Investigating Properties of Kites

Definition:

Kite—a quadrilateral with two distinct pairs of consecutive equal sides (Figure 1).

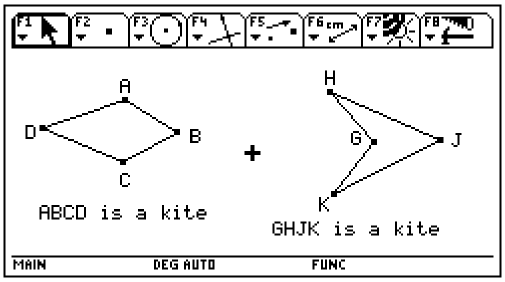

Figure 1

Construct and Investigate:

1. Determine three ways to construct a kite by using the Voyage™ 200. Test your constructions by dragging independent points of the kite to be sure that the figures remain kites under all conditions.
2. For each construction, determine the conditions under which the kite is convex and nonconvex.
3. Conjecture as to the type of quadrilateral formed by connecting consecutive midpoints of the four sides of a kite. Test your conjecture by constructing such a quadrilateral, using a kite formed by one of your methods. Does your conjecture hold true when the kite is nonconvex?
4. Draw the diagonals of a kite. List as many properties as possible that appear to be true about the diagonals of kites. Be sure to indicate properties that are true for both convex and nonconvex kites, as well as properties that are true for only convex kites.
5. Investigate the more specialized kites: the rhombus and the square. Verify that all properties of kites also appear to be true for these quadrilaterals.

Explore:

1. Construct a kite using one of your methods. Measure its area. Is there a relationship between the lengths of the diagonals of a kite and the area of the kite? Test your conjecture on a variety of kites by dragging vertices of the kites. Write an explanation that proves your conjecture is true for all kites. Does your method hold true when the kite is nonconvex? Is this relationship between the diagonals and area true for any other types of quadrilaterals?
2. Inscribe kite ***EFGH*** in rectangle ***ABCD*** so that ***H*** and ***F*** are at the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively (Figure 2).

   What relationship exists between the area of the kite and the area of the rectangle? How does this relationship change as you slide ***E*** and ***G*** along the sides of the rectangle? If the area changes, when does it reach its maximum value? Explain.

   As you slide ***E*** and ***G*** along the sides of the rectangle, how does the perimeter of the kite change? If the perimeter changes, when does it reach its smallest value? Explain.

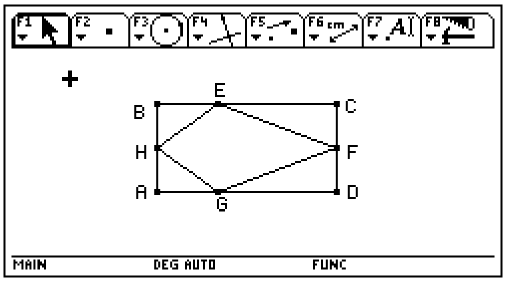

Figure 2

Teacher's Guide: Investigating Properties of Kites

Construct and Investigate:

1. This part of the exploration may be quite challenging for students, depending on their mathematical background, creativity, and ability to persevere during trial and error. Collaborative efforts among groups of students are appropriate for this activity. Finding more than one way to accomplish a task often requires students to think at higher cognitive levels. It also results in a better understanding of the concepts being studied.

   The following are among the methods that can be used to construct a kite:

   - Construct two intersecting circles. The quadrilateral that connects the centers of the circles to the points of intersection of the circles is a kite.
   - Construct a segment and its perpendicular bisector. Place two distinct points on the perpendicular bisector, and connect these points with the endpoints of the segment to form a kite.
   - Construct two isosceles triangles that share a common base and have two distinct third vertices. The pairs of equal sides of the isosceles triangles form a kite (excluding the base).
   - Construct two segments that share a common endpoint and do not form a right angle. Draw a reflection line through the two endpoints that are not shared between the segments. Reflect the two segments over this line. The resulting quadrilateral is a kite.
   - Construct a rectangle. Choose the midpoints of two opposite sides of the rectangle as two vertices of the kite. Draw a line parallel to the two chosen sides. This line will intersect either the rectangle or the extension of two sides of the rectangle. The quadrilateral formed by connecting these intersection points to the midpoints of the two sides of the rectangle is a kite.
   - Reflect any triangle over any one of its sides. The two remaining sides and their reflections form a kite.

