Entropy – ID: 9263

Topic: Heat and Thermodynamics

- *Describe and calculate the entropy of a system.*
- *Describe the relationship between entropy and disorder.*

Activity Overview

In this activity, students will investigate entropy from the point of view of microstates, macrostates, and probability. The goal of the activity is to help students better understand the relationship between probability and entropy. Students will learn that, in the case of the second law of thermodynamics, it is not about what is possible, but rather what is most probable.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes, Data & Statistics

Teacher Preparation

Before carrying out this activity, review the differences between microstates and macrostates. Go over the relationship between entropy and disorder. Discuss how disorder is related to probability. Review the second law of thermodynamics.

- *The screenshots on pages 2–6 demonstrate expected student results. Refer to the screenshots on pages 7–9 for a preview of the student TI-Nspire document (.tns file.)*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9263" in the search box.***

Classroom Management

- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:
- How is it that microstates have equal probabilities and macrostates have unequal probabilities?
- What is the relationship between the number of microstates and probability?
- What is the relationship between the number of microstates and entropy?
- In what way is the second law of thermodynamics all about probabilities?

Students will determine number of microstates and macrostates for different numbers of particles in a box. They will then graph the exponential increase of microstates as the number of particles increases. Finally, they will relate entropy, disorder, and the most probable macrostate of a system.

Part 1 – Microstates and macrostates

Step 1: Students should open the file **PhysWeek03_Entropy.tns** and read the first three pages. Page 1.4 shows a box divided into two sides with a particle on each side. Students should manipulate the particles in the box and then answer questions 1 and 2.
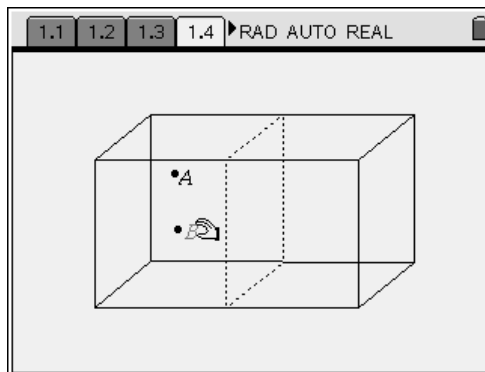
Q1. In terms of the two sides of the box, how many different possible arrangements are there for the two particles? What are they?

A. *There are four possible arrangements: the initial state, with A on the left and B on the right; B on the left, A on the right; A and B both on the right; and A and B both on the left.*

Q2. Which microstate is most probable?

A. *Each microstate is equally probable.*

Step 2: Next, students should read pages 1.6 and 1.7, which describe the possible macrostates for the two-particle scenario. Then, they should move to page 1.8, which contains a *Lists & Spreadsheet* application. In the spreadsheet, each row describes a different macrostate. Note: Students may struggle to understand how there can be a different number of microstates than macrostates for this situation. To help them grasp this concept, you may have them delete the labels on the particles on page 1.4. Then, have them describe the arrangement of the particles in each microstate. Guide them to realize that when the particles are interchangeable, the number of possible states decreases.

Step 3: Students should enter the number of microstates that correspond to each of the macrostates in the table. For example, because there is only one possible microstate that corresponds to the macrostate of having both particles on the right, students should enter "1" in row 3, column C. After students have entered the correct number of microstates in column C, they should answer questions 3 and 4.

Q3. Which macrostate is most probable?

A. *The second macrostate, with one gas particle on each side, is the most probable. If students have difficulty recognizing this, remind them that the probability that a given macrostate will occur is equal to the number of microstates corresponding to that macrostate divided by the total possible number of microstates.*

Q4. How many more times more likely is it to have one particle on each side than to have two particles on the left side of the box?

A. *two times more likely*

Step 4: Next, students should read page 1.10 before moving to page 1.11, which shows a similar two-part box containing four particles. The students should move the particles to produce the most likely arrangement of particles in the box. Encourage student discussion of the most probable arrangement. Also, encourage students to discuss how the interchangeability of the particles makes many of the microstates equivalent.
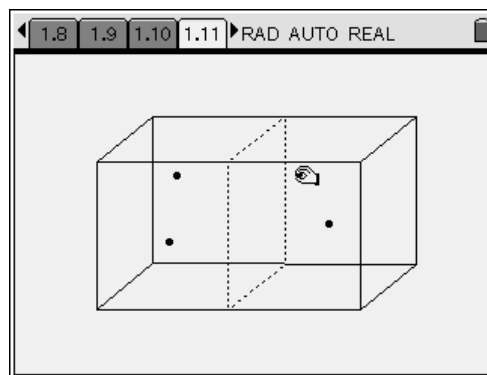
Step 5: After manipulating the four particles on page 1.11, students should read page 1.12 and then move to page 1.13, which contains a *Lists & Spreadsheet* application similar to the one on page 1.8. Students should enter the number of microstates corresponding to each macrostate in the spreadsheet. Then, they should answer questions 5 and 6.

