NUMB3RS Activity: Percolation Theory
Episode: "Soft Target"

Topic: Percolation Theory **Grade Level:** 9 - 12

Objective: In this activity, students will use the TI-Navigator™ system and probability theory to analyze the random movements of an object.

Time: 25 - 30 minutes

Materials: TI-83/84 Plus graphing calculators, TI-Navigator system, and the following activity settings files: *Percolation.act, Galton4.act, Galton5.act*
To download these files, go to **http://education.ti.com/exchange** and search for "6584."

Introduction

In "Soft Target," Charlie talks about how *Percolation Theory* can be used to model the random movements of a disoriented criminal trying to escape FBI agents during a terrorist attack drill. He uses an analogy to a porous material in which liquid is poured on top. The question is: Will the liquid be able to make its way from hole to hole and reach the bottom?

In Part I of this activity, students will use the random number generator on their calculators to simulate the movements of a disoriented criminal, following a random path to an escape route, much like water making its way through a porous object.

In Parts II and III of this activity, students will study the Galton Box, a mathematical device that has a relationship to Percolation Theory and Pascal's triangle.

Part I: Finding an Escape Route

1. a. Students should work in pairs during this part of the activity.

b. Launch TI-Navigator on the computer and press **Begin Class** to start the session.

c. Have one student from each pair log into `NavNet` on their calculators. The other students will use the regular graphing calculator functions.

2. a. Load the ***Percolation.act*** activity settings file into Activity Center. This will display on each calculator a 7 × 7 grid containing the numbers 1 through 49. This also sets up Activity Center so that students can submit 1 point. They will be able to **Mark** the point, but it will not be displayed in Activity Center until the **Send** command is selected.

b. Press **▶ Start Activity**. Instruct each pair of students to find a random integer from 1 to 49 by executing the command **randInt(1,49)** on the calculator that is not logged into the Navigator network. (The **randInt(** command can be found by pressing [MATH] [◄] [5].)

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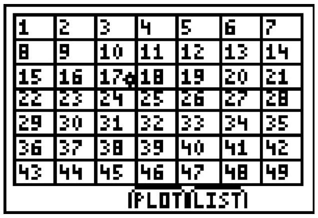
c. Using the calculator that is logged into the Navigator network, locate the box containing the random integer found in step 2b, **Mark** a point in the box, and **Send** it to Activity Center. (Make sure students mark the point in the open space on the right hand side of the box.) The screen shot below shows a point marked in the box for 17.



3. a. After all of the points have been submitted, tell students that the criminal can pass from one box to another only if each box shares an edge.

b. Ask students if they think the criminal can pass through the diagram after this first round. Use **Quick Poll** (with *Yes/No*) to collect students' answers.

c. If the answer is 'No,' press **Stop Activity** and then press **Start Activity**. Now have students repeat Step 2, one pair of students at a time, until an escape route appears.

d. When an escape route is found, ask one student to name the boxes through which the criminal passes.

e. Press **Stop Activity**.

Part II: The Galton Box

4. a. Students should work in pairs during this part of the activity.

b. Load the ***Galton4.act*** activity settings file into Activity Center.

c. Press **Start Activity**. Click the 'List - Graph' tab in Activity Center. Students will see a *Galton Box* with four rows of pins in Activity Center along with the list L_1 on their calculators.

d. Explain to students that a ball is dropped through the top of the box as indicated by the arrow. Each time the ball comes to a pin (marked by a point), it has a 50% chance of going to the left or to the right. They should see that this situation is similar to Part I, except that the path of the ball is more predictable than that of the disoriented criminal.

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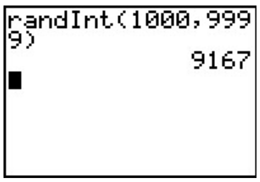
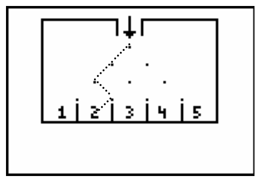
e. For each pair of students, have the student who is not logged into `NavNet` execute the command **randInt(1000,9999)**. This number will simulate the movement of the ball. An odd digit will represent the ball moving to the left and an even digit will represent the ball moving to the right. For example, the number 9167 would be interpreted as follows.

