer to your answer to #3.

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the student

- Consider ground water flow as one application of percolation theory. What could the landscapes and patches represent in this scenario? What is moving through the landscape?
- Animal habitations are also modeled using percolation theory. What do the landscapes and patches represent now? What might be modeled as it moves through the landscape?
- Think of or find other practical applications for percolation theory.

Additional Resources

On the web:

- Applications of Percolation Theory: **http://www1.coe.neu.edu/~emelas/apps.htm**
- Conservation Ecology: **http://www.ecologyandsociety.org/vol1/iss1/art4**
- Introduction to Percolation Theory: **http://www.people.fas.harvard.edu/~wu2/paper1/**
- Percolation Characteristic Functions: **http://ibiblio.org/e-notes/Perc/distr.htm**

In print:

- Stauffer, D., and A. Aharony, *Introduction to Percolation Theory*. London: Taylor & Francis, 1992.
- Gardner, R. H., B.T. Milne, M. G. Turner, and R. V. O'Neill, "Neutral Models for the Analysis of Broad Landscape Pattern," *Landscape Ecology* 1: 19-28, 1987.
- Gardner, R. H., R. V. O'Neill, M. G. Turner, and V. T. Dale, "Quantifying Scael-dependent Effects of Animal Movement with Simple Percolation Models," *Landscape Ecology* 3: 217-227, 1989.
- Allen, T. F. H. and T. W. Hoekstra, *Toward a Unified Ecology*. New York: Columbia University Press, 1993.

Related Topic: Galton Box

Prior to the development of Percolation Theory, the Galton Box was used to simulate random movement through a landscape. Sir Francis Galton was a British statistician whose work was used to make calculations about the likelihood of events occurring. The Galton Box, or *quincunx*, is similar to Pascal's triangle, where the binomial coefficients are replaced by pegs. These form a lattice of random walks for balls falling from the top to the bottom row. It can show how nature produces the binomial coefficients from Pascal's Triangle and their relation to a Gaussian bell-shaped curve. To see how the Galton Box operates, a simulation activity of the Galton Box can be viewed at **http://ccl.northwestern.edu/netlogo/models/GaltonBox**. As a project, research and compare the use of the Galton Box to the use of Percolation Theory.

Beth Glassman, Cedar Park High School, TX