Q5. Which macrostate is most probable?

A. *The third macrostate, with two gas particles on each side, is most probable.*

Q6. How many more times more likely is it to have two particles on each side than to have four particles on the left side of the box?

A. *six times more likely*

Step 6: Next, students should read page 1.15. They should then move to page 1.16 and complete the spreadsheet for a situation involving 12 particles. Students will need to use the equation relating the number of microstates to the number of particles on the left- and right-hand sides to calculate the number of microstates for each macrostate. This equation is given below:

$$W = \frac{N!}{(N_{\text{right}}!)(N_{\text{left}}!)}$$

Students should enter this equation into the formula bar (gray box) in column C. The factorial sign (!) is found in the **Symbols** menu (ctrl ⌨). After completing the spreadsheet, students should answer questions 7–9.

Q7. Which macrostate is most probable?

> **A.** *The seventh macrostate, with six gas particles on each side, is the most probable.*

Q8. How many more times more likely is it to have 6 particles on each side than to have all 12 particles on the left side of the box?

> **A.** *924 times more likely*

Q9. Make a general statement about the most likely macrostate for a two-section box containing *n* particles, where *n* is an even number.

> **A.** *The most likely macrostate is one in which there are $\frac{n}{2}$ particles on each side of the box.*

Part 2 – Exponential growth in number of microstates in the most probable macrostate

Step 1: Next, students should read page 1.19 and examine the data on page 1.20. Page 1.20 is a *Lists & Spreadsheet* application containing data on the number of microstates in the most probable macrostate (**wnum**) for systems containing different numbers of particles (**num**). Encourage students to hypothesize about the relationship between **wnum** and **num**.

Step 2: Next, students should plot **wnum** vs. **num** on the *Data & Statistics* application on page 1.21. They should attempt to fit various regression lines to the data, using the **Regression** tool (**Menu > Actions > Regression**). Encourage students to predict which regression line will produce the best fit, and then test their predictions. After they have found the best-fit line, students should answer questions 10–11.
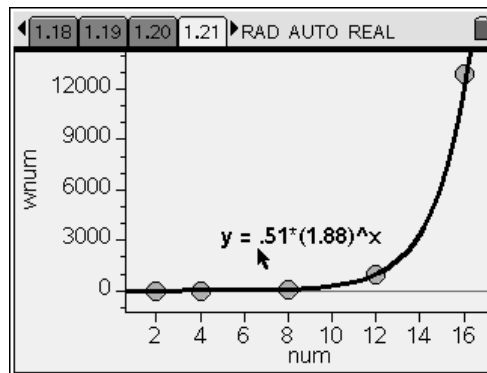
Q10. What type of relationship is there between **wnum** and **num**?

> **A.** *The relationship is closest to an exponential curve.*

Q11. Write the equation relating the number of particles in the system to the number of microstates in the most probable macrostate.

> **A.** $\textbf{wnum} = \textbf{(0.51)(1.88)}^{\textbf{num}}$

Part 3 – Entropy and the second law of thermodynamics

Step 1: Next, students should read page 1.23, which gives a formal definition of entropy. They should then answer questions 12–15.

Q12. What is the entropy of the macrostate with all the particles on the left side of the box?

A. *Zero; if students have trouble grasping this concept, remind them that there is only one microstate corresponding to this macrostate, and that the natural logarithm of 1 is 0.*

Q13. Is it possible for all the particles to spontaneously move to the same side of the box?

A. *Yes.*

Q14. Is it likely that all the particles would spontaneously move to the same side of the box?

A. *No.*

Q15. Which macrostate corresponds to the state with the highest entropy?

A. *The most probable macrostate has the greatest entropy.*

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(Student)TI-Nspire File: *PhysWeek03_Entropy.tns*

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL
ENTROPY
Physics
Thermodynamics

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL
Ludwig Boltzmann was one of the most important scientists of the late 19th century. On Boltzmann's tombstone is the equation $S = k \ln W$. In this activity, you will learn about S and W and discover why this equation is worthy of placement on a tombstone.

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL
On the next page is a box containing two particles. One of the particles is in the left side of the box, and the other is on the right. The particles are constantly moving. They collide with the walls of the container and each other. Therefore, the location of a particle at any point in the box is random.

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL ctrl

1.2 1.3 1.4 1.5 ▶RAD AUTO REAL
1. In terms of the two sides of the box, how many different possible arrangements are there for the two particles? What are they?
2. Which microstate is most probable?

