Specialist Mathematics Unit 3
IMPLICIT DIFFERENTIATION TI *n*-spire CAS.

1. Find $\frac{dy}{dx}$ by implicit differentiation for each of the following relationships:

a. $x = y^3$ b. $x^3 = y^2$ c. $xy = 2x+1$

d. $x^2 + y^2 = 1$ d. $2x^2 - 2xy + y^2 = 5$

e. $xSin^{-1}(y) = e^{2y}$

2. Given that $xy - y - x^2 = 0$, find $\frac{dy}{dx}$

a. by explicit differentiation (making y the subject).
b. by implicit differentiation.

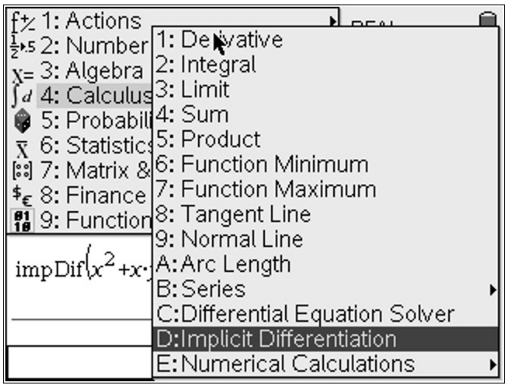
3. Find the equation of the tangent to the curve at the indicated point:

a. $y^2 = 8x$ at (2, -4) b. $x^2 - 9y^2 = 9$ at $(5, \frac{4}{3})$

c. $xy - y^2 = 1$ at $(\frac{17}{4}, 4)$ d. $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at (0,-3)

4. Using TI-nspire CAS calculator:

To differentiate implicitly, in Home screen select :



To find the gradient at a given point type the conditions after using 'given that' sign as shown in the screen to the right.

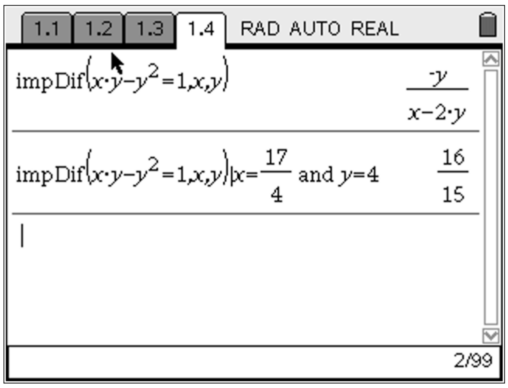
impDif($xy - y \wedge 2 = 1, x, y) \big| x = 17/4 \, and \; y = 4$

If you type: impDif($xy - y \wedge 2 = 1, x, y)$, you will get $\frac{dx}{dy}$.

Type the equation you want to differentiate:

impDif($xy - y \wedge 2 = 1, x, y)$

Note that the syntax above gives $\frac{dy}{dx}$.



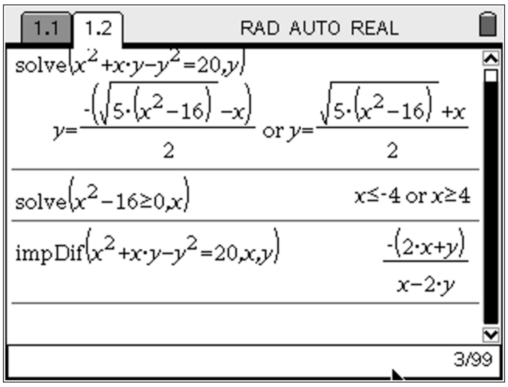
PROBLEM ONE:

Consider the conic section with equation $x^2 + xy - y^2 = 20$.

a. Make y the subject of the equation.

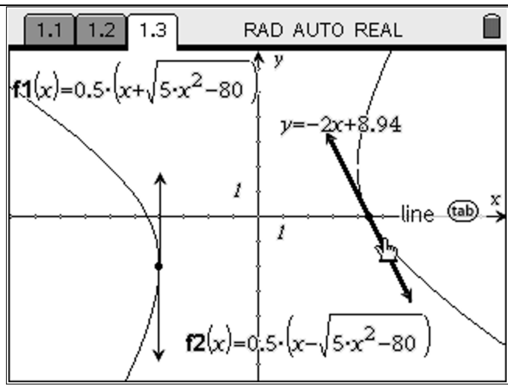
b. Show that the domain is $(-\infty, -4] \cup [4, \infty)$.

c. Find an expression for $\frac{dy}{dx}$.

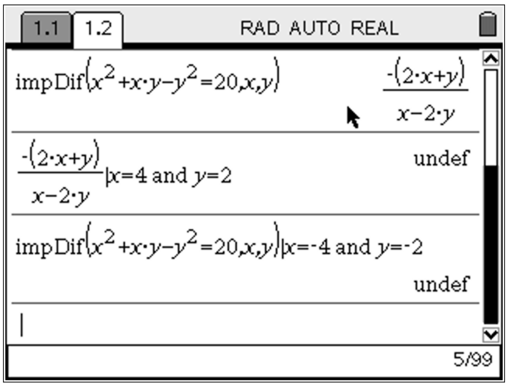


d. Sketch the graph.

You can also draw equations of tangents to the graph and find their equations.



e. Find the equations of the vertical tangents.