2. Referring to the methods described in part 1 above, the kite will be nonconvex *if*

   - the centers of the circles are on the same side of the segment that connects their points of intersection.
   - the two points on the perpendicular bisector are on the same side of the segment.
   - the two isosceles triangles share a common base, but one lies inside the other.
   - the two noncommon endpoints are on the same side of the line that is perpendicular to the reflection line that contains the common endpoint of the segments.
   - the parallel line intersects the extensions of the sides of the rectangle and not the sides themselves.
   - the altitude to the reflection side crosses an extension of the side of the triangle and not the side itself.

3. The quadrilateral that connects consecutive midpoints of a kite will always be a rectangle (Figures 3 and 4).

   Draw either diagonal of a kite. The midsegments of the triangles formed by the diagonals of a kite are parallel to the base (the diagonal) and, therefore, are parallel to each other. This argument holds true for both diagonals, producing two pairs of congruent parallel lines. The result is an inscribed parallelogram.

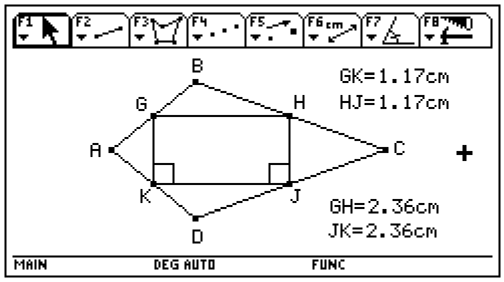

Figure 3

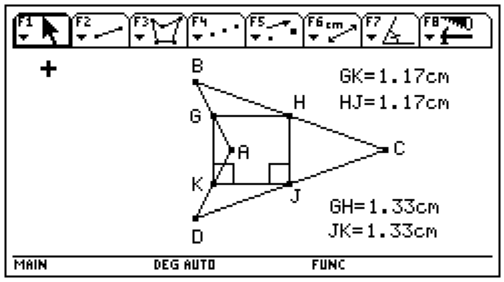

Figure 4

   One way to define a kite is the sides of two isosceles triangles sharing a common base. By this definition, the diagonal connecting the third vertex of each isosceles triangle is the perpendicular bisector of the base, the other diagonal. Therefore, the diagonals are perpendicular and the parallelogram formed by the midsegments is a rectangle (Figures 5 and 6).

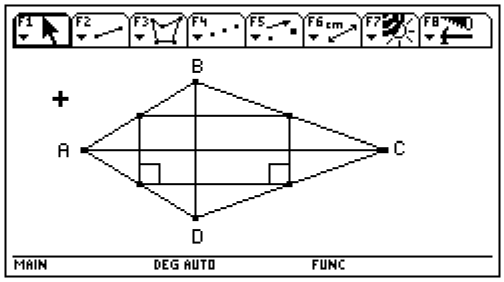

Figure 5

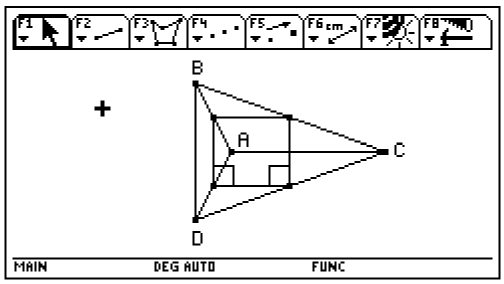

Figure 6

4. The diagonals of a kite are always perpendicular. In kite ***ABCD***, $\overline{AC}$ bisects $\triangle BAD$, $\triangle BCD$, and $\overline{BD}$, and lies on the line of symmetry of the kite. One-half the product of the lengths of the diagonals equals the area of the kite (Figure 7).

5. All properties of kites are also properties of rhombi and squares because these figures are special cases of kites. (Students may wish to find properties of these figures that are not properties of kites.)

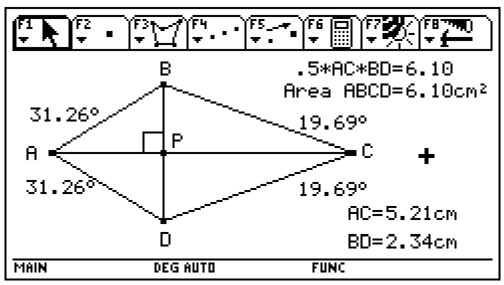

Figure 7

Explore:

1. The area of a kite is always equal to one-half the product of the lengths of its diagonals (Figures 7 and 8). Because $\triangle ABC \cong \triangle ADC$, the two triangles have the same area. Together these two triangles make up the area of kite ***ABCD***.

