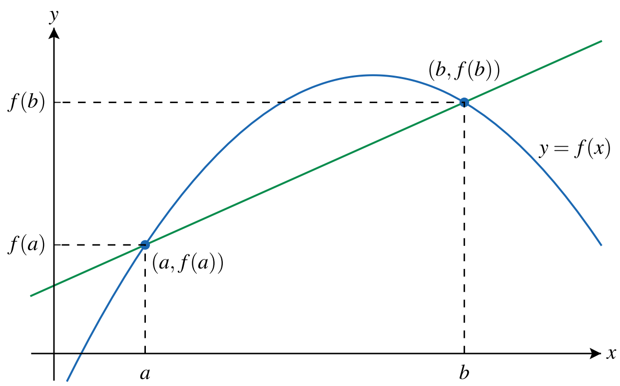
Thursday Night PreCalculus, November 30, 2023

Difference Quotients and Average Rates of Change

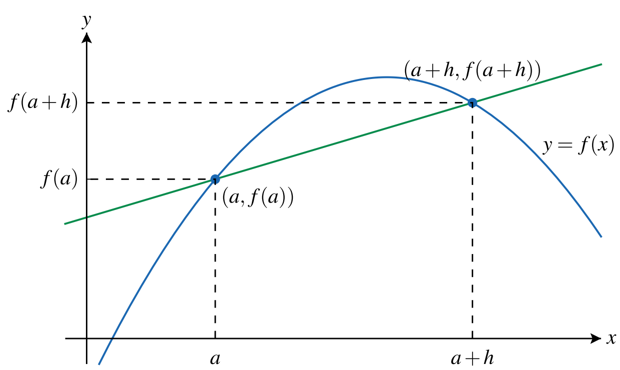
A Few Geometric Interpretations

Average rate of change: the change in y divided by the change in x.

$$\text{Ave Rate of Change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$



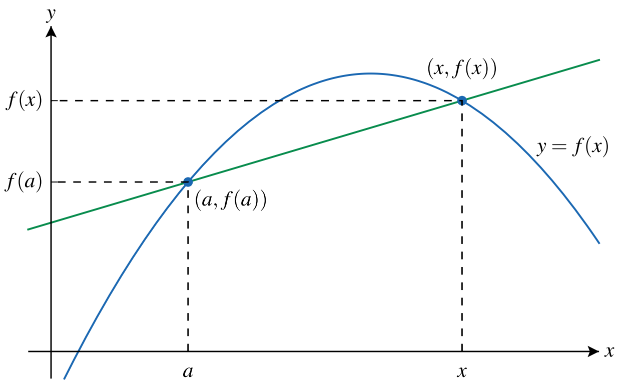
Difference Quotient: A measure of the average rate of change of a function over an interval of length h.



$$\text{DQ} = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}$$

$$\text{DQ} = \frac{f(x+h) - f(x)}{h}$$

Another perspective:



$$\text{DQ} = \frac{f(x) - f(a)}{x - a}$$

Problems

1. For each of the following functions, simplify the expression

$$\frac{f(x+h)-f(x)}{h}, \qquad h \neq 0$$

as far as possible.

(a) $f(x) = 3x^2 - 5x$

(b) $f(x) = \sqrt{x^2 - 1}$

(c) $f(x) = \dfrac{1}{x^2}$

(d) $f(x) = \dfrac{x}{1 + x^2}$

2. The number of pounds (in millions) of lobster caught by Maine commercial fisherman is given by $P(t)$, where t is measured in years. Selected values for $P(t)$ are given in the table.

t	2010	2012	2014	2016	2018	2020	2022
$P(t)$	255.8	318.3	306.9	302.9	285.8	205.1	107.1

Find the average rate of change in pounds of lobsters caught **(i)** from 2016 to 2018; **(ii)** from 2018 to 2020.

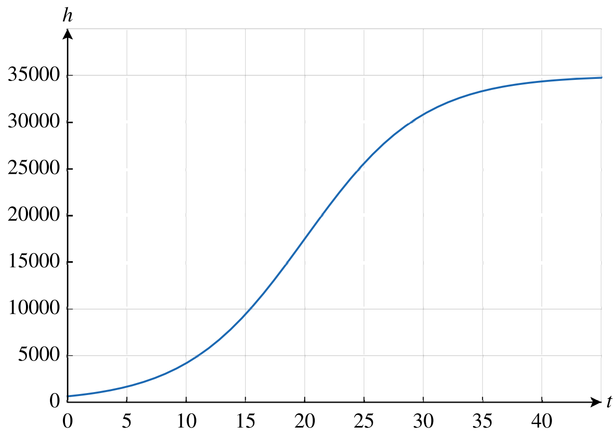
Indicate the unites of measure. What does your answers suggest about the change in the number of pounds of lobster caught in recent years?

3. A particle moves along a horizontal number line. Its position at time $t \geq 0$ is given by $s(t) = t^2 - 7t + 2$ where t is measured in seconds and s is measured in meters.

(a) Find the average rate of change in the particle's position from $t = 0$ to $t = 8$ seconds.

(b) Use your answer in part (a) to determine if the particle is to the left or the right of its initial position at time $t = 8$.

4. The figure shows the graph of the altitude of a plane (h, in feet) from takeoff, $t = 0$ minutes, to $t = 45$ minutes.



Use the graph to determine on which five-minute interval, 0-5, 5-10, 10-15, etc, the average rate of change in height is greatest.

5. For each of the following functions, simplify the expression

$$\frac{f(x+h) - f(x-h)}{2h}$$

(a) $f(x) = 2x + 5$

(b) $f(x) = x^2 + 3x + 4$