# Exploring Space Through MATH
*Applications in Algebra 1*

EDUCATOR EDITION

## Space Shuttle Ascent: Mass vs. Time

Note: This problem is related to the Algebra 1 problem, *Space Shuttle Ascent: Altitude vs. Time*, in the Exploring Space Through Math series.

### Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, Evaluate) will be used to accomplish the following objectives.

Students will
- create scatter plots from a data table;
- determine correlation and interpret its meaning;
- find linear regression equations;
- find the slope and *y*-intercept from a linear equation; and
- communicate the meanings of slope and *y*-intercept as they relate to a real-world problem.

### Prerequisites

Students should have prior knowledge of scatter plots, types of correlations, linear equations (slope and *y*-intercept) and linear graphs. Students should also have experience using a graphing calculator or spreadsheet application to create scatter plots and to find linear regression equations.

### Background

*This problem is part of a series that applies algebraic principles in NASA's human spaceflight.*

The Space Shuttle Mission Control Center (MCC) and the International Space Station (ISS) Control Center use some of the most sophisticated technology and communication equipment in the world. Teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle and the ISS. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support each mission and crew during normal operations and any unexpected events.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the ISS. NASA plans to retire the space shuttle by 2010, but until then space exploration depends on the continued success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.

**Key Concept**
Modeling data with linear regression equations

**Problem Duration**
90 minutes

**Technology**
TI-84 graphing calculator or spreadsheet application, computer with projector, video of space shuttle launch

**Materials**
- Student Edition
- Space Shuttle Launch Video (provided)

**Degree of Difficulty**
Moderate to Difficult

**Skills**
Use a graphing calculator to create scatter plots, find regression equations; interpret graphs and equations of linear data

**NCTM Standards**
- Number and Operations
- Algebra
- Data Analysis and Probability
- Problem Solving
- Communication
- Connections

The ascent phase begins at liftoff and ends when the space shuttle reaches Earth's orbit. The space shuttle must accelerate from zero to approximately 7,850 meters per second (which is approximately 17,500 miles per hour) in eight and a half minutes to reach the minimum altitude required to orbit Earth. It takes a very unique vehicle to accomplish this task.

There are three main components of the space shuttle that enable the launch into orbit (Figure 1). The main component is the orbiter. It not only serves as the crew's home in space and is equipped to dock with the ISS, but it also contains maneuvering engines for finalizing the orbital trajectory, or flight path. The External Tank (ET), the largest component of the space shuttle, supplies the propellant (liquid oxygen and liquid hydrogen) to the Space Shuttle Main Engines (SSMEs) which are liquid propellant rocket engines. The third component is a pair of Solid Rocket Boosters (SRBs) which are reusable. They are attached to the sides of the ET and provide the main thrust at launch (Figure 2).

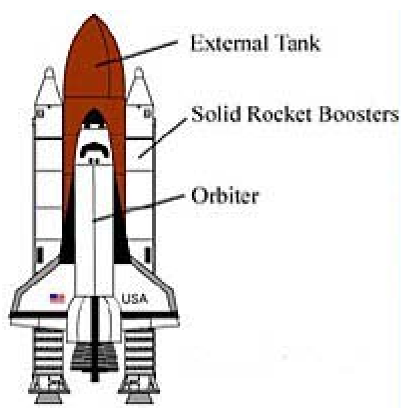



*Figure 1: Main components of the space shuttle*

*Figure 2: Space Shuttle Discovery at liftoff*

One of the flight controllers in the Space Shuttle Mission Control Center is the Booster Engineer. This position, or console, known as Booster, is in charge of monitoring the SSMEs, the SRBs, and the ET during the countdown and the ascent phase, until all of those systems are safe. The components of the space shuttle experience changes in position, velocity, and acceleration during the ascent into space. These changes can be seen by taking a closer look at the entire ascent process (Figure 3).