1.3 1.4 1.5 1.6 ▶RAD AUTO REAL
In a macrostate, the identity of individual particles is unknown. Therefore, there are three possible macrostates for this system, as described below:
1) two particles on the left
2) one particle on each side
3) two particles on the right

1.4 1.5 1.6 1.7 ▶RAD AUTO REAL
The spreadsheet on the next page identifies the macrostates in terms of the number of particles on each side. In the third column, enter the number of microstates (**w2**) possible for each macrostate.

1.5 1.6 1.7 1.8 ▶RAD AUTO REAL
A n2left · B n2right · C w2 · D · E
1	2	0
2	1	1
3	0	2
A1	

1.6 1.7 1.8 1.9 ▶RAD AUTO REAL
3. Which macrostate is most probable?
4. How many more times more likely is it to have one particle on each side than to have two particles on the left side of the box?

1.7 1.8 1.9 1.10 ▶RAD AUTO REAL
Next, consider the case of four particles. Move the particles on the next page to produce the most likely arrangement of four particles. This should be one of the microstates of the most probable macrostate.

1.8 1.9 1.10 1.11 ▶RAD AUTO REAL ctrl

1.9 1.10 1.11 1.12 ▶RAD AUTO REAL
Enter into the third column on the next page the number of microstates for each macrostate of four particles. (Note that the total number of microstates should be $2^4 = 16$.)

Screen 1 (1.10 1.11 1.12 1.13 RAD AUTO REAL)

	A n4left	B n4right	C w4	D	E
1	4	0			
2	3	1			
3	2	2			
4	1	3			
5	0	4			

A1 |

Screen 2 (1.11 1.12 1.13 1.14 RAD AUTO REAL)

5. Which macrostate is most probable?

6. How many more times more likely is it to have two particles on each side than to have four particles on the left side of the box?

Screen 3 (1.12 1.13 1.14 1.15 RAD AUTO REAL)

Enter into the next spreadsheet the number of microstates for each macrostate of 12 particles. Use the following equation for calculating number of microstates:

$$W = \frac{N!}{(Nleft!)(Nright!)}$$

In this equation, **N** is the total number of particles (**N = Nleft + Nright**).

Screen 4 (1.13 1.14 1.15 1.16 RAD AUTO REAL)

	A n12left	B n12right	C w12	D
1	12	0		
2	11	1		
3	10	2		
4	9	3		
5	8	4		

A1 |

Screen 5 (1.14 1.15 1.16 1.17 RAD AUTO REAL)

7. Which macrostate is most probable?

8. How many more times more likely is it to have six particle on each side than to have all twelve particles on the left side of the box?

Screen 6 (1.15 1.16 1.17 1.18 RAD AUTO REAL)

9. Make a general statement about the most likely macrostate for a two-section box containing n particles, where n is an even number.

Screen 7 (1.16 1.17 1.18 1.19 RAD AUTO REAL)

The next page shows the relationship between the number of particles in the system (**num**) and the number of microstates in the most probable macrostate (**wnum**). On page 1.21, plot the number of microstates in the most probable macrostate vs. the number of particles in the system. Then, fit a regression line to the data.

Screen 8 (1.17 1.18 1.19 1.20 RAD AUTO REAL)

	A num	B wnum	C	D	E	F
1	2	2				
2	4	6				
3	8	70				
4	12	924				
5	16	12870				

A1 |

Screen 9 (1.18 1.19 1.20 1.21 RAD AUTO REAL ctrl)

Click to add variable
Click to add variable

Screen 10 (1.19 1.20 1.21 1.22 RAD AUTO REAL)

10. What type of relationship is there between **wnum** and **num**?

11. Write the equation relating the number of particles in the system to the number of microstates in the most probable macrostate.

Screen 11 (1.20 1.21 1.22 1.23 RAD AUTO REAL)

The entropy, S, of a given macrostate is determined by the number of microstates in the macrostate. This is represented by the equation $S = k \ln W$, where k is the Boltzmann constant, 1.381×10^{-23} J/K, and W is the number of microstates in the macrostate.

Screen 12 (1.21 1.22 1.23 1.24 RAD AUTO REAL)

12. What is the entropy of the macrostate with all the particles on the left side of the box?

13. Is it possible for all the particles to spontaneously move to the same side of the box?

◀ 1.22 1.23 1.24 1.25 RAD AUTO REAL

14. Is it likely that all the particles would spontaneously move to the same side of the box?

15. Which macrostate corresponds to the state with the highest entropy?

◀ 1.23 1.24 1.25 1.26 RAD AUTO REAL

As the number of air particles in a box increases, it becomes more and more unlikely for all the particles to be on one side of the box. Closed systems tend to reach maximum entropy because maximum entropy corresponds to the highest probability macrostate. This is the basis for the second law of thermodynamics.