NUMB3RS Activity: No Desk Left Behind
Episode: "Democracy"

Topic: Probability and simulation **Grade Level:** 9 - 10
Objective: Students use probability theory to simulate the likelihood of an event.
Time: 20 - 30 minutes
Materials: There are two versions of this activity. The first version, described on the Teacher pages, uses the TI-Navigator™ system. It also requires TI-83 Plus/TI-84 Plus graphing calculators. The second version, on the Student Pages, requires TI-83 Plus/TI-84 Plus graphing calculators but does not require the TI-Navigator system.

Introduction

In "Democracy," Charlie calculates the likelihood that four people who knew each other would have died by accident in a two-week period. He estimates that the odds are about 10 million to 1. He later learns that there were actually five people who all knew each other and who died in apparent accidents in that two-week period. He redoes the calculation and gets 700 million to 1.

Charlie does not explain the method he uses or what assumptions he makes to estimate these probabilities. One method that could be used to estimate likelihood of an event is a simulation. In this activity, students estimate, using simulation, the likelihood that more than 4 students in a class of 30 are left-handed when it is assumed that only 10% of students are left-handed.

1. a. Launch TI-Navigator and press **Begin Class** to start the session.

b. Have students log into NavNet on their calculators.

2. a. Ask the students if they are left-handed. Use **Quick Poll** (with Yes/No) to collect students' answers.

b. Click on **Poll Summary** to display the results of the poll. Explain that, based on this poll, the probability that a randomly selected student in the class picked at random is left-handed is the "Number who answered Yes" divided by the "Total Number of Yes and No answers."

3. a. Describe the following situation to the students: A school is purchasing new classroom furniture. Each classroom will have 30 student desks. There are two models of desks: right-handed and universal. Either left or right-handed students can conveniently use the universal desks, but they cost four times as much as right-handed desks. The principal believes that on average 10% of the population is left-handed. She wants to accommodate the needs of the students but also has a limited budget. How many universal desks should she order for each classroom?

b. Use **Quick Poll** (with Multiple Choice A – E) to collect students' answers. Tell students to select A for less than two universal desks, B for 2 universal desks, C for 3 universal desks, D for 4 universal desks, and E for 5 or more universal desks per classroom.

c. Click on **Poll Summary** to display the results of the poll. Ask students to explain why they made their choices.

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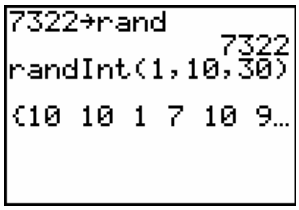
4. a. Explain that one way a student could help the principal make an informed decision is to simulate the situation. The student could create a list of 30 random numbers from the set 1 through 10. Since the principal believes that 10% of the students are left-handed, the student could let the number 1 signify a left-handed student and count the number of 1's that occur among the 30 random numbers.

b. Have students exit NavNet by pressing **4:Exit App**.

c. So that each student generates a different list of random numbers, have them seed the random number generator by storing a number, like the last four digits of their phone number, to the variable **rand** as shown at right. Have each student enter **randInt(1, 10, 30)** in the calculator and press ENTER to get a list of 30 random integers from 1 to 10. The syntax of the command is **randInt(**smallest, largest, number of number of numbers).



d. Ask students to count how many 1s there are in the list as they scroll to the right.

e. Have students log back into NavNet and share their results with the class.

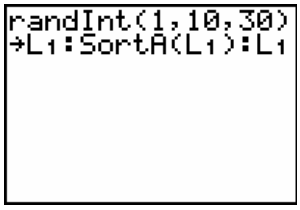
f. Use **Quick Poll** (with Multiple Choice A – E) to collect students' answers. Tell students to select A for fewer than two universal desks, B for two universal desks, C for three universal desks, D for four universal desks, and E for five or more universal desks per classroom.

g. Click on **Poll Summary** to display the results. Ask students if the simulation confirms or contradicts their opinions of how many universal desks the principal should order per classroom.

5. a. Suppose the principal decided to order four universal desks per classroom assuming that about 10% of the 30 students will be left-handed and ordering one extra desk just in case. By how much is the number of left-handed students likely to vary from what you might expect? Will one extra desk be enough? A simulation can be used to estimate this likelihood.

b. Have students exit NavNet by pressing **4:Exit App**.

c. Have students enter the three-part command shown at right in their calculator. Explain that these steps sort the list of 30 numbers so that all of the 1s come first. In this way, it will be easier to count the number of 1s. Remind students that a 1 represents a left-handed student, and that having more than four 1s means that some students do not get the appropriate desk.