The ascent process begins with the liftoff from the launch pad. Propellant burns from the SRBs and the ET causing the space shuttle to accelerate very quickly. This high-rate of acceleration causes a rapid increase in dynamic pressure, known as *Q* in aeronautics (sometimes called velocity pressure). As the space shuttle breaks the sound barrier, its structure can only withstand a certain level of dynamic pressure before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 67% of full power to avoid damage. About 50 seconds after liftoff, the dynamic pressure reaches its maximum aerodynamic load (Max *Q*). The air density then drops rapidly due to the thinning atmosphere, and the space shuttle can be throttled to full power without fear of structural damage. The command is given, "Go at throttle up!"

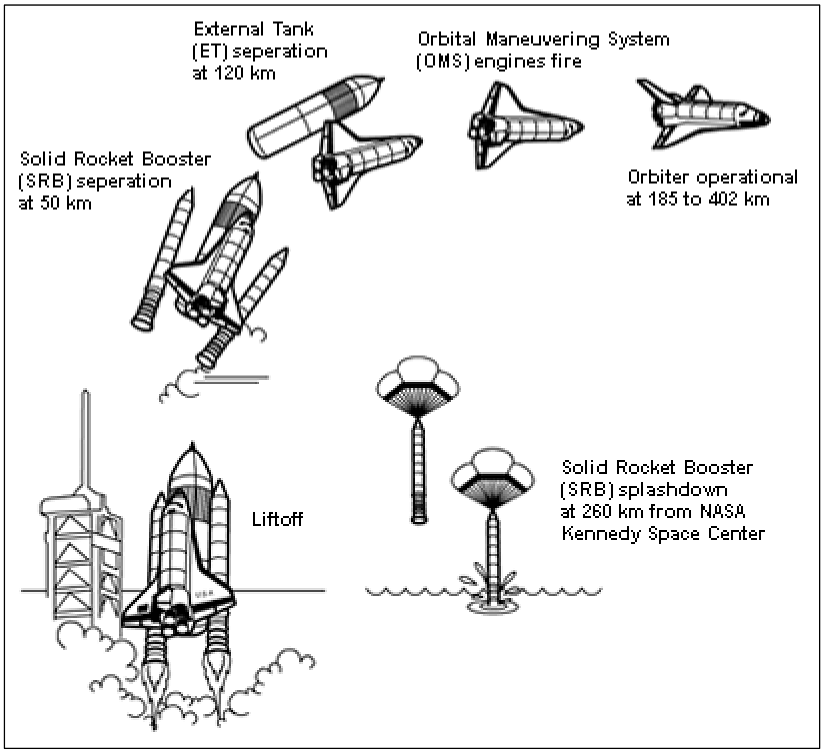



*Figure 3: Space shuttle ascent process*

As the space shuttle climbs, the velocity is increasing and the density of the air is decreasing. About 2 minutes after liftoff the atmosphere is so thin that the dynamic pressure drops to near zero. The SRBs, having used their propellant, are commanded by the space shuttle's onboard computer to separate from the ET. The jettisoning of these booster rockets marks the end of the first ascent stage and the beginning of the second. The spent SRBs fall into the ocean and are recovered, refurbished, reloaded with propellant, and reused for several missions. The second stage of ascent lasts about six and a half minutes, during which time the Booster flight controller continues to monitor the ET and the SSMEs until MECO (Main Engine Cut Off) and ET Separation. The ET re-enters the Earth's atmosphere, breaking up before impact in the ocean, and the space shuttle maneuvers into orbit. This lesson focuses on the first ascent stage, which occurs during the first two minutes after liftoff.

## NCTM Principles and Standards

### Number and Operations

- Develop a deeper understanding of very large and very small numbers and of various representations of them.

### Algebra

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Use symbolic algebra to represent and explain mathematical relationships.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.

- Approximate and interpret rates of change from graphical and numerical data.

**Data Analysis and Probability**

- For bivariate measurement data, be able to display a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

**Problem Solving**

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

**Communication**

- Use the language of mathematics to express mathematical ideas precisely.

**Connections**

- Recognize and apply mathematics in contexts outside of mathematics.

## Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

**1 – Engage** (20 minutes)

- Either assign or let students choose small groups of 3-4 in order to work through this activity. Each group should review the background section for several minutes to be sure that they understand the material. Ask if any groups need clarification.
- Distribute the worksheet, Video – Space Shuttle Ascent: Mass vs. Time.
- This video shows the launch of Space Shuttle Discovery STS 121 mission on July 4, 2006. To access the video, follow the link provided and choose the video titled: "The Rocket's Red Glare." The video lasts 10 minutes and covers both ascent phases. To view only the first ascent phase, stop the video at 3.5 minutes. http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/sts121/launch/sts121-allvideos.html.
- Instruct students to write their answers to the questions on the worksheet as they watch the video.
- In their small groups, students discuss and share their answers to the questions. Circulate to help facilitate discussion in small groups.

**2 – Explore** (20 minutes)

- Distribute graphing calculators (a TI-84 is used for this problem). If necessary, review some of the steps on entering lists and finding regression equations. As an alternative to the calculator, a spreadsheet application may be used to analyze the data.
- Distribute the worksheet, Regression Equations.
- Ask students to work through questions 1-3 as a team.
- Call on students to give their answers and discuss. If available, students can use presentation technology to demonstrate the graphing calculator aspects of the questions.

**3 – Explain** (15 minutes)

- Have the students work with their team to answer questions 4-5.
- Call on students to give their answers and discuss. If available, students can use presentation technology to demonstrate the graphing calculator aspects of the questions.

**4 – Extend** (15 minutes)
- Have the students work with their team to answer question 6.
- Call on students to give their answers and discuss. If available, students can use presentation technology to demonstrate the graphing calculator aspects of the questions.

**5 – Evaluate** (20 minutes)
- Have students complete questions 7-10 individually.

## ENGAGE

## Video – Space Shuttle Ascent: Mass vs. Time

### Solution Key

View the video, *Space Shuttle Ascent-Clip_ STS-121*, and reference the Background section to answer the following questions:

1. In the launch of the Space Shuttle Discovery what does ”auto sequence start” mean?
   Note: In the video, right after the statement, “And we have a go for auto sequence start.”, the next statement we hear is, “Discovery's onboard computers have primary control of all the vehicle's critical functions.”

2. What are the 3 main components of the space shuttle?
   The orbiter, the external tank (ET), and the solid rocket boosters (SRBs).

3. Which component is the first to ignite? Which component ignites next and what is the result?
   The orbiter’s main engines are the first to ignite. Then the solid rocket boosters ignite causing liftoff.

4. At about 40 seconds after launch, due to the velocity of Discovery, what occurs?
   Discovery breaks the sound barrier.

5. At about 40 seconds after launch, why do the main engines throttle back to about 67% of rated performance?
   To reduce the structural stress on the space shuttle as it breaks the sound barrier.

6. At about 1 minute into the flight, what command is given, and what does this mean?
   “Discovery Houston, Go at throttle up!” meaning the Discovery engines return to full power.

7. How is it possible for Discovery to fly at full power again after the 1 minute point?
   The atmosphere is so thin that there is no danger of structural damage due to dynamic pressure.

8. What occurs at approximately 2 minutes into flight?
   SRB separation.

9. What happens to the depleted SRBs?

   The SRBs are recovered from the ocean, refurbished, reloaded with propellant, and reused for several missions.

10. What powers Discovery after SRB separation?

    The orbiter's 3 main engines burning propellant from the external tank.

## EXPLORE

## Regression Equations

Solution Key

### Problem

On July 4, 2006 Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121, to begin a rendezvous with the International Space Station, or ISS. Before each mission, the projected data is compiled to assist in the launch of the space shuttle to ensure safety and success during the ascent. To complete this data, flight design specialists take into consideration a multitude of factors such as space shuttle mass, propellant used, mass of payload being carried to space, and mass of payload returning. They must also factor in atmospheric density, which is changing throughout the year. After running multiple tests, information is compiled in a table showing exactly what should happen each second of the ascent.

The Booster flight controller monitors system health for the Space Shuttle Main Engines (SSMEs), the Solid Rocket Boosters (SRBs), and the External Tank (ET). They monitor pressures, temperatures, propellant flow rates, and valve positions that show that the engines are running and controlling properly. Booster also monitors all the pipes and valves that move propellant from the ET to the SSMEs. Propellant flow rates are significant because they determine how mass changes over time which affects acceleration.

