Application of the Unit Circle

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TEACHER NOTES

Math Objectives

- Understand the relationship between the unit circle and the sine and cosine functions.
- Use the unit circle and its relationship to the sine and cosine functions to solve a real-world application.

Vocabulary

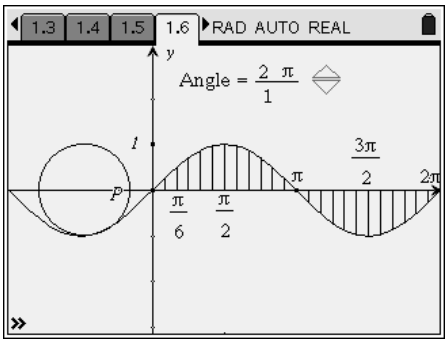
- unit circle
- sine function
- cosine function

About the Lesson

- This lesson is a follow-up lesson to the *Unit Circle* activity.
- This lesson involves students using the unit circle and its relationship to the sine and cosine functions to solve a real-world application.

Related Lessons

- Prior to this lesson: Unit Circle
- After this lesson: Trigonometric Transformations



TI-Nspire™ Technology Skills:

- Download TI-Nspire document
- Open a document
- Move between pages
- Measure the length of a segment

Tech Tips:

- *Make sure the **font size** on your TI-Nspire handhelds is set to Medium.*

Lesson Materials:

Student Activity

Application_of_the_Unit_Circle_Student.PDF

Application_of_the_Unit_Circle_Student.DOC

TI-Nspire document

Application_of_the_Unit_Circle.tns

Discussion Points and Possible Answers:

1. How many teeth will be machined around the cogwheel?

Since a circle has 360 degrees and the teeth are at intervals of 15 degrees, divide 360 by 15. There will be 24 teeth on the cogwheel.

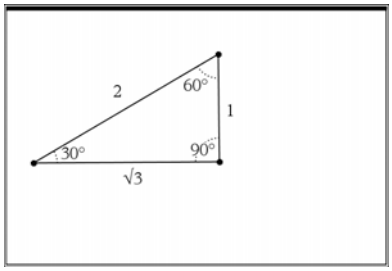
2. The angle measures on page 1.6 are given in radians. Determine all the degree measures of the angles where the teeth will be positioned around the cogwheel. Record the angle measures in degrees and radians in the table below.

Angle Measure (in Degrees)	Angle Measure (in Radians)	*x*-Coordinate	*y*-Coordinate
15	$\pi / 12$	0.966	0.259
30	$\pi / 6$	0.866	0.5
45	$\pi / 4$	0.707	0.707
60	$\pi / 3$	0.5	0.866
75	$5\pi / 12$	0.259	0.966
90	$\pi / 2$	0	1
105	$7\pi / 12$	–0.259	0.966
120	$2\pi / 3$	–0.5	0.866
135	$3\pi / 4$	–0.707	0.707
150	$5\pi / 6$	–0.866	0.5
165	$11\pi / 12$	–0.966	0.259
180	π	–1	1
195	$13\pi / 12$	–0.966	–0.259
210	$7\pi / 6$	–0.866	–0.5
225	$5\pi / 4$	–0.707	–0.707
240	$4\pi / 3$	–0.5	–0.866
255	$17\pi / 12$	–0.259	–0.966
270	$3\pi / 2$	0	–1
285	$19\pi / 12$	0.259	–0.966
300	$5\pi / 3$	0.5	–0.866
315	$7\pi / 4$	0.707	–0.707
330	$11\pi / 6$	0.866	–0.5
345	$23\pi / 12$	0.966	–0.259
360	2π	1	0

Teacher Tip: *Students may need to be reminded that to convert angle degree measure to radian measure, multiply the degree measure by* $\frac{\pi}{180}$.

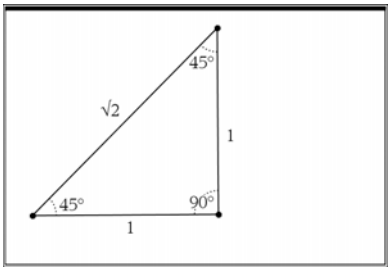
3. The diagram on page 1.6 shows the relationship between the unit circle and the sine function. How can the sine function help determine the ordered pairs of point *P* for each angle?	**The sine function can be used to represent the *y*-coordinate of point *P* for each angle.** ***Teacher Tip:*** *Students should notice that the highlighted leg in the triangles created in the unit circle when pressing the arrow buttons is vertical.*
4. On the graph on page 1.6, use the **Coordinates and Equations** tool (**MENU > Actions > Coordinates and Equations**) to find the *y*-coordinates for each ordered pair for point *P*. Record your findings (to three decimal places) in the table for Question 2.	**See table from Question 2.**
5. The diagram on page 2.2 shows the relationship between the unit circle and the cosine function. How can the cosine function help determine the ordered pairs of point *P* for each angle?	**The cosine function can be used to represent the *x*-coordinate of point *P* for each angle.** ***Teacher Tip:*** *Students should notice that the highlighted leg in the triangles created in the unit circle when pressing the arrow buttons is horizontal.*
6. On the graph on page 2.2, use the **Coordinates and Equations** tool to find the *x*-coordinates for each ordered pair for point *P*. Record your findings (to three decimal places) in the table for Question 2.	**See table from Question 2.**
7. Now that you have the measurements, you input them into the cogwheel machine. However, you notice the measurements are not quite right. The teeth are not exactly evenly spaced around the wheel. Why would that be?	**Some of the answers are not exact values, because the answers are rounded to three decimal places.**

8. Use your knowledge of a 30°-60°-90° triangle to check your results for the 30° angle. Show your work in the space below.



Using the special triangle and the sine ratio of opposite leg/hypotenuse, the sine of 30 degrees is $\frac{1}{2}$, or 0.5. The cosine ratio is adjacent leg/hypotenuse; therefore, the cosine of 30 degrees is $\frac{\sqrt{3}}{2}$, which is approximately 0.866.

9. Use your knowledge of a 45°-45°-90° triangle to check your results for the 45° angle. Show your work in the space below.



Using the special triangle and the sine ratio of opposite leg/hypotenuse, the sine of 45 degrees is $\frac{1}{\sqrt{2}}$, or $\frac{\sqrt{2}}{2}$, which is approximately 0.707. The cosine ratio is $\frac{\text{adjacent leg}}{\text{hypotenuse}}$; therefore, the cosine of 45 degrees is the same.

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students are able to:

- Understand the relationship between the unit circle and the sine and cosine functions.
- Understand how the unit circle and its relationship to the sine and cosine functions can be used to solve a real-world application.