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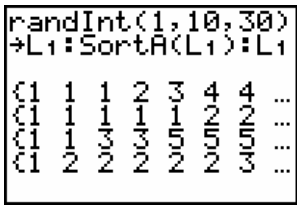
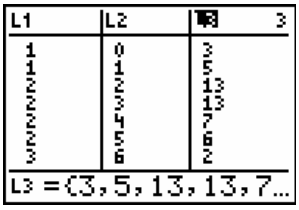
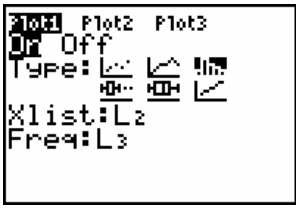
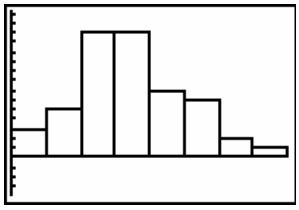
d. Tell students to press ENTER. Each time they do so, another set of 30 numbers is displayed. Have them press ENTER a total of 50 times while tallying how many 1's appear in each list. Each student should have 50 sets of tallies when they are finished.



e. Have students enter their tallies in lists L_2 and L_3 by listing each distinct outcome in L_2 and its frequency of the outcome in L_3. Have them each plot a histogram of this tally as shown.







When students are finished, display the histograms using **Screen Capture** to share results with the class. Discuss the histograms. Are the shapes the same? Are they spread out as far? Are they centered about the same number?

f. Use **Quick Poll** (with Open Response and Numeric checked) to collect students' results on how many times four universal desks in a room were not enough for all of the left-handers.

g. Click on **Poll Summary** to display the results. Ask students to count the number of responses and find the sum of the responses. Use these values to estimate the probability that four universal desks will be sufficient for a class of 30 students. Discuss the results and solicit opinions as to whether the principal made the correct decision.

Student Page Answers:
The sample lists and results on the teacher and student pages are representative of the results of the activity. No actual answers are provided due to the variable nature of the results.

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Name: _____ Date: _____

NUMB3RS Activity: No Desk Left Behind

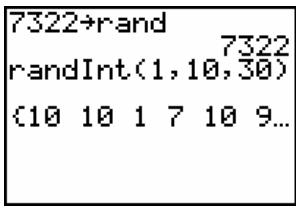
In "Democracy," Charlie calculates the likelihood that four people who knew each other would have died by accident in a two-week period. He determines the result to be 10 million to 1. He later learns that there were actually five people who all knew each other and who died in apparent accidents in that two-week period. He redoes the calculation and gets 700 million to 1. Charlie does not explain the method he uses or what assumptions he makes to estimate these probabilities. One method that could be used to estimate likelihood of an event is simulation. In this activity students estimate, using simulation, the likelihood that more than four students in a class of 30 are left-handed when it is assumed that only 10% of students are left-handed.

1. How many students in your class are left-handed? What is the probability that if a student is picked at random from your class, he or she will be a left-handed student? Estimate the percentage of students in your school who are left-handed.

2. A school is purchasing new classroom furniture. Each classroom will have 30 student desks. There are two models of desks: right-handed and universal. Either left- or right-handed students can conveniently use the universal desks, but they cost four times as much as right-handed desks. The principal believes that on average 10% of the population is left-handed. She wants to accommodate the needs of the students, but also has a limited budget. In your opinion, how many universal desks should she order per classroom?

3. One way to help the principal make an informed decision is to simulate the situation. A student could generate a list of 30 random numbers from the set 1 through 10. Given that 10% of the worldwide population is left-handed, it seems reasonable to expect that about 10% of the students will be left-handed. Students could let the number 1 represent a left-handed student and count the number of 1s that occur among the 30 random numbers.

So that all students generate different sets of random numbers, seed the random number generator by storing a number, like the last four digits of your phone number, to the variable `rand` as shown at right. Press MATH, select **PRB** and choose **5:randInt(**. Enter **randInt(1,10,30)**. This command generates a list of 30 random integers between 1 and 10. The syntax of the command is **randInt(**smallest, largest, number of numbers).



Count the number of 1s that occur in your list. Scroll to the right to see all of the numbers. How does this number compare to your answer to Question 2?

4. Suppose the principal decided to order 4 universal desks per classroom assuming that about 10% of the 30 students will be left-handed, with one extra desk just in case. By how much is the number of left-handed students likely to vary from what you might expect? Will one extra desk be enough? A simulation can be used to estimate this likelihood.