*Table 1: STS-121 Discovery Ascent data (total mass)*

| **Time** (s) | **Space Shuttle Total Mass** (kg) |
|---|---|
| 0 | 2,051,113 |
| 10 | 1,935,155 |
| 20 | 1,799,290 |
| 30 | 1,681,120 |
| 40 | 1,567,611 |
| 50 | 1,475,282 |
| 60 | 1,376,301 |
| 70 | 1,277,921 |
| 80 | 1,177,704 |
| 90 | 1,075,683 |
| 100 | 991,872 |
| 110 | 913,254 |
| 120 | 880,377 |

Table 1 shows the total mass of Discovery for mission STS-121 every 10 seconds from liftoff to SRB separation. Total mass includes the orbiter, SRBs, ET, propellant, and payload. It is during the first stage of the ascent, that the space shuttle is burning the greatest amount of propellant. You can see in the table that the space shuttle has a total mass of 2,051,113 kg at $t = 0$. After 2 minutes its total mass is only 880,377 kg, or 43 % of the original mass. The burning of this vast amount of propellant is needed to get the space shuttle through Earth's atmosphere and into orbit.

***Directions: Answer questions 1 – 6 in your group. Discuss answers to be sure everyone understands and agrees on the solutions.***

Use the graphing calculator to analyze the data from flight STS-121. To enter the data press the **STAT** button and select the option **1: Edit**. Enter the times in seconds into **L1** and enter the total mass values in kilograms in **L2**. (Directions are for a TI-84 series calculator. Consult user manual for other models.)

1. Determine appropriate ranges and scales for the viewing window.
    a. Look at the range of values in the Time column. What are reasonable numbers for **Xmin** and **Xmax**?

       **Xmin** = 0 and **Xmax** = 120.

    b. Considering the difference between any two consecutive times, what is a reasonable **Xscl** value?

       **Xscl** = 10.

    c. Look at the range of values in the Total Mass column. What are reasonable numbers for **Ymin** and **Ymax**?

       **Ymin** = 850,000 and **Ymax** = 2,100,000.

    d. Since these numbers are quite large and in order to have visible space between the tick marks on the *y*-axis, what is a reasonably large number for **Yscl**?

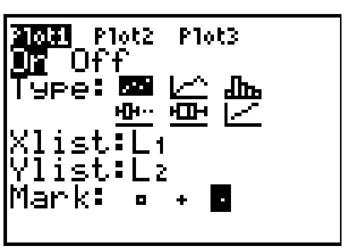
       **Yscl** = 100,000.

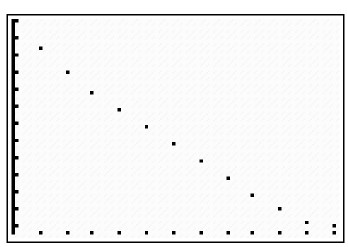
    e. Based on the data in Table 1, predict how the graph will look.

       Answers will vary. The graph will fall from left to right.

2. To create a scatter plot of the total mass vs. time, go to **STAT PLOT** (**2ND**, **Y=**). Select **Plot 1** and press **ENTER.** Select On by pressing **ENTER,** and select scatterplot for Type. Choose a period for Mark, and press **GRAPH**. Describe the scatter plot and explain why it could be represented by a linear function.

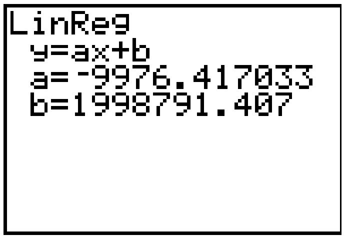
   The points lie in a line that is falling from left to right. Because the points are in a straight line, the data is linear.



3. Find the equation of the line that best fits the data. On a TI graphing calculator you will find the equation by pressing the **STAT** key and selecting the **CALC** menu. Since the data is linear,

select option **4: LinReg(ax+b)** and press **ENTER**. Use the given information to write the function of mass vs. time in function notation. To represent time use the variable *t* and round coefficients and constants to the nearest whole number.

