Relating Rates

ID: 11643

Time Required
45 minutes

Activity Overview

In this activity, students are introduced to the topic or related rates. The process is explained and dynamic visuals are provided. Self-check multiple choice questions help students build understanding and put what they learned into practice with homework or extension questions.

Topic: Related Rates

- *Applications of implicit differentiation*
- *Formula relating to real world phenomena*

Teacher Preparation and Notes

- *Students can write their responses directly into the TI-Nspire handheld or on the accompanying handout. On self-check questions, students enter their response and press **MENU** > **Check Answer** (or press ctrl +▲).*
- *This activity can be completed with or without the use of CAS functionality. If CAS functionality is not used, then students should complete the implicit differentiation and unit conversion by hand.*
- *The activity is designed to be a student-centered discovery and instruction of related rates. At the conclusion of the activity, reinforce the method for solving these problems. After completing the activity, students should be more successful with AP* questions like 2002 formB AB6, 1999 AB6, 96BC5&6, 95AB5 BC3, 94AB5 BC2, 92AB6, 92 BC5, and multiple -choice questions 1998AB90, AB&BC78*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "11643" in the quick search box.***

Associated Materials

- *RelatingRates_Student.doc*
- *RelatingRates.tns*

Suggested Related Activities

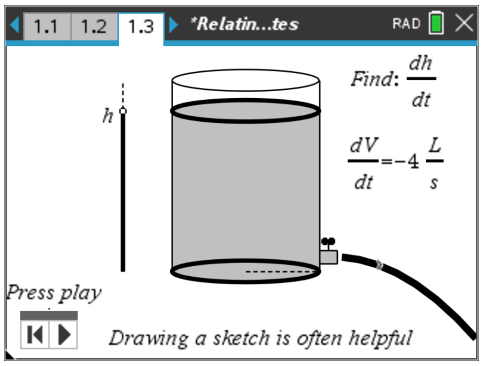
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *The Falling Ladder (TI-Nspire CAS technology) — 8318*
- *Related Rates (Learning Check) — 7080*

Problem 1 – Example & Explanation

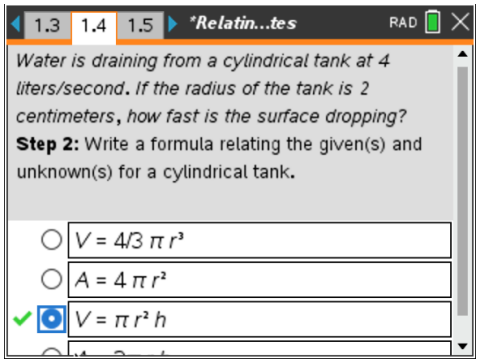
Students are given a situation of water draining out of a cylindrical tank. This problem serves as the basis for explaining the process of solving related rates questions.

Students have difficulty realizing that if the volume is decreasing, the rate is negative.

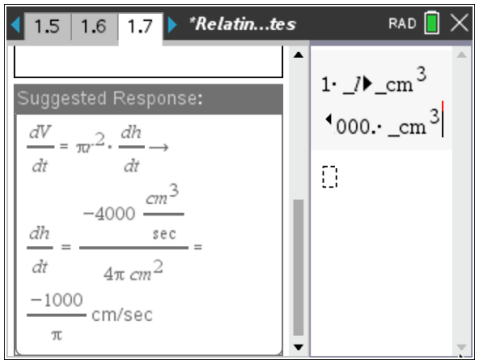


Another difficulty students have is remembering an appropriate formula that relates what they know and what they want to find. The multiple-choice question on page 1.4 can serve as a review of other formulae.

Students often either want to treat *r* and *h* as variables when one of them is not changing, or they treat one as a constant when it is really changing with respect to time. The question associated with Step 3 brings that out.



Another issue students need to pay attention to is units. The unit conversion capabilities of the CAS functionality are utilized on page 1.7.