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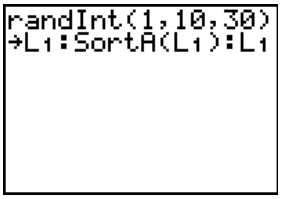
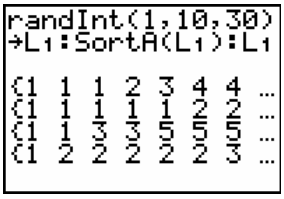
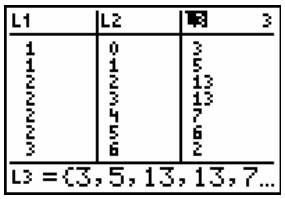
Enter the command shown at right. This command has three steps; it generates a list of 30 random numbers from 1 to 10, sorts them in ascending order, and displays them.

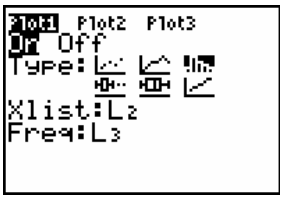


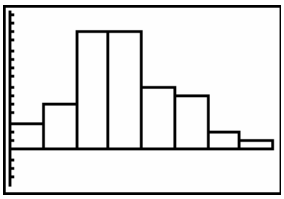
Press ENTER and count the number of 1s appear in the list. Continue to press ENTER for a total of 50 times while making a tally of how many 1s appear in each list. You should have 50 sets of tallies when you are finished.



Enter your tally in lists L_2 and L_3 by listing each distinct outcome in L_2 and its frequency in L_3. Plot a histogram of this tally as shown at right.







Compare your histogram to your classmates' histograms. Are the shapes the same? Are they spread out as far? Are they centered about the same number? How many times, out of 50, were there more than four 1s? Compare this value to the values from your classmates' simulations. Combine the values for all of your classmates and compute an estimation of the probability that four universal desks will be sufficient for a class of 30 students.

5. Based on your results in Question 4, do you think the principal made the correct decision? What would have happened if she had ordered only three left-handed desks?

6. Do you think that if your class repeated this activity you would get the same results?

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Charlie does not actually explain how he calculates the probability of four (and then five) people who know each other dying accidentally in a two-week period. He may have used a process similar to the following.

The accidental death rate in 1997 for 25 – 44 year olds was 31.3 per 100,000; the rate for 45 – 64 year olds was 29.6 per 100,000. Consider 30 per 100,000 to be a good approximation. If there were 1,500 people who knew each other well, then an estimate of the probability of one of them dying by accident in a two-week period is: $1,500 \cdot \frac{30}{100,000} \cdot \frac{2}{52} \approx 0.0173 \approx \frac{1}{58}$. Using this value, the probability of four of them dying in a two-week period is $\left(\frac{1}{58}\right)^4 = \frac{1}{11,316,596} \approx \frac{1}{10,000,000}$, and the probability of five of them dying in a two-week period is $\left(\frac{1}{58}\right)^5 = \frac{1}{656,356,768} \approx \frac{1}{700,000,000}$, which are consistent with Charlie's results.

(Source: **http://gosset.wharton.upenn.edu/mortality/DataSet/AF8.HTM**)

For the Student

In this activity, the principal believed that 10% of the population was left-handed. Others have different opinions. The source listed below states that the most common estimate is that 13% of the population is left-handed. Design a simulation for this activity that would reflect this value. With this value, how many universal desks should the principal order for a classroom of 30 students?

http://www.faqs.org/faqs/lefty-faq

Students in statistics classes often design and study simulations. These simulations help tell which outcomes are likely to occur by chance and what it means to have rare (or surprising) outcomes. For a more thorough exploration of this process using variability, see the *NUMB3RS* activity "Thumbs Up!" for Episode 217, "Mind Games." To download this activity, go to **http://education.ti.com/exchange** and search for "6659."

Additional Resources

Texas Instruments' Probability Simulation application for TI-83 Plus/TI-84 Plus graphing calculators gives another way of generating random integers. If not already on the calculator, it can be downloaded for free from: **http://education.ti.com/us/probsim**

For additional related probability activities, see:

Barrett, Gloria, Rick Billstein, Henry Krandondonk, Roxy Peck, and Michael Shaughnessy. *Navigating Through Probability for Grades 9 – 12*. J. Lott (ed.). Reston, VA: National Council of Teachers of Mathematics, 2004.

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