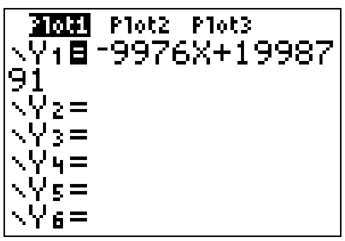


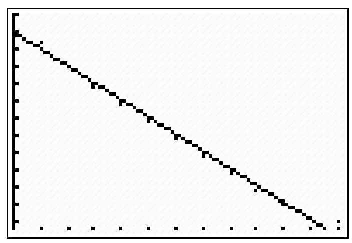

$$f(t) = -9{,}976t + 1{,}998{,}791$$

## EXPLAIN

### Solution Key

4. Enter the function in Y1 and graph it. Compare it to the scatter plot as you answer the following questions.



   a. Does the line fit the data? How can you tell?

      Yes, because the points of the scatter plot appear to be on the line or are very close to it on either side.

   b. What is the correlation of the data (positive, negative, constant, or no correlation)? Explain this in terms of the problem.

      The graph shows a negative correlation. This is because, as time passes, propellant is being burned which causes the mass of the space shuttle to decrease.

   c. How much propellant does Discovery burn per second? Explain what this represents with regard to the graph of the equation.

      The space shuttle burns 9,976 kg of propellant per second. This represents the slope of the line is which is –9,976.

   d. What is the *y*-intercept of the equation found in question 3? Explain what this represents with regard to the space shuttle.

      The *y*-intercept is 1,998,791. This represents the mass in kilograms of the space shuttle at liftoff, or when $t = 0$.

5. Table 2 shows several familiar objects and the approximate mass in kilograms of each one.

   a. To gain perspective regarding the magnitude of the propellant consumption of the space shuttle, find the approximate number of each type of object that it would take to equal the mass of the entire space shuttle system at launch. Round to the nearest whole number.

*Table 2: Mass of various Objects*

| Object | Approximate mass (kg) | Approximate number of objects to equal the mass of the space shuttle at launch |
|---|---|---|
| Statue of Liberty | 204,117 | **10** |
| Boeing 747 airplane | 158,757 | **13** |
| Fuel tank truck | 27,216 | **75** |
| School bus | 11,340 | **181** |

b. The slope of the best fit line that you found in question 4c is the amount of propellant in kilograms that is burned per second. If a Boeing 767 airplane burns about 24,500 kg of fuel on a 6 hour flight from New York to Los Angeles, about how much time would it take the space shuttle to burn an equivalent amount of propellant?

$$\frac{24{,}500}{9{,}976} \approx 2.5 \text{ seconds}$$

**EXTEND**

Solution Key

6. Consider the data set in Table 1. When time is zero seconds, the total mass shown in the table is the actual value of the mass at liftoff. The equation of the line of best fit that you found In question 3 models the data in Table 1. However, a model of any data set contains some error.

   a. Use the equation for the line of best fit that you found in question 3 to find the estimated value of the mass in kilograms when $t = 0$ at liftoff. Label your answer "estimated value".

   $$f(t) = -9{,}976t + 1{,}998{,}791$$
   $$\text{f}(0) = -9{,}976(0) + 1{,}998{,}791$$
   $$\text{f}(0) = 1{,}998{,}791 \quad \text{estimated value}$$

   b. Use the following formula to determine the percent error in the equation of the line of best fit. Round to the nearest tenth.

   $$\text{Percent Error} = \left|\frac{\text{Actual value} - \text{Estimated value}}{\text{Actual value}}\right| \bullet 100$$

   $$\text{Percent Error} = \left|\frac{2{,}051{,}113 - 1{,}998{,}791}{2{,}051{,}113}\right| \bullet 100$$

   $$\text{Percent Error} = 2.6\%$$

## EVALUATE

### Solution Key

***Directions: Answer questions 7-10 independently.***

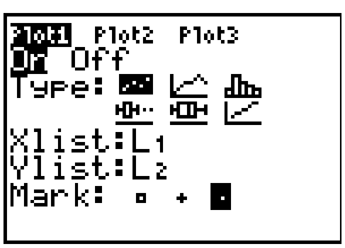
On May 12, 2009, Space Shuttle Atlantis launched from Kennedy Space Center on mission STS-125 to repair the Hubble Space Telescope. Astronauts installed two new instruments, repaired two inactive ones, and performed the component replacements to keep the telescope functioning into at least 2014. Table 3 shows the total mass of Atlantis for mission STS-125 every 10 seconds from liftoff to SRB separation.