Student Solutions

Step 1: Variables: height *h*, radius *r*, volume *V*, time *t*

Given information: $\frac{dV}{dt} = -4$ L/s, $r = 2$

Unknown(s): $\frac{dh}{dt}$

Step 2: $V = \pi r^2 h$

Step 3: No, *r* is constant; $\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$

Step 4: $\frac{-1000}{\pi}$ cm/sec

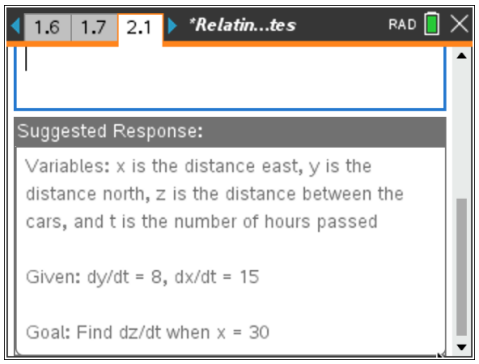
TI-Nspire Navigator Opportunity: *Screen Capture*

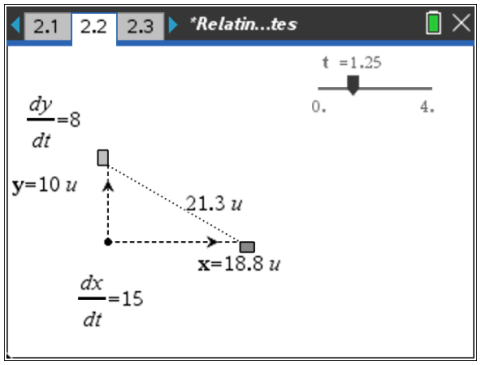
See Note 1 at the end of this lesson.

Problem 2 – Additional Example & Explanation

Implicit differentiation and the steps for related rates questions are exemplified again. The steps are presented in an open response self-check format. The example of two cars leaving a single point is a typical Pythagorean problem that needs to be carefully differentiated implicitly so as not to forget the Chain Rule.

To add more depth to this question, ask the students if they would get the same answer if the car going to the east actually started from 60 units away and traveled west at 15 units/hour. Ask them to explain their reason.





Student Solutions

Step 1: Variables: x is the distance east, y is the distance north, z is the distance between the cars, and t is the number of hours passed

Given information: $\frac{dy}{dt} = 8$, $\frac{dx}{dt} = 15$

Goal: $\frac{dz}{dt}$ when $x = 30$

Step 2: $x^2 + y^2 = z^2$

Step 3: $2x \cdot x' + 2y \cdot y' = 2z \cdot z'$

Step 4: $2 \cdot 30 \cdot 15 + 2 \cdot 16 \cdot 8 = 2 \cdot 34 \cdot \frac{dz}{dt}$

17 units

TI-Nspire Navigator Opportunity: *Quick Poll*

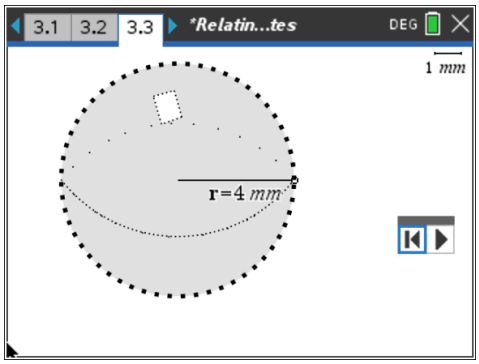
See Note 2 at the end of this lesson.

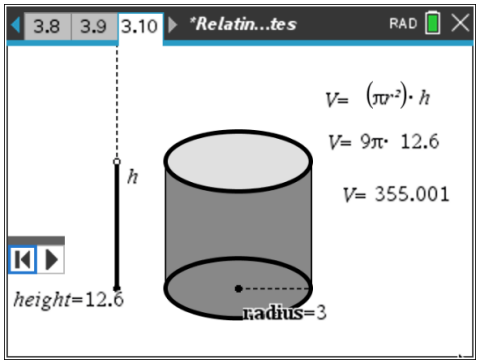
Problem 3 – Homework/Extension

Students apply what was learned to various real-world and graphical problems.

Exercises 1, 3, and 6 have dynamic illustrations constructed. These constructions can be used to either confirm solutions or to enhance conceptual understanding of the questions.

If needed, students can press ⌨ to use the *Scratchpad* to help them with their calculations. Solutions are available for self-check.





Student Solutions

1. $\frac{1}{4\pi}$ mm/sec
2. decreasing 0.5 units/sec
3. –36
4. -160π cm^2/hr
5. approximately 28.83 mph
6. 70.686 mL/s

TI-Nspire Navigator Opportunities

Note 1
Problems 1-3, *Screen Capture*
Throughout the lesson, you may use screen capture to verify students are able to run the applications and correctly answer the questions.

Note 2
Problems 1-3, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.