   Because $\overline{AC}$ bisects $\overline{BD}$, $BP = ½ BD$.

   The area of $\triangle ABC = ½ AC * BP$ (Figure 7).

   Thus, the area of kite $ABCD = 2 *$ the area of $\triangle ABC = 2 * ½ * AC * ½ * BD = ½ AC * BD$.

   This is true for convex as well as nonconvex kites. Because both a rhombus and a square are kites, the areas of these figures equal one-half the product of their diagonals. You can prove this algebraically using symbolic algebra on the Voyage™ 200.

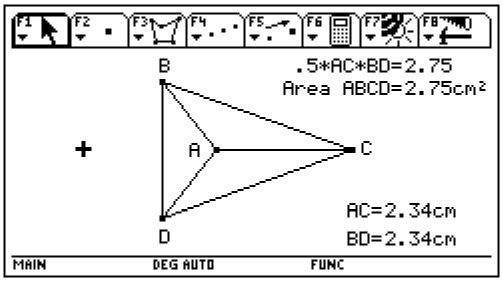

Figure 8

2. The area of the kite is always one-half the area of the rectangle, regardless of the location of the moving vertices of the kite (Figure 9). In this construction, $\overline{HF}$ is parallel and equal to $\overline{AD}$, the length of the rectangle; $\overline{EG}$ is parallel and equal to $\overline{AB}$, the width of the rectangle.

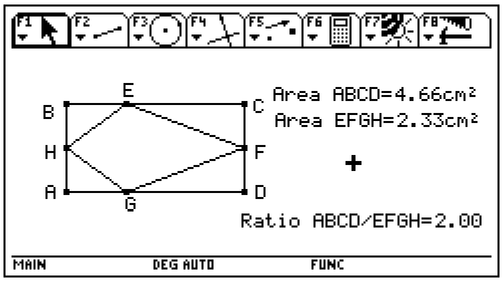

Figure 9

   By substitution, the area of kite $EFGH = ½ * AD * AB$. This is also exactly half the area of rectangle ***ABCD*** because the area of rectangle $ABCD = AD * AB$.

   The perimeter of kite ***EFGH*** will change as vertices ***E*** and ***G*** are moved. The perimeter minimizes when these vertices are moved to the midpoints of the two sides of the rectangle, forming a rhombus (Figure 10). The perimeter maximizes when ***G*** coincides with ***A*** or ***D*** as the kite becomes an isosceles triangle.

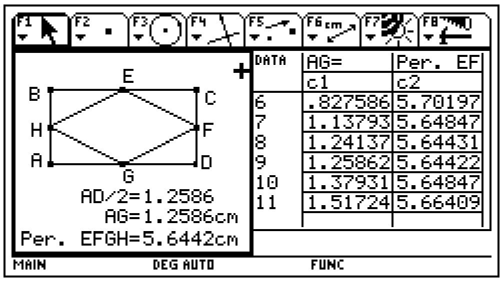

Figure 10

   Figure 11 shows a quadratic function (computed using the **QuadReg** tool) that models the data gathered on the length of $\overline{AG}$ and the perimeter of ***EFGH***.

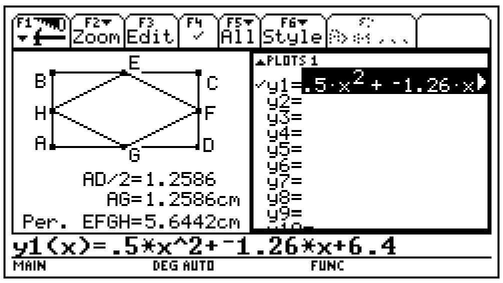

Figure 11

   Figure 12 shows the graph of the quadratic function in Figure 11, drawn over the scatter plot of the data. The minimum perimeter of kite ***EFGH*** is approximated by the minimum point of the function.

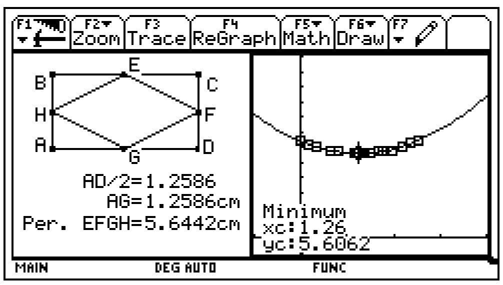

Figure 12