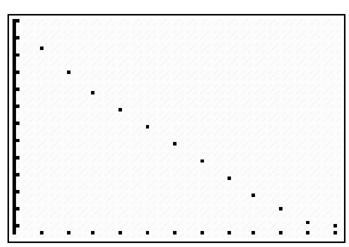
*Table 3: STS-125 Atlantis Ascent data (total mass)*

| **Time** (s) | **Space Shuttle Total Mass** (kg) |
|---|---|
| 0 | 2,049,780 |
| 10 | 1,932,475 |
| 20 | 1,795,086 |
| 30 | 1,676,053 |
| 40 | 1,562,508 |
| 50 | 1,468,886 |
| 60 | 1,374,449 |
| 70 | 1,264,663 |
| 80 | 1,163,639 |
| 90 | 1,061,679 |
| 100 | 978,131 |
| 110 | 902,427 |
| 120 | 874,457 |

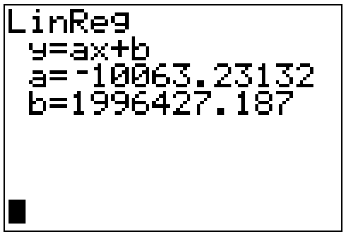
7. Enter the data from Table 3 in your calculator. Adjust your viewing window to accommodate the range of entries in the table. Create a scatter plot of the total mass vs. time and graph it. What type of function would best fit the data? Describe the scatter plot.

   It is a linear function. The points lie in a straight line that is falling from left to right.



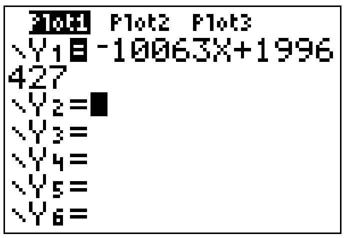



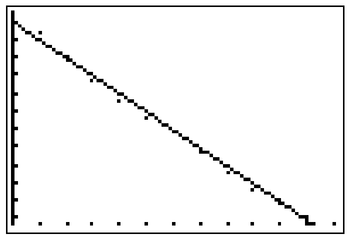
8. Find the equation that best fits the data. Using function notation, write this as function of total mass vs. time? Use the variable *t* to represent time and round coefficients and constants to the nearest whole number.




$f(t) = -10{,}063t + 1{,}996{,}427$

9. Enter the function in Y1 and graph it. Compare it to the scatter plot to answer the following questions.



   a. Does the line fit the data? How can you tell?

      The line fits the data because the points of the scatter plot appear to be on the line or are very close to it on either side.

   b. What is the correlation of the data (positive, negative, constant, or no correlation)? Explain this in terms of the problem.

      The graph shows a negative correlation. This is because, as time passes, propellant is being burned which causes the mass of the space shuttle to decrease.

   c. What is the slope of the equation found in question 8? Explain what this represents with regard to Atlantis.

      The slope is -10,063. This means that the space shuttle is burning 10,063 kg of propellant per second.

   d. What is the *y*-intercept of the equation found in question 8? Explain what this represents with regard to the space shuttle.

      The *y*-intercept is 1,996,427. This represents the mass in kilograms of the space shuttle at liftoff, or when $t = 0$.

10. Compare the rate of burn of propellant (slope) for missions STS-121 (Discovery) and STS-125 (Atlantis) in questions 4c and 9c.

   a. Which space shuttle has the fastest burn rate?

      Atlantis.

   b. What do you think the reason might be?

      Answers will vary. The payload in Atlantis is heavier.

## Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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