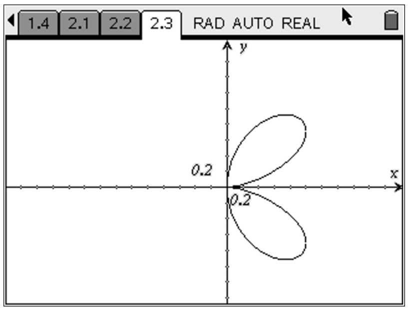


So the equations of the vertical tangents are $x = 4$ *and* $x = -4$.

f. Use (a) to eliminate y from your expression for $\frac{dy}{dx}$.

g. Hence prove that as $x \to \pm\infty, \frac{dy}{dx} \to \frac{5 \pm \sqrt{5}}{2\sqrt{5}}$.

PROBLEM TWO:



The graph of the curve $(x^2 + y^2)^2 = 4xy^2$ is shown alongside.

a. Find the gradient of the curve at the point where $x = 1$. Explain your result.

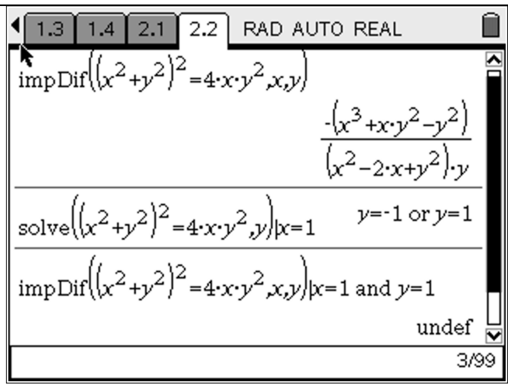
b. Find the gradients of the curve where $y = \frac{1}{2}$, giving your answers to 2 decimal places.

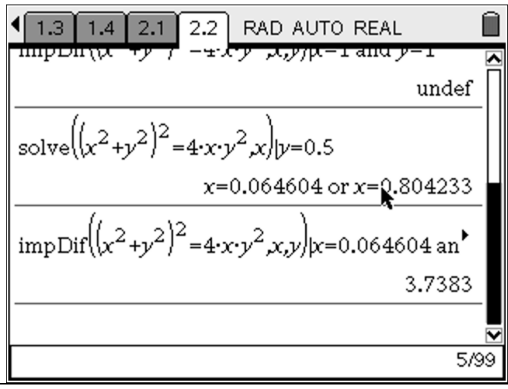
c. Sketch the graph. Check your gradient values graphically.

Differentiate implicitly.

Find the y-values when $x = 1$ and the x-values when $y = \frac{1}{2}$,

It can be seen that $\frac{dy}{dx}$ is undefined for $y = 0$ and also at (1,1) and at (1,-1) – it makes the denominator equal to zero.

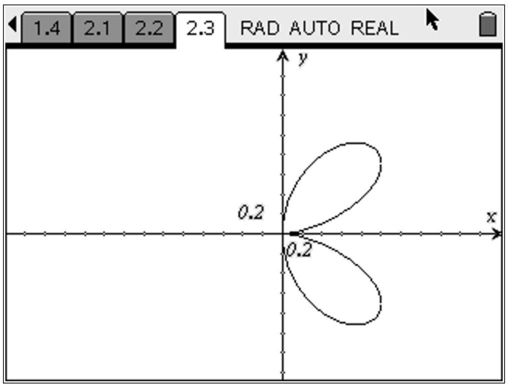




To sketch the graph we need to find expressions for y in terms of x:

$$y^2 = \frac{4x - 2x^2 \pm \sqrt{(2x^2 - 4x)^2 - 4x^4}}{2}$$

And enter as $\pm\sqrt{\ }$ of the above which means that we need to enter 4 equations.



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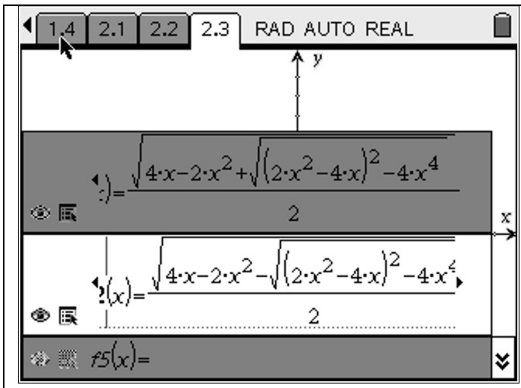
<td>
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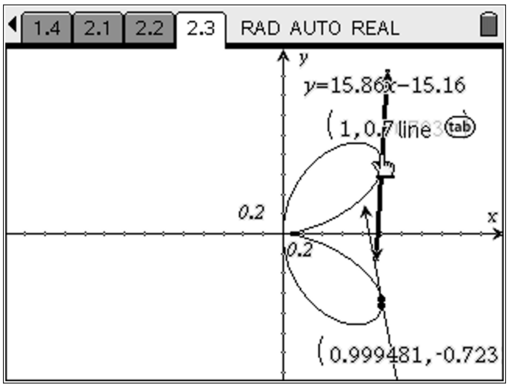
<td>You can also draw tangents to the curve at those values of x.

So the equation of the tangent to the graph at (0.804233, 0.5) is $y = 1.32428x - .565029$.</td>
<td>
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