

Overlapping Squares

Teacher

7 8 9 10 **11** 12



Introduction

This document and the associated TI-Nspire file is designed for the classroom teacher. The TI-Nspire document can be used as a demonstration tool by using the TI-Nspire software and a data projector to display the interactive geometric object to students. The purpose is to provide a dynamic visual for a ‘simple’ geometry problem and to engage students in a discussion about ‘proof’. The first visual for the solution is not accompanied by any level of proof but is extraordinarily simple, the second visual is more complicated by provides access to a much easier proof.

Observation

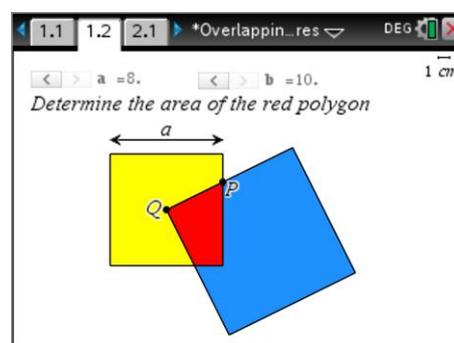
Open the TI-Nspire document: “Overlapping Squares” and display the file for students to see. Make sure page 2.1 is not visible on the page sorter. (Display the keypad or tools menu on the side.)

Navigate to page 1.2, grab point P and drag it around the perimeter of the yellow square.

Pose the problem to students:

“Determine an expression for the area for the red polygon.”

The size of the yellow square and blue square can be set using the corresponding sliders.



Solutions

Navigate to page 2.1.

The ‘simplest’ solution is to extend the sides of the blue square and simply show that the yellow square is divided into four equal polygons, therefore the red area is $\frac{1}{4}$ the area of the yellow square.

Discussion

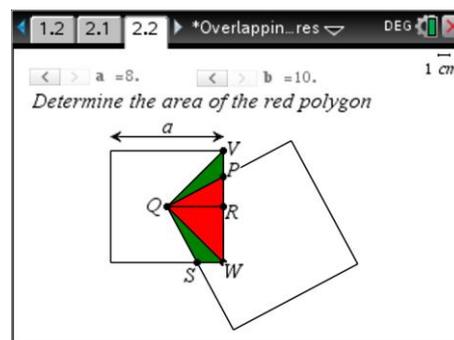
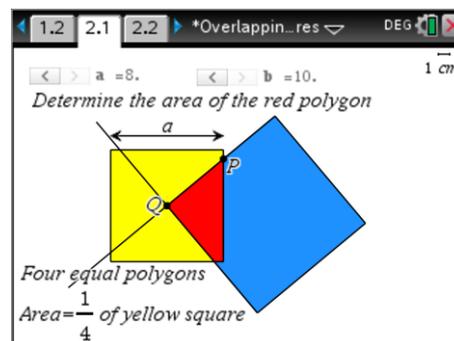
- Can we just accept that the four polygons will be equal in area?
- What conditions do we need to meet to show that two quadrilaterals (polygons) are congruent?

Navigate to page 2.2

This is an alternative approach to that shown on page 2.1. Note that part of the original red polygon now has a green triangle QSW overlaid.

- The two green triangles QPV and QSW are congruent. How do we prove that these triangles are congruent?

ASA



Use the reflection tool to show students that the two triangles ‘appear’ to be congruent.

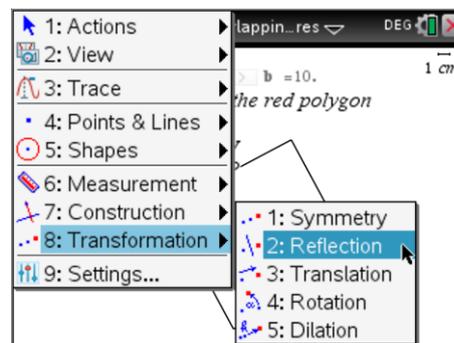
From the Menu: Transformation > Reflection

Place the mouse over the ‘area’ of QSW and the “Triangle QSW” will be displayed. Next, select side QW as the mirror by clicking on this line.

A reflected triangle Q’S’W’ will be displayed.

Now reflect Q’S’W’ on the line QR. Triangle Q”S”W” will appear over the top of triangle QPV.

This double reflection helps students match sides and angles in preparation for their proof by ASA that the two triangles are congruent, but also assists in the overall proof that the area of the ‘red’ polygon is constant.



Discussion

The purpose of this activity is to introduce students to the idea of proof. The area of the red polygon on page 1.2 can be measured using some of the calculator tools, this provides us with an insight to the solution, but is not acceptable as ‘proof’.

Also worthy of discussion is the necessary condition that the red polygon area remains constant:

$$b > \frac{a}{\sqrt{2}}.$$

The first visual ‘solution’ provided illustrates that the area of the red polygon is constant provided we are willing to accept that the four polygons will always be the same? Is observation and intuition enough? Euclidean Geometry relied on an assumption about the angles bounded by a transversal and two parallel lines. This was an acceptable geometry for many centuries, however other geometries such as hyperbolic and elliptical geometries have since been constructed that do not accept this parallel postulate or ‘observation’ and serves as a constant reminder of the power of a valid proof.