9 1 6 7
↓ ↓ ↓ ↓
L L R L

The screens below show how to simulate a ball going left, left, right, and left, landing in Slot 2.





f. Each time this command is executed, the student that is still logged into Activity Center should enter the *slot number* that the ball lands in list L_1.

g. Each pair of students should repeat this experiment 20 times. Have students **Send** their data to Activity Center.

h. After all the data has been collected, press [**Stop Activity**]. Then click ‘Configure’ and select ‘Existing activity lists.’

i. Press [**Start Activity**]. Each calculator will receive the entire class data set.

5. a. Students should then exit `NavNet`. Have students press [2nd] [QUIT] to return to TI-Navigator Home, and select `4:EXIT` APP.

b. Have students create a histogram displaying the frequency that the ball landed in each slot. To create a histogram, press [2nd] [STAT PLOT] [1]. Then, turn the plot on, and select the histogram icon. Press [WINDOW] and change the settings as follows: `Xmin = 0`, `Xmax = 7`, `Xscl = 1`, and `Ymin = 0`. The values of `Ymax` and `Yscl` will need to be adjusted based on the number of students (and number of data points) in the class. For a class of 25 students, you may want to let `Ymax = 200` and `Yscl = 10`. Finally, press [GRAPH] to draw the histogram.

c. Use **Screen Capture** to view the students’ histograms. Discuss the results with students. Calculate experimental probabilities (students can use [TRACE] to find the number of occurrences for each slot), which should convince students that Slot 3 is the most likely outcome and Slots 1 and 5 are the least likely outcomes.

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Part III: Determining the Theoretical Probability

6. a. Have students log back into `NavNet`. Ask students to think of all the different ways a ball can travel through the Galton Box to the slots at the bottom of the box. For example, LLRL is one such arrangement.

b. Use **Quick Poll** (with *Open Response*) to collect students' answers. Tell students to submit the four letters without any spaces or punctuation marks. (This will allow the results to be sorted.) There are 16 different arrangements which are listed below:

LLLL	LRLL	RLLL	RRLL
LLLR	LRLR	RLLR	RRLR
LLRL	LRRL	RLRL	RRRL
LLRR	LRRR	RLRR	RRRR

7. a. Write all 16 arrangements on the board and have students determine the slot the ball lands in for each arrangement. Students should determine the results below.

Movement	Slot
4 lefts	1
3 lefts and 1 right	2
2 lefts and 2 rights	3
1 left and 3 rights	4
4 rights	5

b. Have students calculate the theoretical probabilities from their analysis of the organized list. Discuss with students the connection between the theoretical results and the experimental results from Part II.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

- This problem can be extended in several different ways. For example in 16 trials, the ball should land (theoretically) in the bottom row slots, from left to right, 1, 4, 6, 4, 1 times. This sequence of numbers can be found in the 5th row of Pascal's triangle. Using this observation, students can predict the probabilities for Galton Boxes with additional slots and rows of pins.

  You can illustrate this point by loading the activity settings file ***Galton5.act***. This time, use **randInt(10000,99999)** to simulate the movement of the ball. You may realize that the 6th row of Pascal's triangle, with the numbers 1, 5, 10, 10, 5, 1, correlates to the theoretical distribution of 32 balls in a Galton Box with 5 rows of pins.

  You can also repeat Part II of the activity to find an experimental distribution that supports the theoretical results described above.

- Combinations can also be used to determine the theoretical distributions in a Galton Box. The expression ${}_nC_{(r-1)}$ gives the theoretical distribution of results for a Galton Box with *n* rows of pins, where *r* represents the slot number, for 2^n trials. For example, for a Galton Box with 6 rows of pins, the number of times a ball will land in slot 3 after 64 trials is ${}_6C_{(3-1)} = 15$.

- Additional *NUMB3RS* activities for "Soft Target" can be downloaded for free from the Web sites listed below.

  http://www.cbs.com/primetime/numb3rs/ti/activities/Act1_IsThisSeatTaken_SoftTarget_final.pdf

  http://www.cbs.com/primetime/numb3rs/ti/activities/Act2_Escape_SoftTarget_final.pdf

  http://www.cbs.com/primetime/numb3rs/ti/activities/Act3_AreYouSure_Target_final.pdf

- If you would like to learn more about TI-Navigator, visit **http://education.ti.com/navigator**.

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